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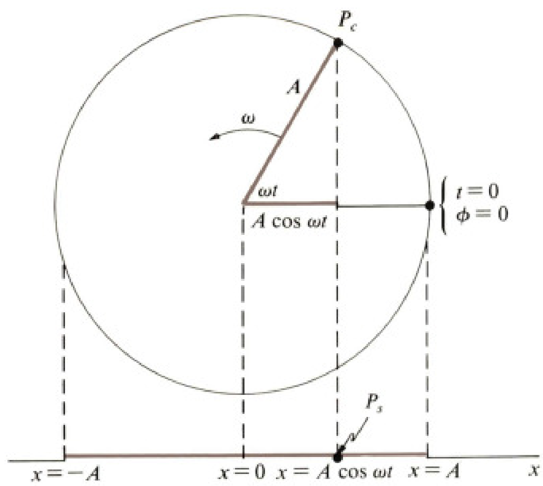
Chapter 15: Oscillation

Summary: The movement of certain objects is periodic, repeating over a period of time. In this section, we look at movements that oscillate back and forth, called simple harmonic motion.

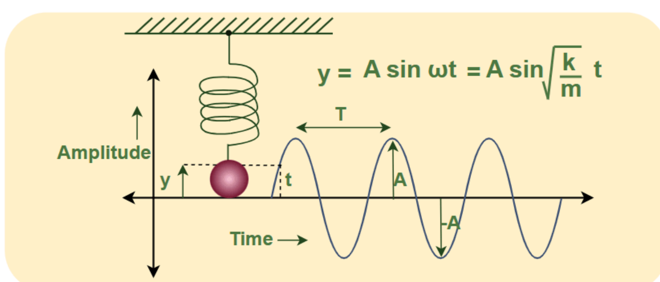
Key Points and Terms

- Consider a circle as a starting point for understanding simple harmonic motion (SHM)



  - A = radius of the circle or amplitude of our simple harmonic motion
  - T = period, time it takes for one complete circular revolution or cycle of SHM
  - ϕ = phi, otherwise known as the initial starting angular position, is the phase constant in SHM
  - ω = angular speed, or angular frequency in SHM

- To describe the x position of that point **P**, we use the equation: $x = Acos(\omega t + \phi)$
- **SHM =** a way to describe linear motion over time for an object in rotational motion



- *T (period)* = time for a single rotation or cycle for a particle moving back and forth in SHM, in SI units per second (per cycle)
  - $T = \frac{2\pi}{\omega}$
- *F (frequency)* = the inverse of the period; number of cycles a second

    - $f = \frac{1}{T} = \frac{\omega}{2\pi}$
- Velocity of a particle in SHM
    - $v = -\omega A sin(\omega t + \phi)$
- Acceleration of a particle in SHM
    - $a = -\omega^2 A cos(\omega t + \phi)$ OR $a = -\omega^2 x$
    - Acceleration of SHM is linearly proportional to the displacement x and in the opposite direction
- In a mass-spring system, the angular velocity ω is
    - $\omega = \sqrt{\frac{k}{m}}$ where k is the spring constant and m is the mass
    - Substituting into the SHM equations: $T = 2\pi\sqrt{\frac{m}{k}}$
    - $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
- **IMPORTANT:** For small angles of θ less than 15 degrees, simple pendulums = Simple Harmonic oscillators.
- In case of small angles, we can use a ω for the pendulum
    - $\omega = \sqrt{\frac{g}{L}}$
    - $T = 2\pi\sqrt{\frac{L}{g}}$
    - $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$
- Oscillation for a physical pendulum using torque analysis
    - $\omega = \sqrt{\frac{rmg}{I}}$

Key Equations:

- $x = A cos(\omega t + \phi)$
- $T = \frac{2\pi}{\omega}$ (general)
- $f = \frac{1}{T} = \frac{\omega}{2\pi}$ (general)
- $v = -\omega A sin(\omega t + \phi)$
- $a = -\omega^2 A cos(\omega t + \phi)$ OR $a = -\omega^2 x$
- $\omega = \sqrt{\frac{k}{m}}$ (spring)

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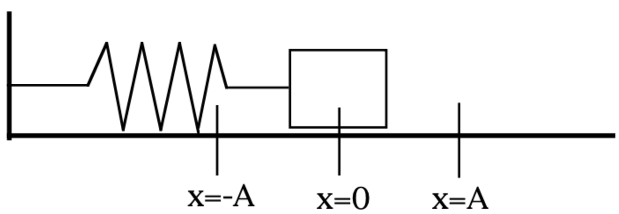
- $T = 2\pi\sqrt{\frac{m}{k}}$ (spring)
- $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (spring)
- $\omega = \sqrt{\frac{g}{L}}$ (massless pendulum)
- $T = 2\pi\sqrt{\frac{L}{g}}$ (massless pendulum)
- $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$ (massless)
- $\omega = \sqrt{\frac{rmg}{I}}$ (pendulum with rod)

1) A block on a horizontal frictionless plane is attached to a spring, as shown below. The block oscillates on the x-axis with simple harmonic motion of amplitude A. Describe the block's acceleration at x=A.



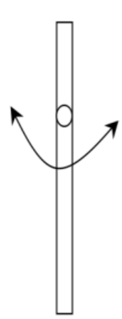
2) A particle of mass 50 grams is attached to a horizontal spring. It is pulled 3 cm from the equilibrium position and released from rest. It then moves in simple harmonic motion with a frequency of 10 oscillations per second.

(a) Write the equation for the position as a function of time for this particle.

(b) Calculate the spring constant of the spring.

(c) Calculate the maximum speed of the particle.

3) A 0.8 kg meter stick pivots back and forth about a point at its 30 cm mark as shown below. Its rotational inertia about its center is $\frac{ml^2}{12}$. What is the frequency of its motion?



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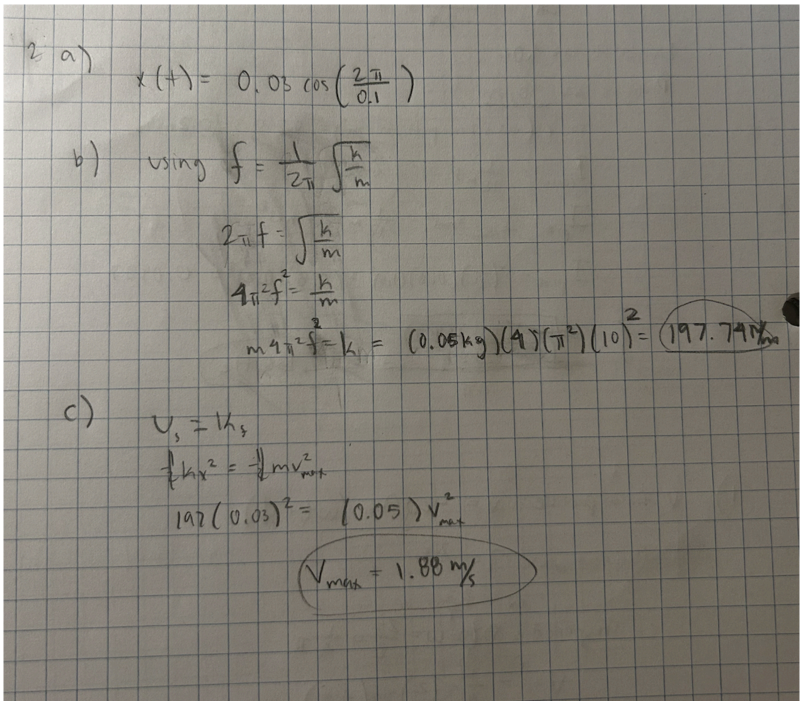
Answers

1) At x=A, the acceleration of the block is at its maximum. Using knowledge of Hooke's law (F=-kx), we can see that the force of the spring is at its greatest at x=A, and so will acceleration

2)



2 a) $x(t) = 0.03\cos\left(\frac{2\pi}{0.1}\right)$

b) using $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$2\pi f = \sqrt{\frac{k}{m}}$

$4\pi^2 f^2 = \frac{k}{m}$

$m4\pi^2 f^2 = k = (0.05kg)(4)(\pi^2)(10)^2 = 197.74 N/m$

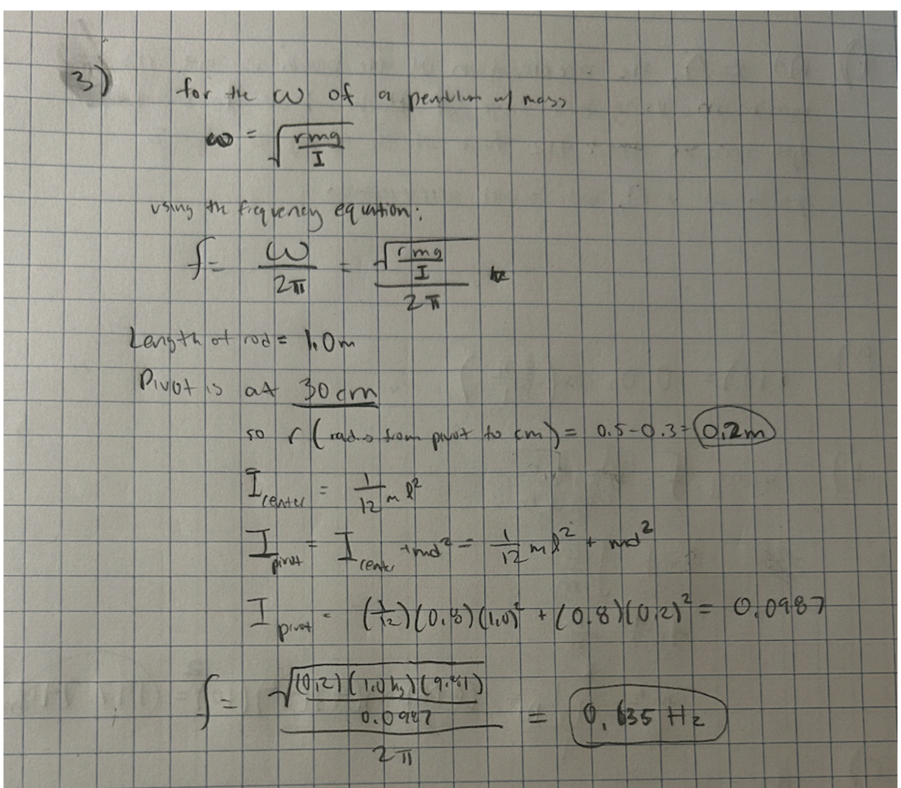
c) $U_s = K_s$

$\frac{1}{2}kx^2 = \frac{1}{2}mv_{max}^2$

$197(0.03)^2 = (0.05)v_{max}^2$

$V_{max} = 1.88 m/s$

3)



3) for the ω of a pendulum w/ mass

$\omega = \sqrt{\frac{rmg}{I}}$

using the frequency equation:

$f = \frac{\omega}{2\pi} = \frac{\sqrt{\frac{rmg}{I}}}{2\pi}$ hz

Length of rod = 1.0m

Pivot is at 30cm

so r (radius from pivot to cm) = 0.5-0.3 = 0.2m

$I_{center} = \frac{1}{12}ml^2$

$I_{pivot} = I_{center} + md^2 = \frac{1}{12}ml^2 + md^2$

$I_{pivot} = (\frac{1}{12})(0.8)(1.0)^2 + (0.8)(0.2)^2 = 0.0987$

$f = \frac{\sqrt{\frac{(0.2)(1.0kg)(9.81)}{0.0987}}}{2\pi} = 0.635 Hz$