Tab 1

Chapter 7: Work and Energy

Background/ Summary

This chapter explains how forces transfer energy to objects when they move through a displacement. It introduces the idea of work, shows how work can change an object's motion through kinetic energy, and connects these ideas through the work-energy theorem. The unit also introduces power, which measures how quickly work is done.

Major Topics Covered

- Definition of work and the dot product
- Work done by constant forces
- Work done by variable forces
- Hooke's Law and spring work
- Kinetic energy
- Work-Kinetic Energy Theorem
- Average and instantaneous power

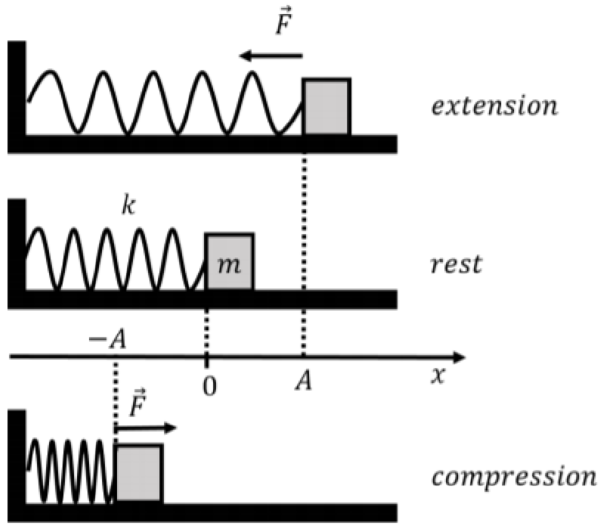
Key Vocabulary

- Work: Energy transferred when a force causes displacement. It is a scalar quantity and depends on the angle between force and motion
- Kinetic Energy: The energy an object has because of its motion.
- Work-Energy Theorem: The net work done on an object equals the change in its kinetic energy.
- Power: The rate at which work is completed or energy is transferred.
- Equilibrium Position: The point where the net force on an object is zero.
- Amplitude: The maximum distance from equilibrium in oscillating motion.

Important Equations

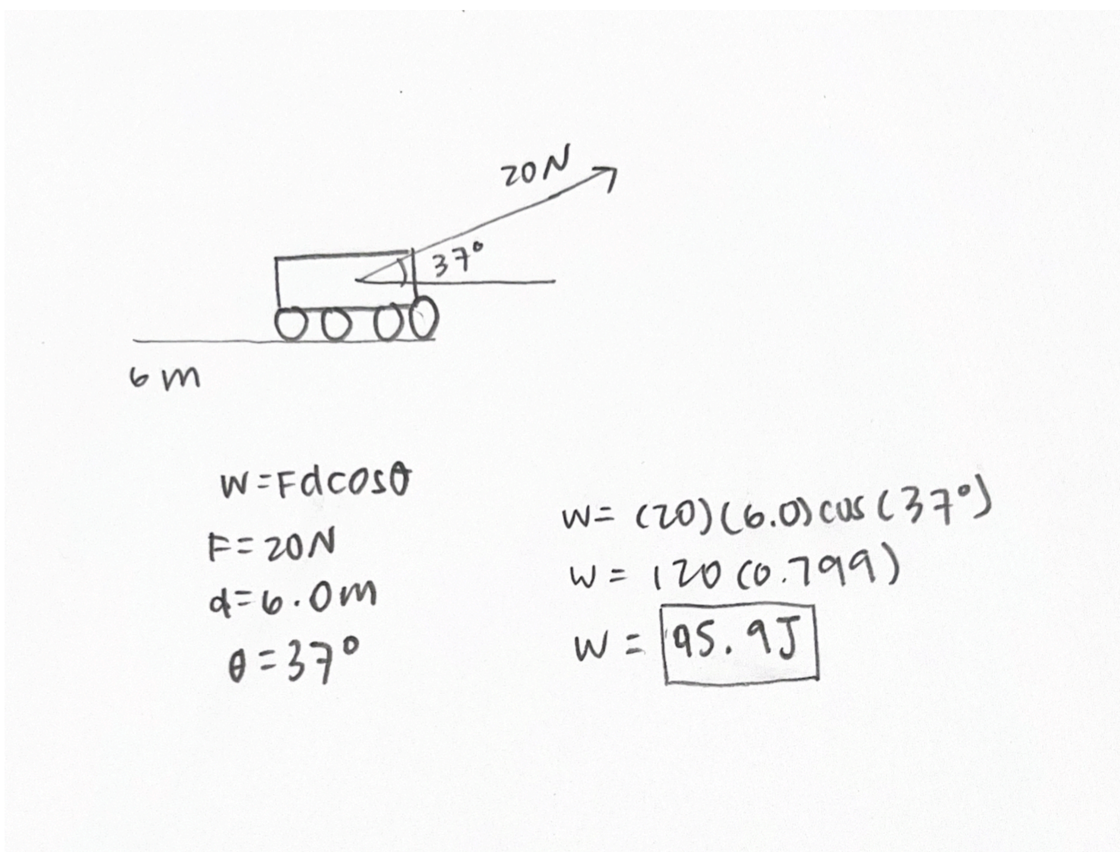
- W = Fcosθ → Used to calculate work done by a constant force at an angle.
- W =∫F dx → Used when force changes with position.
- Fspring=−kxF→ Hooke's Law for spring forces.
- Wspring $=\frac{1}{2}kx^2$→ Work done by or on a spring.
- K $=\frac{1}{2}mv^2$ → Kinetic energy of a moving object.
- Wnet=ΔK → Relates net work to change in kinetic energy.
- Pavg $=\frac{W}{t}$ → Average power
- P=Fv → Instantaneous power when force and velocity are in the same direction.

Spring Compression



FRQ Problems

Easy) A toy cart is pulled a distance of 6.0 m in a straight line across the floor. The force pulling the cart has a magnitude of 20 N and is directed at 37° above the horizontal. What is the work done by this force?



Medium) A 2.0-kg block starts with a speed of 10 m/s at the bottom of a plane inclined at 37° to the horizontal. The coefficient of sliding friction between the block and plane is μk=0.30. (a) Use the work-energy principle to determine how far the block slides along the plane before momentarily coming to rest.

a) $W_{net} = \Delta K$

Forces doing work while block slides up:

$W_g = -mgd\sin\theta$ ← work done by gravity while obj moves on incline

$W_f = -\mu_K mgd\cos\theta$

↑ work done by friction (neg bc friction opposes directoin of motion)

[$-mgd\sin\theta - \mu_k mgd\cos\theta = 0 - \frac{1}{2}mv^2$] work energy theorem

↳ $gd(\sin 37° + \mu_k \cos 37°) = \frac{1}{2}v^2$

$W_{net} = \Delta K$

$d = \frac{\frac{1}{2}(10^2)}{9.8(\sin 37 + 0.30\cos 37)}$

$\Delta K = K_f - K_i$

$K_f = 0$

$K_i = \frac{1}{2}mv^2$

$\Delta K = 0 - \frac{1}{2}mv^2$

$d = \frac{50}{9.8(0.602 + 0.30(0.799))}$

$d = \frac{50}{8.25} = \boxed{6.1\text{ m}}$

(b) After stopping, the block slides back down the plane. What is its speed when it reaches the bottom?

b)

$W_{net} = \Delta K$

$W_g = mgd\sin\theta$ ← pos gravity bc pulls down same direction as block

$W_f = -\mu_K mgd\cos\theta$ ← neg bc opposing motion

work energy theorem [$mgd\sin\theta - \mu_k mgd\cos\theta = \Delta K$]

$mgd\sin\theta - \mu_k mgd\cos\theta = \frac{1}{2}mv_f^2$ ↳

$K_i = 0$

$K_f = \frac{1}{2}mv_f^2$

$\Delta K = \frac{1}{2}mv_f^2 - 0$

$2(gd\sin\theta = \mu_k gd\cos\theta = \frac{1}{2}v_f^2)$

$2(9.8)(6.1)(\sin 37° - 0.30\cos 37°)$

$= 43.2 = v_f^2$

$v_f = \sqrt{43.2} = \boxed{6.6\text{ m/s}}$

Challenge Problem) Constant power P is delivered to a car of mass m by its engine. Show that if air resistance can be ignored, the distance covered in a time t by the car, starting from rest, is given by $s=(8P/9m)^{1/2}t^{3/2}$

$P = \frac{dW}{dt} \Rightarrow \frac{dK}{dt}$ ← work changes to kinetic energy

$P = \frac{d}{dt}\left(\frac{1}{2}mv^2\right)$ ← substitute Kin

$P = mv\frac{dv}{dt}$

$v\,dv = \frac{P}{m}dt$

$\int v\,dv = \int \frac{P}{m}dt$

$\frac{1}{2}v^2 = \frac{P}{m}t$

$v = \sqrt{\frac{2Pt}{m}}$ velocity is rate of change of position

$v = \frac{ds}{dt}$

$\frac{ds}{dt} = \sqrt{\frac{2P}{m}}\,t^{\frac{1}{2}}$ ← integrate again to find displacement

$s = \sqrt{\frac{2P}{m}}\int t^{\frac{1}{2}}dt$

$s = \sqrt{\frac{2P}{m}}\left(\frac{2}{3}t^{3/2}\right)$

$s = \frac{2}{3}\sqrt{\frac{2P}{m}}\,t^{\frac{3}{2}}$

$\frac{2}{3}\sqrt{\frac{2P}{m}} = \sqrt{\frac{8P}{9m}}$

$s = \left(\frac{8P}{9m}\right)^{\frac{1}{2}}t^{\frac{3}{2}}$

Tab 2