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Momentum

Background

Momentum describes the motion of an object as the product of its mass and velocity. The momentum of a system is conserved in the absence of external forces, making it a great tool for analyzing collisions and other interactions between objects. This unit focuses on linear momentum, impulse, and conservation laws in elastic and inelastic collisions.

Key Concepts and Vocabulary

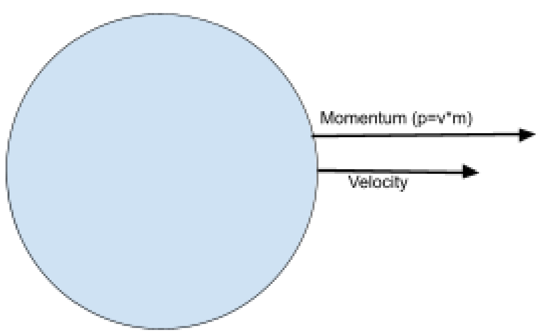
- **Linear Momentum**
  - The quantity of motion of an object is equal to its mass multiplied by its velocity.
- **Impulse and Change in Momentum**
  - Impulse is the product of force and time, and it equals the change in an object's momentum.
- **Conservation of Momentum**
  - In an isolated system with no external forces, the total momentum remains constant.
- **Elastic vs Inelastic Collisions**
  - Elastic collisions conserve both momentum and kinetic energy, while inelastic collisions conserve momentum but not kinetic energy.
- **Elastic Collision**
  - KE conserved
- **Inelastic Collision**
  - KE not conserved
- **Perfectly Inelastic**
  - Objects stick together
- **Center of Mass**
  - Weighted average position
- **External Force**
  - Force from outside the system

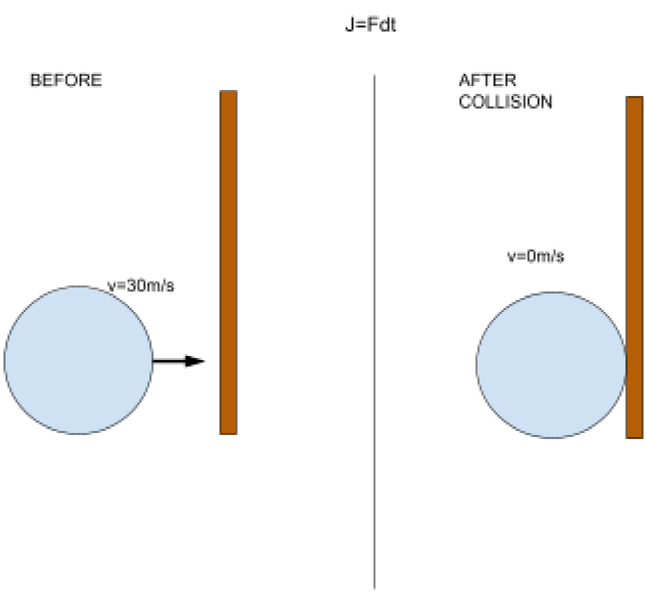
Key Equations

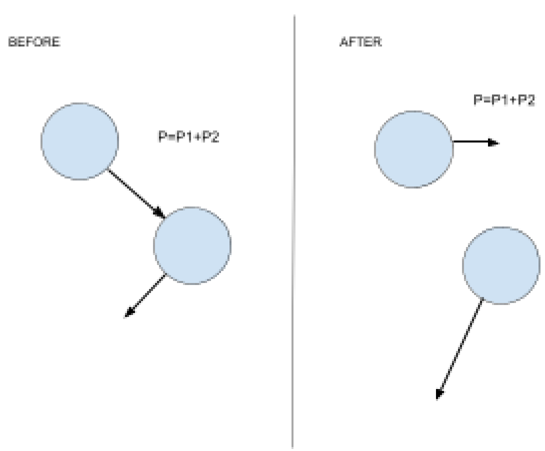
- $\vec{p} = m\vec{v}$
- $\vec{F}_{\text{net}} = \dfrac{d\vec{p}}{dt}$
- $\vec{J} = \int_{t_1}^{t_2} \vec{F}_{\text{net}}(t)\,dt = \Delta\vec{p}$
- $\vec{v}_{\text{cm}} = \dfrac{\sum \vec{p}_i}{\sum m_i} = \dfrac{\sum m_i \vec{v}_i}{\sum m_i}$
- $m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}'_1 + m_2\vec{v}'_2$

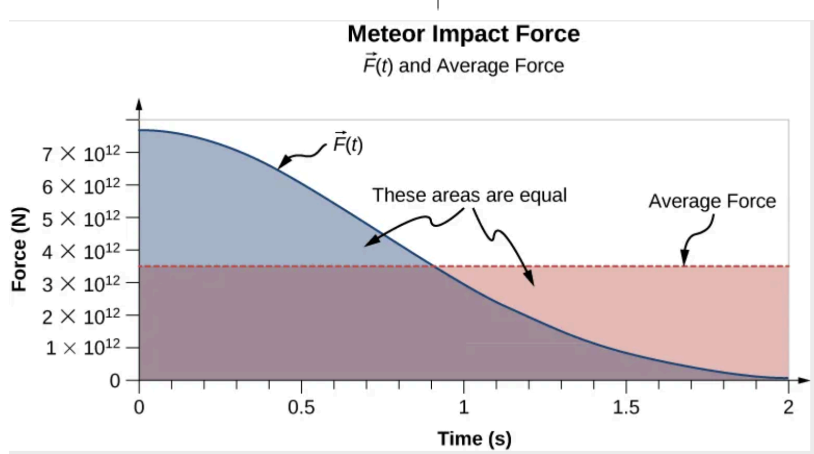
$$\tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}m_1v_1'^2 + \tfrac{1}{2}m_2v_2'^2$$

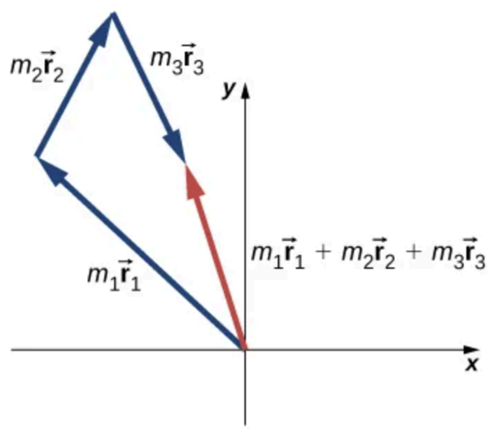
Diagrams











(c)

Practice Problems

1. (easy) A 3 kg cart moving at 4 m/s collides with a 2 kg cart at rest. The carts stick together after the collision. What is their final velocity?

1. Since the carts stick together → **perfectly inelastic collision**
2. Momentum is conserved: $m_1 v_1 + m_2 v_2 = (m_1 + m_2)v_f$
3. $m_1 = 3\,kg,\ v_1 = 4\,m/s,\ m_2 = 2\,kg,\ v_2 = 0\,m/s$
4. Plug into equation: (3)(4)+(2)(0)=(5)v_f
5. $v_f = \frac{12}{5}$ → v_f **= 2.40 m/s**

2. (medium) A 0.20 kg baseball traveling at 30 m/s is hit by a bat and reverses direction, leaving the bat at 40 m/s. If the bat is in contact with the ball for 0.01 s, what is the average force exerted on the ball?

1. Since we know that we are trying to solve for the average force, use the impulse-momentum theorem: $J = F\Delta t = m(v_f - v_i)$
2. $t = .01\,s,\ v_i = 30\,m/s,\ v_f = 40\,m/s$
3. Plug into the equation: F(0.01)=0.20(−40−30) → F(0.01)=0.20(−70)
4. $F = \frac{-14}{.01}$ → F=-1400 N or F = 1400 N in the opposite direction of the initial momentum of the ball

3. (hard) A 5 kg object at rest explodes into two pieces. One piece has a mass of 2 kg and moves at 6 m/s at a 30° angle above the horizontal. What is the velocity (magnitude and direction) of the 3 kg piece?

1. Object starts at rest, so initial momentum is 0 → $p_i = 0$
2. Determine the momentum of the 2 kg piece in the x and y directions
   a. v_x = 6cos(30º) = 6(0.866) =5.20 m/s
      i. p_{1x}= (2)(5.20) = 10.4
   b. v_y = 6sin(30º) = 6(0.5) = 3.00 m/s
      i. p_{1y}= (2)(3.00)=6.0
3. Using the conservation of momentum, the total must remain zero
   a. $p_{1x} + p_{2x} = 0$ → p_{2x} = -10.4
   b. $p_{1y} + p_{2y} = 0$ → p_{2y} = -6.0
4. Now, calculate the velocities of the 3 kg piece
   a. $v_x = \frac{-10.4}{3}$ = -3.47 m/s
   b. $v_y = \frac{-6}{3}$ = -2.0 m/s
5. Find the magnitude of velocity
   a. $v = \sqrt{(-3.47)^2 + (-2.0)^2}$ → v = 4.0 m/s
6. Now determine direction
   a. $tan(\theta) = \frac{2}{3.47}$
   b. $\theta = tan^{-1}(\frac{2}{3.47})$ → $\theta = 30º$
7. v = 4.0 m/s 30º below the horizontal (opposite direction)