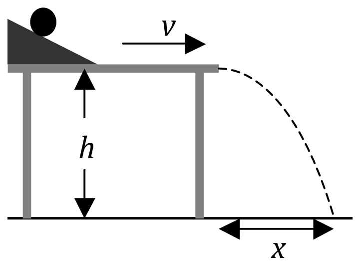
This test covers vectors using both polar coordinates and **i**-**j** notation, radial and tangential acceleration, and two-dimensional motion including projectiles.

Part I. Multiple Choice

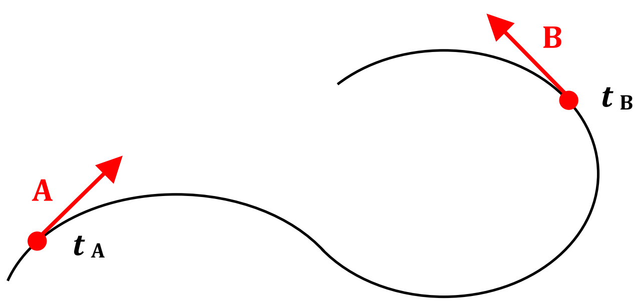
1.



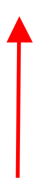
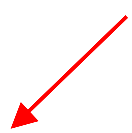
In a lab experiment, a ball is rolled down a ramp so that it leaves the edge of the table with a horizontal velocity *v*. If the table has a height *h* above the ground, how far away from the edge of the table, a distance *x*, does the ball land? You may neglect air friction in this problem.

a. $\frac{2v^2}{g}$
b. $v\sqrt{\frac{2h}{g}}$
c. $\frac{2vh}{g}$
d. $\frac{2h}{g}$
e. none of these

2.



An object travels along a path shown above, with changing velocity as indicated by vectors **A** and **B**. Which vector best represents the net acceleration of the object from time t_A to t_B?

a. 
b. 
c. 
d. 
e. 

3. A circus cannon fires an acrobat into the air at an angle of 45° above the horizontal, and the acrobat reaches a maximum height *y* above her original launch height. The cannon is now aimed so that it fires straight up into the air at an angle of 90° to the horizontal. What is the maximum height reached by the same acrobat now?

a. y
b. $\frac{y}{2}$
c. $2y$
d. $y\sqrt{2}$
e. $\frac{2y}{\sqrt{2}}$

4. A particle moves along the *x*-axis with an acceleration of $a = 18t$, where *a* has units of m/s^2. If the particle at time $t = 0$ is at the origin with a velocity of -12 m/s, what is its position at $t = 4.0$s?

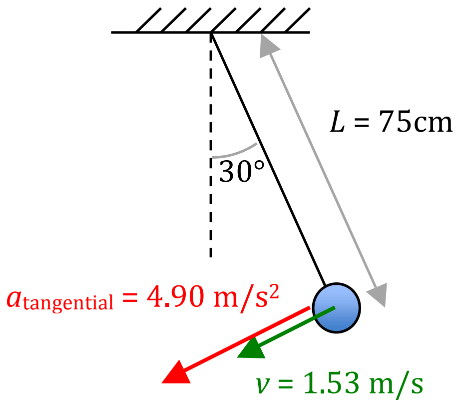
a. 12m
b. 72m
c. 60m
d. 144m
e. 196m

5. Consider a ball thrown up from the surface of the earth into the air at an angle of 30° above the horizontal. Air friction is negligible. Just *after* the ball is released, its acceleration is:

a. Upwards at 9.8 m/s^2
b. Upwards at 4.9 m/s^2
c. Downwards at 9.8 m/s^2
d. 0 m/s^2
e. None of these

Part II. Free Response

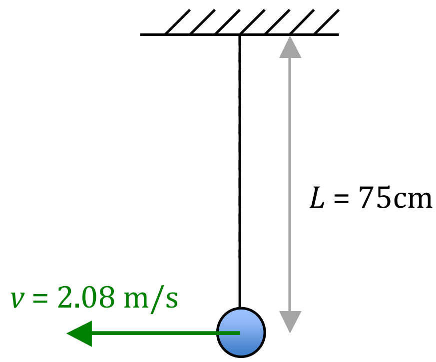
6.



A mass is attached to the end of a 75.0-centimeter long light string which is then attached to the ceiling, and allowed to swing back and forth as a simple pendulum. When the pendulum makes an angle of 30.0° with the vertical, it has a tangential acceleration of 4.90 m/s^2 and a tangential velocity of 1.53 m/s, as shown.

a. Calculate the *radial* acceleration of the mass at this position.

b. Calculate the *net* acceleration of the mass at this position.



At the bottom of its swing, the ball is traveling 2.08 m/s when the string suddenly breaks.

c. What is the radial acceleration of the ball just *before* the string snaps?

d. What is the acceleration of the ball just *after* the string breaks?

e. If the ball is 1.75 meters above the floor when the string breaks, how far away horizontally does the ball land from where it began to fall?

7. A cannonball with an initial height of 1.00 meter above the ground is fired from a cannon at an angle of 50° above the horizontal. The cannonball travels a horizontal distance of 60.0 meters to a 10.0 meter high castle wall, and passes over 2.00 meters above the highest point of the wall. (Assume air friction is negligible.)

a. Determine how much time it takes for the ball to reach the wall.

b. Determine the initial speed of the cannonball as it is fired.

c. Determine the velocity (magnitude and direction) of the cannonball as it passes over the castle wall.

d. If the same cannonball is fired with the same initial speed at an angle of 55.5° above the horizontal, determine whether or not the ball will still clear the castle wall.

8. A large cat, running at a constant velocity of 5.0 m/s in the positive-x direction, runs past a small dog that is initially at rest. Just as the cat passes the dog, the dog begins accelerating at 0.5 m/s^2 in the positive-x direction.

a. How much time passes before the dog catches up to the cat?

b. How far has the dog traveled at this point?

c. How fast is the dog traveling at this point?

In a different situation, the cat passes the dog as before, traveling in the positive-x direction at 5.0 m/s. Now, as the cat passes, the dog begins accelerating at 0.5 m/s^2 in the positive-y direction.

d. What is the cat's acceleration relative to the dog?

e. What is the cat's velocity relative to the dog at time t = 5.0 seconds after the dog begins running?

f. What is the cat's position relative to the dog at time t = 5.0 seconds after the dog begins running?

1. The correct answer is ***b***. The ball takes a time *t* to fall from the table, as determined here:

$$\Delta y = v_0 t + \tfrac{1}{2}at^2$$

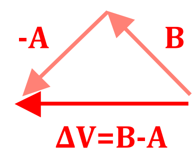
$$t = \sqrt{\frac{2\Delta y}{-g}} = \sqrt{\frac{2h}{g}}$$

Horizontally, during that time the ball travels at constant velocity:

$$\Delta x = vt$$

$$x = v\sqrt{\frac{2h}{g}}$$

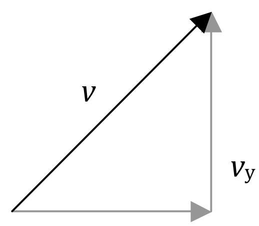
2. The correct answer is *d*. The direction of acceleration is the same as the direction of the change in velocity, according to $a = \frac{v_f - v_i}{t}$. Because $\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$, we can determine Δv graphically by adding $\mathbf{v}_f$ to the negative of $\mathbf{v}_i$, or B+(-A). Placing the B vector "tip-to-tail" with the –A vector gives a direction for $\Delta\mathbf{v}$ (and therefore, **a**) to the left.



3. The correct answer is ***c***. The acrobat reaches her height in the first instance based on the initial vertical component of velocity, v_y:

$$v_f^{\,2} = v_i^{\,2} - 2ay$$

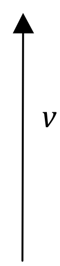
$$y = \frac{0 - v_i^{\,2}}{-2g} = \frac{v_i^{\,2}}{2g}$$



For the second situation, the vertical velocity *v* is greater than v_y from before, by a factor of $\sqrt{2}$. Using this information:

$$y' = \frac{(v_i')^2}{2g}$$

$$y' = \frac{(v\sqrt{2})^2}{2g} = \frac{2v^2}{2g} = 2y$$



4. The correct answer is *d*. The particle's displacement as a function of time can be determined by analyzing the antiderivative of the acceleration and velocity:

$$v = \int a\ \ dt, \text{ and } x = \int v\ \ dt$$

To get the velocity as a function of time:

$v = \int 18t\ \ dt = 9t^2 + C = 9t^2 + -12$, where we've given *C* the value -12, which represents the velocity of the particle at time *t* = 0.

Continuing:

$$x = \int 9t^2 + -12\ \ dt = 3t^3 - 12t + C = 3t^3 - 12t$$

In this case, *C* = 0 because the location of the particle at time *t* = 0 was the origin.

Now, solve with t = 4.0s:

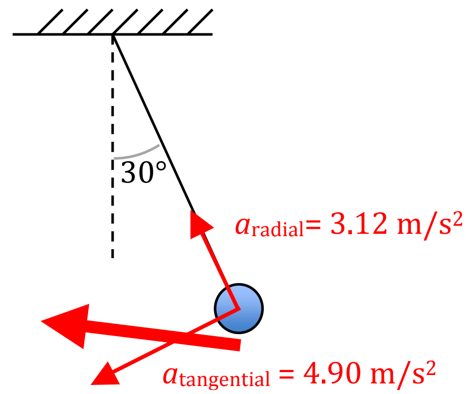
$$x = 3t^3 - 12t = 3(4)^3 - 12t = 144m$$

5. The correct answer is *c*. The ball, even as it moves upwards and sideways through the air, experiences a force of gravity acting on it, which causes it to accelerate downwards at **g**.

6.

a. Radial acceleration is calculated as follows:

$$a_c = \frac{v^2}{r} = \frac{(1.53m/s)^2}{(0.75m)} = 3.12m/s^2$$



b. The net acceleration of the ball is determined by combining the radial and tangential accelerations, which are at right angles to each other. The magnitude of this acceleration is:

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{3.12^2 + 4.90^2} = 5.81m/s^2$$

The direction of that net acceleration at this particular position (relative to 0° east) is:

$$\theta\ (relative\ \ to\ \ radial) = \tan^{-1}\left(\frac{a_{tangential}}{a_{radial}}\right)$$

$$\theta = \tan^{-1}\left(\frac{4.90}{3.12}\right) = 57.5° + 90 + 30 = 177.5°$$

One might also give the net acceleration as the sum of tangential and radial unit vectors:

$$a_{net} = (-4.90\hat{\theta} - 3.12\hat{\mathbf{r}})\ \ m/s^2$$

where the tangential component is negative because the acceleration is in the counterclockwise direction, and the radial component is negative because it is directed toward the center of the circular path.

c. The radial acceleration of the ball is calculated just as in (a) above: $a_c = \frac{v^2}{r} = \frac{2.08^2}{0.75} = 5.77m/s^2$

d. Just after the string snaps, there is nothing to keep the ball moving in a circle. It is now in "free-fall," with an acceleration of 9.80 m/s² downward.

e. The ball is behaving as a projectile, with an initial horizontal velocity of 2.08 m/s and initial vertical velocity of 0. Apply a projectile analysis to determine how far away horizontally the ball lands:

$$\text{Vertically:}\quad \Delta y = v_i t + \tfrac{1}{2}at^2$$

$$-1.75 = 0t - 4.9t^2$$

$$t = 0.598s$$

$$\text{Horizontally:}\quad \Delta x = vt$$

$$\Delta x = (2.08)(0.598) = 1.24m$$

7.

a. The time it takes for the ball to reach the specified height above the wall can be determined by analyzing the ball's vertical motion:

$$y_i = 1.0m;\ y_f = 12.0m;\ a = -9.8m/s^2;\ t = ?;\ v_i = v\sin 50$$

$$\Delta y = v_i t + \tfrac{1}{2}at^2$$

$$11 = v\sin 50t - 4.9t^2$$

We have a single equation with two unknowns—v_i and t—so we turn to the horizontal motion to get a second equation with the same two unknowns:

$$x = 60m;\ v_x = v\cos 50;\ t = ?$$

$$v_x = \frac{x}{t}$$

$$v\cos 50 = \frac{60}{t},\ so\ \ v = \frac{60}{\cos 50t}$$

Substituting the second equation into the first, we get:

$$11 = \left(\frac{60}{t\cos 50}\right)t\sin 50 - 4.9t^2$$

$$11 - 60\tan 50 = -4.9t^2$$

$$t = \sqrt{12.3} = 3.51s$$

b. Now that we known how much time elapsed, substitute back in one of the initial equations in (a) above to determine the initial speed

$$v = \frac{60}{\cos 50t} = \frac{60}{3.51(\cos 50)} = 26.6m/s$$

c. As the cannonball passes over the castle wall, its velocity will be the combination of horizontal and vertical velocities:

$$v_x = \frac{x}{t} = \frac{60.0}{3.51} = 17.1m/s$$

$$v_{y-final} = v_{y-initial} + at = 20.8\sin 50 - 9.8(3.51) = -14.0m/s$$

$$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{17.1^2 + (14.0)^2} = 22.1m/s$$

$$\theta = \tan^{-1}\left(\frac{-14.0}{17.1}\right) = -39.3° \text{ (ie. 39.3° below the horizontal)}$$

d. Performing an analysis similar to the one above, the Δy for the cannon ball is 9.71m. Because the ball began its motion from a height of 1.00m, its height when it reaches the wall is 10.7m, which is enough to clear it.

$$v_x = 26.6\cos 55.5 = 15.1m/s;\ v_y = 26.6\sin 55.5 = 21.9m/s$$

$$t = \frac{x}{v_x} = \frac{60}{15.1} = 3.97s$$

$$y_f - y_i = v_{y-initial}t + \tfrac{1}{2}at^2$$

$$y_f - 1.00 = (21.9)(3.97) - 4.9(3.97)^2$$

$$y_f = 10.7m$$

8.

a. The cat's position as a function of time is given by $\Delta x_{cat} = vt = (5.0m/s)t$. The dog's position as a function of time is given by $\Delta x_{dog} = v_i t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(0.5m/s^2)t^2$. The dog will have caught up with the cat when the displacement of both of them (Δx) is equal. Therefore:

$$5t = 0.25t^2$$

This is a quadratic equation that has two roots: 0 and 20. The 0 seconds refers to the initial time when the dog and cat are next to each other. The 20 seconds is our chosen solution—it takes 20 seconds for the dog to catch up to the cat.

b. Use the answer from (a) to determine Δx: $\Delta x_{cat} = vt = (5.0m/s)(20s) = 100m$

c. Use kinematics to determine the dog's velocity at this point:

$$v_f = v_i + at$$

$$v_f = 0 + (0.5)(20) = 10m/s$$

d. The cat's acceleration relative to the dog can be determined with vector analysis, and using the cat's and dog's acceleration relative to the ground as a reference point.

$$a_{cat-ground} = 0\mathbf{i}$$

$$a_{dog-ground} = 0.5\mathbf{j}$$

$$a_{cat-dog} = 0\mathbf{i} - 0.5\mathbf{j} = (-0.5\mathbf{j})m/s^2$$

The interpretation of this is that the cat is accelerating downward ($-y$ direction) relative to the dog, but not accelerating in the $+x$ direction relative to the dog.

e. One way to solve this is using **i-j** notation:

$$v_{cat-dog} = v_{cat-dog-initial} + a_{cat-dog}t$$

$$v_{cat-dog} = (5\mathbf{i} + 0\mathbf{j}) + (0\mathbf{i} - 0.5\mathbf{j})(5)$$

$$v_{cat-dog} = (5\mathbf{i} - 2.5\mathbf{j}) \ \ m/s$$

You could also get the dog's velocity relative to the ground at 5.0 seconds (2.5 m/s in the y-direction) and the cat's velocity relative to the ground (0.5 m/s in the x-direction) and do a vector analysis similar to (d) above.

f. The position of the cat relative to the dog can again be determined using **i-j** notation:

$$\Delta r = v_i t + \frac{1}{2}at^2$$

$$\Delta r = (5\mathbf{i} + 0\mathbf{j})(5) + \frac{1}{2}(0\mathbf{i} - 0.5\mathbf{j})(5)^2$$

$$\Delta r = (25\mathbf{i} - 6.25\mathbf{j}) = (25\mathbf{i} - 6.25\mathbf{j})\,m$$