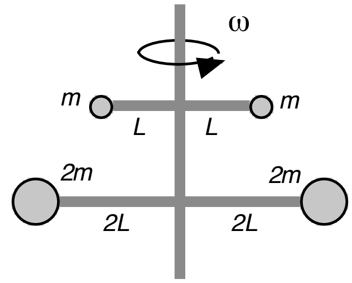
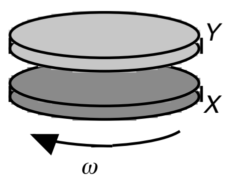
AP Physics
Rotation of a Rigid Body about a Fixed Axis; Rolling Motion, Angular Momentum, and Torque

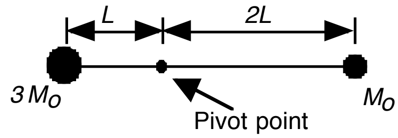
Part I. Multiple Choice (5 points each)
Choose the one best answer to each of the following problems.



1 (AP).
The rigid body shown in the diagram above consists of a vertical support post and two horizontal crossbars with spheres attached. The masses of the spheres and the lengths of the crossbars are indicated in the diagram. the body rotates about a vertical axis along the support post with constant angular speed ω. If the masses of the support post and the crossbars are negligible, what is the ratio of the angular momentum of the two upper spheres to that of the two lower spheres?
a) 2/1 b) 1/1 c) 1/2 d) 1/4 e) 1/8



2. (AP).
A disk X rotates freely with angular velocity w on a frictionless bearings, as shown above. A second identical disk Y, initially not rotating, is placed on X so that both disks rotate together without slipping. When the disks are rotating together, which of the following is half what it was before?
a) Moment of inertia of X
b) Moment of inertia of Y
c) Angular velocity of X
d) Angular velocity of Y
e) Angular momentum of both disks



3 (AP).
A light rigid rod with masses attached to its ends can pivot about a horizontal axis as shown above. When released from rest in a horizontal orientation, the rod begins to rotate with an angular acceleration of magnitude
a) 1/7 (g/L)
b) 1/5 (g/L)
c) 1/4 (g/L)
d) 5/7 (g/L)
e) g/L

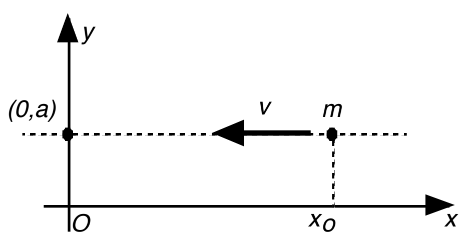
4 (AP). A particle is moving in a circle of radius 2 meters according to the relation $\theta = 3t^2 + 2t$, where θ

is measured in radians and t in seconds. The speed of the particle at $t = 4$ seconds is
a) 13 m/s b) 16 m/s c) 26 m/s d) 52 m/s e) 338 m/s

5 (AP). An ice skater is spinning about a vertical axis with arms fully extended. If the arms are pulled in closer to the body, in which of the following ways are the angular momentum and kinetic energy of the skater affected?

	Angular Momentum	Kinetic Energy
a)	Increases	Increases
b)	Increases	Remains Constant
c)	Remains Constant	Increases
d)	Remains Constant	Remains Constant
e)	Decreases	Remains Constant

6 (AP). A turntable that is initially at rest is set in motion with a constant angular acceleration α. What is the angular velocity of the turntable after it has made one complete revolution?
a. $\sqrt{(2\alpha)}$ b. $\sqrt{(2\pi\alpha)}$ c. $\sqrt{(4\pi\alpha)}$ d. 2α e. $4\pi\alpha$

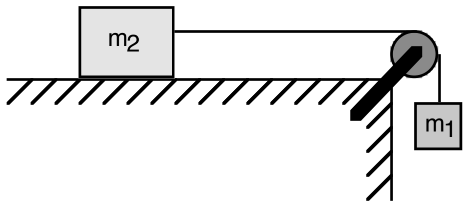


7.

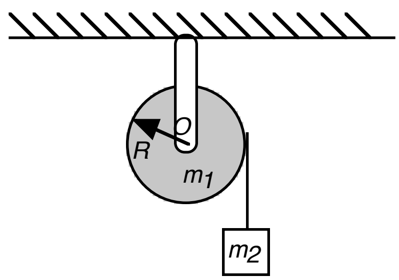
A particle of mass m moves with a constant speed v along the dashed line $y = a$. When the x-coordinate of the particle is x_o, the magnitude of the angular momentum of the particle with respect to the origin of the system is
a) zero b) mva c) mvx_o d) $\sqrt{(mv\ (x_o^2 + a^2\ )\ )}$^ e) $\sqrt{(mva/(x_o^2 + a^2\ ))}$

8. A particle whose mass is 2 kg moves in the xy plane with a constant speed of 3 m/s along the direction $\mathbf{r} = \mathbf{i} + \mathbf{j}$. What is its angular momentum (in kg m^2 /s) relative to the origin?
a) 0 k b) $6\sqrt{2}\ \mathbf{k}$ c) $-6\sqrt{2}\ \mathbf{k}$ d) $6\ \mathbf{k}$ e) $-6\ \mathbf{k}$

Part II. Free Response Questions (15 points each)
Answer each of the following problems in as much detail as possible, being sure to *show all work!*



9.
A mass (m_1 = 5.0 kg) is connected by a light cord to a mass (m_2 = 4.0 kg) which slides on a smooth surface, as shown in the figure above. The pulley (radius = 0.20 m) rotates about a frictionless axle. The acceleration of m_2 is 3.5 m/s^2. What is the moment of inertia of the pulley?



10 (AP).
A uniform solid cylinder of mass m_1 and radius R is mounted on frictionless bearings about a fixed axis through O. The moment of inertia of the cylinder about the axis is $I = 1/2\, m_1 R^2$. A block of mass m_2, suspended by a cord wrapped around the cylinder as shown above, is released at time $t = 0$.
a) On the diagram below, draw and identify all of the forces acting on the cylinder and on the block.

b) In terms of m_1, m_2, R, and g, determine each of the following.

i. The acceleration of the block

ii. The tension in the cord

iii. The angular momentum of the disk as a function of time t

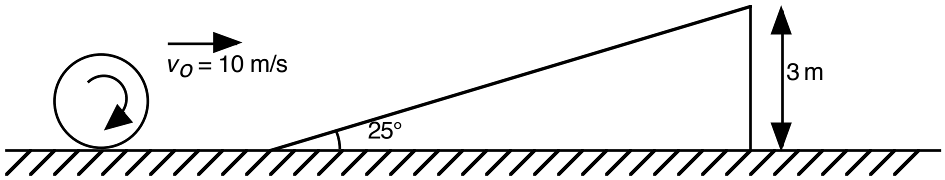


Diagram not drawn to scale

11 (AP).
A large sphere rolls without slipping across a horizontal surface. The sphere has a constant translational speed of 10 meters per second, a mass m of 25 kg, and a radius r of 0.2 meters. The moment of inertia of the sphere about its center of mass is $I = 2/5\ mr^2$. The sphere approaches a 25° incline of height 3 meters as shown above and rolls up the incline without slipping.

a) Calculate the total kinetic energy of the sphere as it rolls along the horizontal surface.

b) i. Calculate the magnitude of the sphere's velocity just as it leaves the top of the incline.

ii. Specify the direction of the sphere's velocity just as it leaves the top of the incline.

12. (Continuation of #11 above)

c) Neglecting air resistance, calculate the horizontal distance from the point where the sphere leaves the incline to the point where the sphere strikes the level surface.

d) Suppose, instead, that the sphere were to roll toward the incline as stated above, but the incline were frictionless. State whether the speed of the sphere just as it leaves the top of the incline would be less than, equal to, or greater than the speed calculated in 11(b) above. Explain *briefly*.

ANSWERS/SOLUTIONS:

1\. e $L_{top}=I\omega = 2ML^2\omega$, and $L_{bottom} = 2(2M(2L)^2)\omega = 16ML_2\omega$

2\. c $I_{disk}=1/2\,MR^2$, $I_{total}=1/2(2M)R^2$…

3\. a   Calculate $I=7M_oL^2$; calculate $\tau_{net} = M_olg$, and $\tau_{net} = I\alpha$, so $\alpha=g/(7l)$

4\. d $\omega = d\theta/dt = 6t+2$; $v=\omega r=52$ m/s

5\. c

6\. c $\omega^2=\omega_o^2+2\alpha\theta$, so $\omega = \sqrt{(4\pi\alpha)}$

7\. b $L=rmv = amv$

8\. a

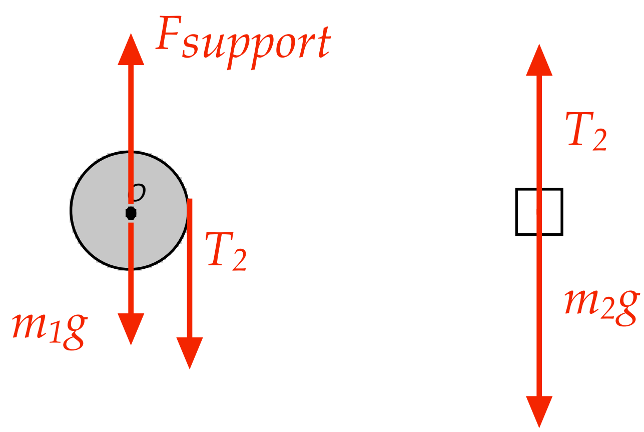
9\. For pulley: $\tau_{net} = I\alpha$, so $I=\tau_{net}/\alpha = \tau_{net}/a/r$

$\tau_{net} = \tau_1 - \tau_2 = r(m_1g-m_1a)-r(m_2a)$

Substitute into first equation to get $I=r^2(m_1g-m_1a-m_2a)/a$.

Sub in known values to get I=0.200 kg•m^2

10\. a.



b. i. Use $F_{net}=mg-T_2$; and $\tau_{net} = I\alpha$. Substitute and solve to get:

$a=(m_2g)/(m_2+1/2\,m_1) = (2m_2g)/(2m_2+m_1)$

ii. Substitute back into a previous equation to get:

$T=(m_1m_2g)/(m_1+2m_2)$

iii. $L=I\omega$, and $\omega=\omega_o+\alpha t$. Substitute to get

$L=(m_1m_2grt)/(2m_2+m_1)$

11\. a. $K_{total}=K_{translational}+K_{rotational}$. Substitute in known values to get

K_{total}=1750 J

b. i. $K_{translational}+K_{rotational}=K_{translational}'+K_{rotational}'+U$

7.62 m/s

ii. 25° above the horizontal (angle at which it leaves ramp)

12\. c. Lots of ways to solve: kinematics is probably the easiest. Solve it as a trajectory problem to get:

x=8.15 m

d. Sphere's speed is *less* at the top of the ramp in this frictionless situation. At bottom of ramp, K_{total} will be the same as before. As sphere goes up ramp, $K_{translational}$ is reduced, but $K_{rotational}$ remains constant because sphere keeps spinning. Increase in sphere's potential energy comes at the expense of kinetic energy, but since there is just as much $K_{rotational}$ at the top, the $K_{translational}$ has to be less than it was before.