

Ch 3 - Vectors



Welcome back...!

- No preLab, just bring printout of handout (download on Tues, typically)
- Engineering Computation Pads!
- Homework Solutions
<http://www.crashwhite.com/ap/materials/hwsolutions/chapter03.pdf>
polyuser, orange1907
- Class picture
- Tests graded (examine, questions, return)
- Chapter 3, in two periods!

Tests

- Rubric used to grade tests (consistent, fair?)
- I keep tests. You're free to come in to examine them at any time, but because AP problems are used on them from year to year, tests stay with me.
- *Drawings are important!* Your primary goal in taking a physics test is *not* to get the correct answer; you're actually trying to clearly communicate how well you understand physics, and drawings/sketches/diagrams/pictures are an important part of that.
- Equations w/variables are important! *Then* develop vertically.
- "CP" = "correct procedure": most or all of points awarded despite incorrect final answer
- "SF" = "significant figures": you've indicated a more-or-less blatant disregard for this concept

Your tests.... Questions?

Vectors

Vectors have two characteristics: magnitude, and direction; and we've talked about three kinds of vector quantities so far:

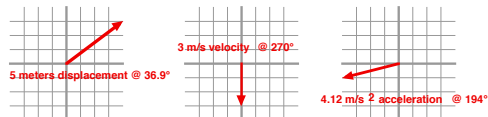
displacement, \mathbf{x}
velocity, \mathbf{v}
acceleration, \mathbf{a}

Up till now, we've considered very simple straight line motion, restricting ourselves to one-dimensional horizontal motion (in the $+\mathbf{x}$ or $-\mathbf{x}$ direction), or vertical motion (in the $+\mathbf{y}$ or $-\mathbf{y}$ direction). We need to expand our abilities now into additional dimensions...

Vectors – Graphical Analysis

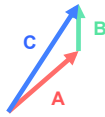
One way of analyzing 2-dimensional situations with vectors is by drawing the vector on paper, according to the following rules.

1. **length** of line drawn is proportional to *magnitude* of vector quantity
2. **direction** that arrow points (relative to an Cartesian coordinate system) indicates *direction* of vector quantity.



Vectors – Why?

It turns out that there are lots of things we can use vectors for: adding vectors, for example, allows us to determine the combined effects of two different motions.

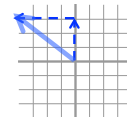


Adding two or more vectors graphically is accomplished by drawing the vectors such that the head (the arrow) of one vector is touching the tail of the next vector, in any order. The net, or resultant vector is found by drawing a line from the tail of the first vector to the head of the last vector.

$$\vec{A} + \vec{B} = \vec{C}$$

Example 1 – Adding Vectors

If I walk 6 blocks north from my house, and 8 blocks west, where do I end up relative to where I started?



This technique is called the “tip-to-tail” technique, or the “triangle” technique, of vector addition.

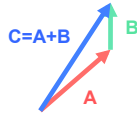
Measuring with a ruler & protractor, the displacement is:

In order to actually determine our final answer here, we’d need to use a ruler & protractor to measure the magnitude and direction of the final, resultant, displacement.

10 blocks, 140°

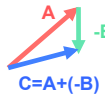
Vector Subtraction?

How would one go about graphically subtracting one vector from another?



$$\vec{A} - \vec{B} = \vec{C}$$

Hint: a negative vector has the same magnitude, but points in the opposite direction.

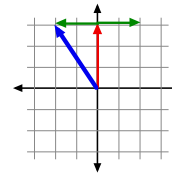


$$C = A + (-B)$$

Example 2 – Subtracting Vectors

A = 3 m/s North
B = 2 m/s East

What is A + -B (magnitude & direction)?

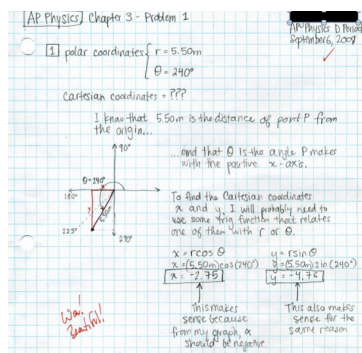


Measure the resultant vector to get your final answer, both magnitude (3-point-something m/s) and direction (~120° ?)

Welcome back...!

Homework Reminders:

- One problem per page
- Sketches, sketches, sketches
- Blurbs
- Start with formula from equation sheet...

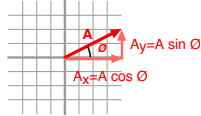


Other ways of adding vectors

Obviously, graphical addition of vectors is cool for sketches, but measuring with a ruler and protractor isn't very precise when it comes to numeric solutions. For that, we have two slightly more complex, but incredibly useful, systems: the polar-notation system, and the unit-vector system.

Example 3 – Trig Review

The vector **A** shown here is pointing at some angle θ measured relative to the x-axis.



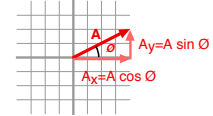
If we know A and θ , how could we calculate A_x and A_y ?

If we knew A_x and A_y , how could we calculate A 's magnitude and direction?

$$A = \sqrt{A_x^2 + A_y^2} \text{ and } \tan \phi = \frac{A_y}{A_x}$$

Polar Notation

This system of describing a vector **A** in terms of its magnitude A , and its polar angle θ is called *polar notation*. Angles are typically given relatively to East, or positive-x, and can be notated as follows:



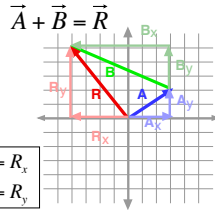
$$5 \text{ m/s @ } 36.9^\circ$$

$$5 \text{ m/s } \angle 36.9^\circ$$

Adding Vectors in PolarNot

Watch the process here:

The important thing to note here is the relationship between all of the **x**-components of the vectors, and all of the **y**-components of the vectors.



General Strategy:

$$\begin{aligned} \text{x - direction: } & A_x + B_x = R_x \\ \text{y - direction: } & A_y + B_y = R_y \end{aligned}$$

1. Sketch & label vectors
2. Find **x**- and **y**- components of all vectors
3. Add **x**-components together to get **x**-component of Resultant. Do the same thing with the **y**-components.
4. Use Pythagorean theorem to get Resultant's magnitude
5. Use trig to get Resultant's direction

Example 4 – Polar Notation

A tortoise crawls 10m SE, then 12m at 60° west of south.

1. Make a sketch of his journey.
2. Find components of each leg of trip
3. Calculate components of resultant
4. Calculate magnitude & direction of resultant

Solution: $A_x = 7.07\text{m}, A_y = -7.07\text{m}$
 $B_x = -10.4\text{m}, B_y = -6.00\text{m}$
 $R_x = A_x + B_x = -3.3\text{m}, R_y = A_y + B_y = -13.07\text{m}$
 Use Pythagoras to get $R = 13.5\text{m}$
 Use \tan^{-1} to get 255°

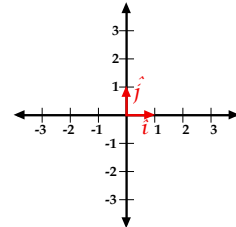
Adding 2-D Vectors

- Graphically
- Polar Notation
 - split vectors into components
 - add components separately
 - recombine component sums to get resultant magnitude and direction
- Unit Vector Notation

Unit Vectors

Yet another way exists to designate, and calculate with, vectors: *unit vectors*. A unit vector is simply a vector along the x- or y-axis that has a value of 1.

The unit vector in the x-direction is given the label **i**, while the y-direction unit vector is labelled **j** (sometimes with a carat ^ over the top of them). These unit vectors have a unit that matches whatever unit we're talking about (meters, m/s, m/s^2 , or whatever).



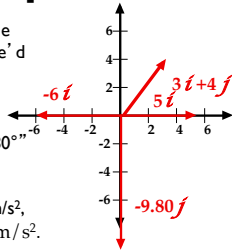
Unit Vector Examples

So, if we want to write the vector for the displacement "5 meters to the east," we'd write it: $(5i)$ m.

The vector for the velocity "6 m/s at 180° " would be... $(-6i)$ m/s.

The vector for the acceleration "9.80 m/s², down" would be... $(-9.80j)$ m/s².

And the vector for the displacement "5 m at 53.1° " is $(3i+4j)$ m.



Why you love unit vectors

The **very cool** thing about unit vectors is that we're essentially dealing with *components*: the x-component of the vector is given to you, and labeled with an *i*, while the y-component is there, and labeled with a *j*. So any vector addition doesn't require that the vector be split into components—it's already been given to you in components.

Example 5 – Another tortoise

A tortoise crawls $(7.07i + -7.07j)$ m, then $(-10.4i + -6.00j)$ m.

1. Make a sketch of his journey.
2. Determine his displacement.

Solution:

$$(7.07i + -7.07j) \text{ m} \\ + (-10.4i + -6.00j) \text{ m}$$

$$(-3.33i + 13.07j) \text{ m}$$

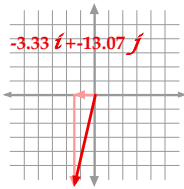
If you're lucky, you can leave it in that form and be done with it. Sometimes, you'll have to take those end vectors and convert them back into polar notation.

Converting from unit-vector to polar!

$$A^2 + B^2 = R^2$$

$$R = \sqrt{A^2 + B^2} = \sqrt{(-3.33)^2 + (-13.07)^2} = 13.50 \text{ m}$$

$$\tan^{-1}\left(\frac{13.07}{3.33}\right) = 75.7^\circ \dots + 180^\circ = 256^\circ$$



Ch 4 Preview

Consider a projectile—a Nerf dart?—that is shot at some angle above the horizontal.

- How can we describe the dart's motion in the *horizontal* direction?
- How can we describe the dart's motion in the *vertical* direction?
- How can we combine our knowledge of these two situations to determine where the dart lands?

Example 6 – Projectile Motion

A long jumper leaves the ground with a velocity of 11.0 m/s at 20.0° above the horizontal.

- a. What is the maximum height he reaches?
- b. How far horizontally does he jump? Solve using components.
- c. Then, once you know time, solve again using *i, j* notation.

