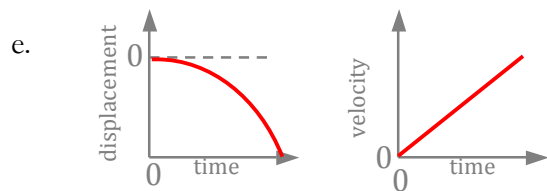
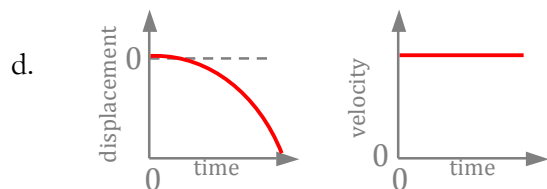
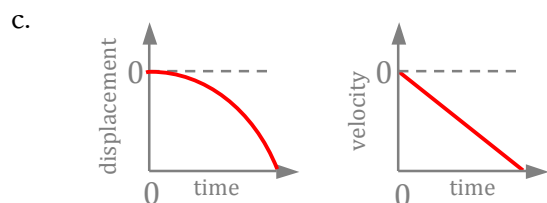
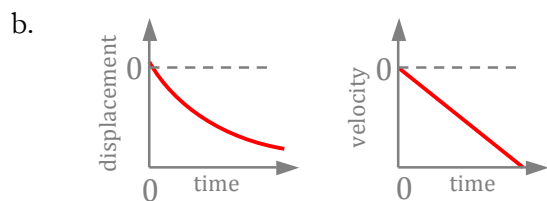
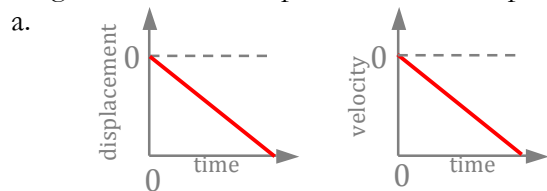


This test covers one-dimensional kinematics, including speed, velocity, acceleration, motion graphs, with some problems requiring a knowledge of basic calculus.

Part I. Multiple Choice

1. A rock is released from rest from the top of a very high cliff, and accelerates downward at g . Approximately how far does the rock travel in the first 7 seconds of its free-fall? (Assume no air friction.)
 - a. 35 m
 - b. 70 m
 - c. 180 m
 - d. 245 m
 - e. 490 m
2. A projectile is fired horizontally from a height of 20 meters above the ground, with an initial velocity of 7.0 m/s. How far does the projectile travel horizontally before it reaches the ground?
 - a. 7m
 - b. 14m
 - c. 140m
 - d. 3.5m
 - e. 20m
3. An object is launched into the air at an angle less than 90° above the surface of the earth. Air resistance is negligible. At the highest point in its path of motion, which of the following statements is true?
 - a. It has no vertical velocity and no vertical acceleration.
 - b. It has no horizontal velocity and no vertical acceleration.
 - c. It has vertical velocity and no horizontal acceleration.
 - d. It has horizontal velocity and no vertical acceleration.
 - e. It has no vertical velocity and no horizontal acceleration.
4. A particle begins from rest at a point +10 meters from the origin at time $t = 0$, and begins accelerating at a constant 2 m/s^2 in the negative direction. At time $t = 4$ seconds, the particle has reached a certain speed; it stops accelerating, and continues traveling with that same speed until $t = 7$ seconds. What is its position relative to the origin at $t = 7$ seconds?
 - a. -6 meters
 - b. -30 meters
 - c. -8 meters
 - d. -40 meters
 - e. -59 meters
5. An object moving along the x -axis has its position given by the equation $x = 2.0t^2 - 3.0t + 4$, with x in meters and t in seconds. What is the acceleration of the object at time $t = 4.0\text{s}$?
 - a. 24 m/s^2
 - b. 46 m/s^2
 - c. 13 m/s^2
 - d. 16 m/s^2
 - e. 4.0 m/s^2

6. A mass is dropped from a height h above the ground, and freely falls under the influence of gravity. Which graphs here correctly describe the displacement and velocity of the object during the time the object is falling? Consider the "up" direction to be positive.

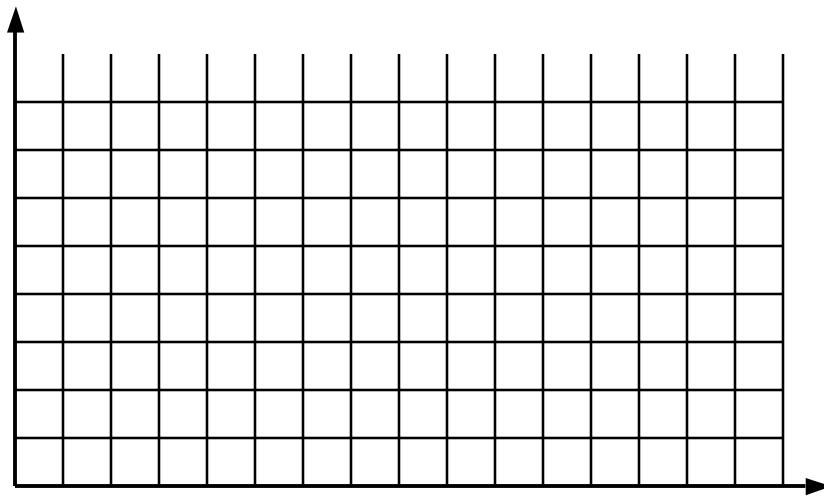


Part II. Free Response

7. A 50-gram superball is thrown horizontally in the negative- x direction against a brick wall so that it bounces directly back after hitting the wall. (The vertical motion of the ball during the throw is negligible.) The horizontal position of the ball's center of mass as a function of time is described in the data table below.

x -position relative to wall (centimeters)	25	20	15	10	6	3	2	3	6	10	14	18	22
t (milliseconds)	0	2	4	6	8	10	12	14	16	18	20	22	24

- a. Draw a graph of the ball's position as a function of time.



- b. Determine the ball's initial velocity before coming into contact with the wall?
- c. Determine the ball's final velocity after bouncing off the wall?
- d. What is the ball's average acceleration during the time period when the ball's velocity is changing, from 6 to 16 ms?

- c. Create appropriate data table(s) with clearly-labeled headers that you would use to record the data from your experiment.
- d. Clearly describe how you will analyze the data you have collected for each part of the experiment (ramp and horizontal surface).

1. The correct answer is *d*. The rock is accelerating constantly at 10 m/s^2 , so its displacement can be calculated using simple kinematics:

$$\Delta y = v_i t + \frac{1}{2} a t^2$$

$$\Delta y = 0 + \frac{1}{2}(-10 \text{ m/s}^2)(7 \text{ s})^2$$

$$\Delta y = 245 \text{ m}$$

It is arguably easier to calculate this quickly by determining the average velocity during the seven seconds of falling— 0 m/s to 70 m/s , the average velocity is 35 m/s —and multiplying this value by the total time of 7 seconds: $7 \times 35 = 245 \text{ m}$.

2. The correct answer is *b*. We begin by finding how much time it takes the object to fall the 20m:

$$\Delta y = v_i t + \frac{1}{2} a t^2$$

$$-20 \text{ m} = 0 t + \frac{1}{2}(-10) t^2$$

$$t = \sqrt{4} = 2 \text{ s}$$

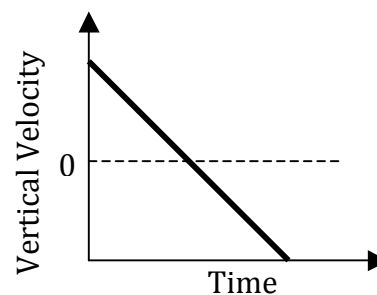
Then, determine how far the ball travels horizontally during that time:

$$\Delta x = v_x t$$

$$\Delta x = (7 \text{ m/s})(2 \text{ s}) = 14 \text{ m}$$

3. The correct answer is *e*. The object flying through the air, at the highest point in its motion (the apex), has no vertical velocity at that instant—it only has horizontal velocity. The force of gravity, acting vertically, is causing it to accelerate vertically, at $\mathbf{g}=9.80 \text{ m/s}^2$ in the downward direction, even though at that particular instant it has no vertical velocity. There is no horizontal force causing it to accelerate in that direction.

It may help to understand how an object can have no vertical velocity, even as it's accelerating, by considering the vertical velocity-time graph for an object launched into the air. One can see that the slope of the graph is constant, representing the constant acceleration the object experiences due to the constant force of gravity. The velocity curve has to cross from positive to negative velocity at some point, and it is at that moment in time that the instantaneous velocity of the object is 0.



4. The correct answer is *b*. We can determine the displacement of the particle relative to the origin by examining its motion in two separate steps.

From $t = 0$ to 4 seconds (particle accelerating):

$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$

$$x_f = +10 \text{ m} + 0 + \frac{1}{2}(-2 \text{ m/s}^2)(4 \text{ s})^2 = -6 \text{ m}$$

To get the displacement for the next part, we need to know how fast the particle is traveling after the 4 seconds have passed:

$$v_f = v_i + a t$$

$$v_f = 0 + (-2 \text{ m/s}^2)(4 \text{ s}) = -8 \text{ m/s}$$

From $t = 4$ to 7 seconds (particle at constant velocity):

$$\Delta x = vt$$

$$x_f = x_i + vt$$

$$x_f = -6m + (-8m/s)(3s)$$

$$x_f = -30m$$

5. The correct answer is *e*. An object's acceleration is given by the second derivative of its position function:

$$x = 2.0t^2 - 3.0t + 4$$

$$v = \frac{dx}{dt} = \frac{d}{dt}(2.0t^2 - 3.0t + 4) = 4.0t - 3.0$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(4.0t - 3.0) = 4.0$$

6. The correct answer is *c*. The object begins to fall from a height *b* in the negative direction, accelerating as it falls, so it's covering a greater and greater distance per unit time. This is consistent with the displacement graphs a, c, d, and e. The object's speed increases with time, but its velocity is in the downward, or negative direction, as indicated in the velocity-time graph for answer *c*.

7.

a. See graph at right.

b. Calculate velocity according to

$$v = \frac{\Delta x}{\Delta t}$$

Choosing values from the data table:

$$v = \frac{x_f - x_i}{t_f - t_i}$$

$$v = \frac{20 - 25}{2 - 0} = \frac{-5}{2}$$

$$v = -2.5 \text{ cm/ms}$$

$$v = -25 \text{ m/s}$$

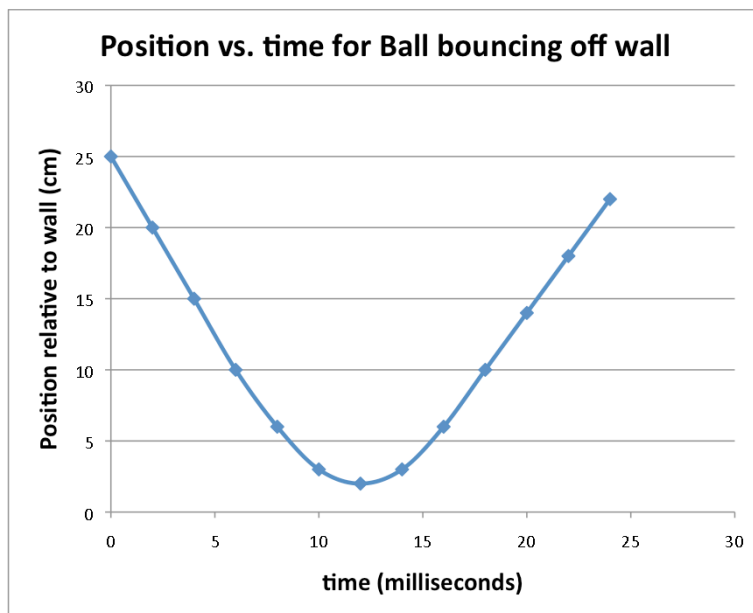
c. Again, choosing data values after the ball has changed direction:

$$v = \frac{x_f - x_i}{t_f - t_i}$$

$$v = \frac{22 - 18}{24 - 22} = \frac{4}{2}$$

$$v = +2 \text{ cm/ms}$$

$$v = +20 \text{ m/s}$$



- d. Using the formula for acceleration:

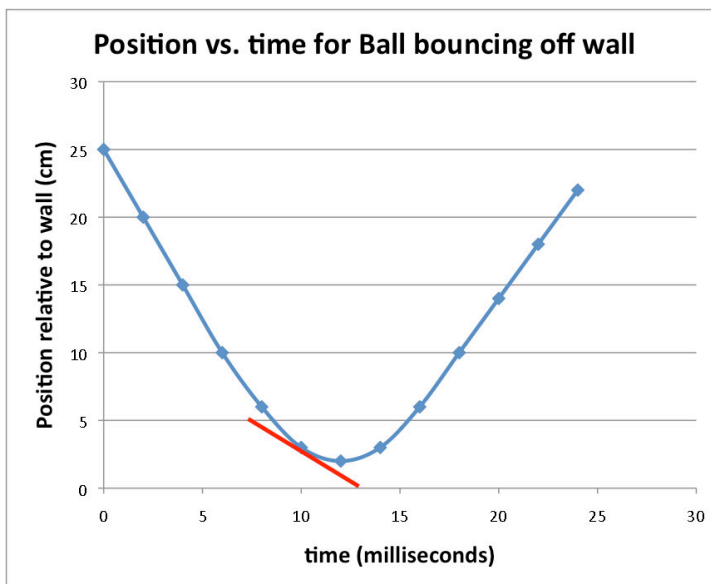
$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

$$a = \frac{20 - (-25)}{16 - 6} = \frac{45}{10} = 4.5 \text{ cm/ms}^2 = 4.5 \text{ e4 m/s}^2$$

- e. To determine the instantaneous velocity at time $t = 10$ seconds, it is not sufficient simply to use the data points on either side of $t = 10$; the velocity is in the process of changing in a way that may not be accurately reflected by averaging those two points. Ideally one would have a function that one could use to solve for v at $t = 10$ ms. In this case, the problem suggests we use the graph, so your answer should read something like this: "By using the tangent to the curve on a position-time graph, we can determine the instantaneous velocity at a point in time. Here:

$$v_{inst} = \frac{x_f - x_i}{t_f - t_i} = \frac{0 - 5}{13 - 7}$$

$$v_{inst} = -0.83 \text{ cm/ms} = -8.3 \text{ m/s}$$



8.

- a. Rocket accelerates from rest at 12.0 m/s^2 for a displacement of 1000 m . To find v_{final} :

$$v_i = 0; \Delta x = 1000 \text{ m}; a = 12.0 \text{ m/s}^2; v_f = ?$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$v_f = \sqrt{0^2 + (2)(12 \text{ m/s}^2)(1000 \text{ m})} = 155 \text{ m/s}$$

- b. Rocket travels additional distance upwards without the benefit of thrust, so its acceleration is only that due to gravity:

$$v_i = 155 \text{ m/s}; a = -9.8 \text{ m/s}^2; v_f = 0; \Delta x = ?$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$\Delta x = \frac{v_f^2 - v_i^2}{2a} = \frac{0^2 - 155^2}{(2)(-9.8)} = 1225 \text{ m}$$

So total height above the ground is the original $1000 \text{ m} + 1225 \text{ m} = 2225 \text{ m}$.

- c. Free-fall from height of 2225 m :

$$v_i = 0 \text{ m/s}; a = -9.8 \text{ m/s}^2; \Delta x = -2225 \text{ m}; v_f = ?$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$v_f = \sqrt{0^2 + 2(-9.8)(-2225)} = \pm 209 \text{ m/s}$$

The math gives us either a positive or negative value. Based on our knowledge that the rocket is traveling downward just before it hits, we'll select -209 m/s as the correct answer.

- d. Because the acceleration has changed during the course of this problem, we'll have to do at least two different calculations to get the time for those different conditions. You could also do it in three steps, but two steps is faster.

For the 12.0m/s^2 time:

$$v_f = v_i + at$$

$$t = \frac{v_f - v_i}{a} = \frac{155\text{m/s} - 0}{12.0\text{m/s}^2} = 12.9\text{s}$$

For the rest of the flight, going up and coming down:

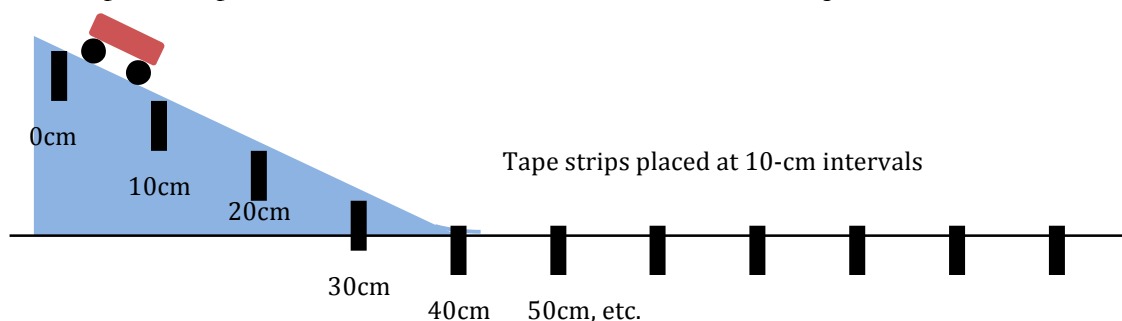
$$t = \frac{v_f - v_i}{a} = \frac{-209\text{m/s} - 155\text{m/s}}{-9.8\text{m/s}^2} = 37.1\text{s}$$

Total time in the air is 50.0 seconds.

9. This question is based on a lab that you may or may not have actually performed in your class. Regardless, you would be expected to determine what experimental procedure would be appropriate for this task.

Acceptable answers may vary somewhat, but the ones given below are typical, and appropriate.

- Materials/equipment that could be used to measure the motion of the car include a meter stick (for measuring the position of the car), masking tape (for marking various positions on the ramp and the floor), a stopwatch (for recording the time-elapsed at different positions).
- The experimental procedure might be:
 - Use the meter stick to measure 10-centimeter intervals down the ramp and along the floor, and mark these positions with masking tape.
 - Release the car from the top of the ramp at the same time that the stopwatch is started. Stop the stopwatch when the car reaches the first 10-cm tape. Repeat this for several trials and record elapsed time.
 - Repeat this process for the 20-cm distance, and for all subsequent distances.



- c. Data tables:

Position (m)	Elapsed time (s) Trial #1	Elapsed time (s) Trial #2	Elapsed time (s) Trial #3	Average time (s)
0				
0.10				
0.20				
0.30				

d. Analysis

By graphing the *position vs. time* for the car, we should be able to clearly identify where the car's velocity is increasing (based on a concave upward curve for that part of the graph) and where the car has a constant velocity (where the slope of the position-time graph is constant, and represents the velocity of the car at that point).

By using the final velocity of the car at the bottom of the ramp and the time it takes for the car to get there, we can calculate the acceleration of the car using $a = \frac{v_f - v_i}{t}$. Or, we could do a regression on the data collected for the accelerating car and use $x = \frac{1}{2}at^2$ to identify the acceleration a of our regression.