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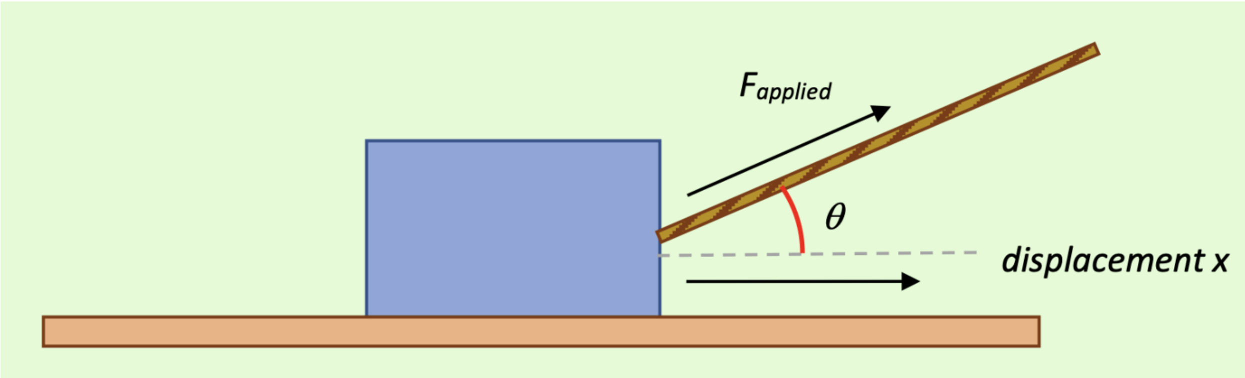
Unit 7: Energy and Energy Transfer

Summary: Since we've looked at force on an object, we can expand our thinking into force applied over a distance, which is *Work.* This unit covers *Work* and it's relevant theorems and further applications like *Power*, and a short stint of the behavior of springs.

Key Points/Topics

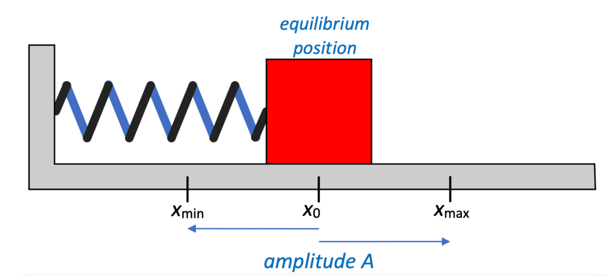
- ***Work*** = a force applied to an object over some displacement, remember it's a scalar quantity (no direction), use $W = Fxcos\theta$
    - *Positive work* = force increases energy of object
    - *Negative work* = force decreases energy of object



- ***Friction*** can do either positive or negative work, depending on the situation (i.e., a waiter carrying a serving tray with a drink on it vs. a box sliding on a rough surface)
- ***Dot Product*** = type of vector multiplication, to find work if given work and displacement in *i* and *j* notation
    - $C = \overline{A} \bullet \overline{B}$ (pretend those are vector arrows) or $C = (A_x \bullet B_x) + (A_y \bullet B_y)$
- If *force* on the object varies as distance changes (hint: they give you an equation for force instead of a value), use different strategies
    - 1st method: $dW = F \bullet dx \rightarrow W = \int_{x_i}^{x_f} F \bullet dx$
    - The equation above = total work on an object done by a force that varies with displacement

- 2nd method: if provided a graph of force over a distance(x), find the integral/total area under the curve to find *Work*

- ***Hooke's Law****:* shows behavior of springs where the force applied by the spring is linearly proportional and opposite to displacement
  - $F_{spring} = -kx$
  - The negative means that the Force of the spring is opposite the direction of displacement x
- ***Equilibrium position of spring system:*** x_0, where a mass attached to a spring experiences 0 net force, while x_{min} and x_{max} show areas with the highest force from the spring.



- ***Work Energy Theorem****: a* method relating energy using work
  - $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$, or $W = K_f - K_i$
  - Positive work inrcreases the kinetic energy of object, while negative decreases
- ***Power****:* how much Work done over time; the more quickly work is done, the more Power is needed to do Work
  - $P_{avg} = \frac{\Delta Work}{\Delta time}$
  - $P_{inst} = \frac{dW}{dt}$
  - $P_{inst} = F \bullet v$ (Force times velocity)

Important Equations:

- $W = Fx\cos\theta$
- $C = \vec{A} \cdot \vec{B}$ OR $C = (\vec{A}_x \cdot \vec{B}_x) + (\vec{A}_y \cdot \vec{B}_y)$

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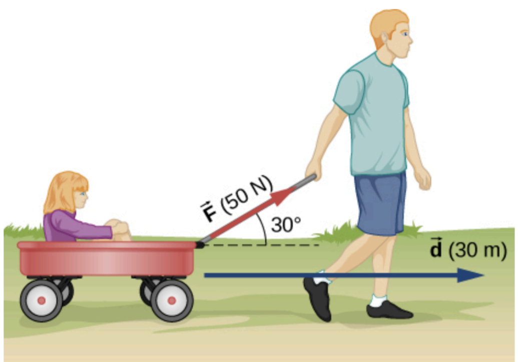
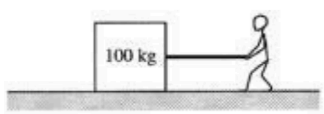
- $W = \int_{x_i}^{x_f} F \bullet dx$
- $F_{spring} = -kx$
- $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$
- $P_{avg} = \frac{\Delta Work}{\Delta time}$
- $P_{inst} = \frac{dW}{dt}$
- $P_{inst} = F \bullet v$

Practice

28. How much work is done by the boy pulling his sister 30.0 m in a wagon as shown below?

1) 



2003M1. The 100 kg box shown above is being pulled along the x-axis by a student. The box slides across a rough surface, and its position x varies with time t according to the equation $x = 0.5t^3 + 2t$, where x is in meters and t is in seconds.

a. Determine the speed of the box at time $t = 0$.
b. Determine the following as functions of time t.
   i. The kinetic energy of the box
   ii. The net force acting on the box
   iii. The power being delivered to the box
c. Calculate the net work done on the box in the interval $t = 0$ to $t = 2$s.
d. Indicate below whether the work done on the box by the student in the interval $t = 0$ to $t = 2$s would be greater than, less than, or equal to the answer in part (c).
   _____Greater than _____Less than _____Equal to
   Justify your answer

2)

1986M3 (modified) A special spring is constructed in which the restoring force is in the opposite direction to the displacement, but is proportional to the *cube of the displacement;* i.e., $F = -kx^3$
This spring is placed on a horizontal frictionless surface. One end of the spring is fixed, and the other end is fastened to a mass M. The mass is moved so that the spring is stretched a distance A and then released. Determine each of the following in terms of k, A, and M.

a. The potential energy in the spring at the instant the mass is released
b. The maximum speed of the mass
c. The displacement of the mass at the point where the potential energy of the spring and the kinetic energy of the mass are equal

3)

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Answers

1.

given $F = 50\text{N}$, $\theta = 30^\circ$, and $x = 30\text{m}$

$W = Fx\cos\theta$

$W = 50(30)\cos(30) = 1299.04\text{ J}$

2.

a) Given $x = 0.5t^3 + 2t$

$v = \frac{dx}{dt} = 1.5t^2 + 2$

$v(0) = 0^2 + 2 = 2\text{ m/s}$

b) i) using $K = \frac{1}{2}mv^2$

$K = \frac{1}{2}m(1.5t^2+2)^2$

ii) $F_{net} = ma = m\frac{dv}{dt} = m3t = 300t$

iii) $P = F_{net}v = (300t)(1.5t^2+2) = 450t^3 + 600t$

c) $W_{net} = \Delta K = K_f - K_i$

$v(2) = 8\text{ m/s}$

$v(0) = 2\text{ m/s}$

$\rightarrow W = \frac{1}{2}(100)(8)^2 - \frac{1}{2}(100)(2)^2 = 3000\text{ J}$

d) Greater than, since the student had to perform work against friction.

3.

a) Using the formula $F = -\frac{dU}{dx}$, so $\Delta U = -\int F\,dx$

$U = -\int_0^A -kx^3\,dx$

$U = \int_0^A kx^3\,dx$

$U = \frac{kx^4}{4}\Big|_0^A$

$U = \frac{kA^4}{4}$

b) using energy analysis...

$U = K_f$

$\frac{kA^4}{4} = \frac{1}{2}mv_{max}^2$

$\sqrt{\frac{2kA^4}{4m}} = v_{max}$

$v_{max} = A^2\sqrt{\frac{k}{2m}}$

c) $E_{total} = K + U$ when $K = U$

$E_{total} = U + U = 2U$

$U_{spring} = \frac{1}{2}E_{total}$

Where E_{total} is the max energy from the potential energy calculation in part (a). To find x, use the integral from part A that gets you $\frac{kx^4}{4}$ (without the A substituting in).

$\frac{1}{4}(kx^4) = \frac{1}{2}\left(\frac{kA^4}{4}\right)$

$x^4 = \frac{1}{2}A^4$

$x = A(2^{-1/4})$