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Chapter 7: Energy & Energy Transfer

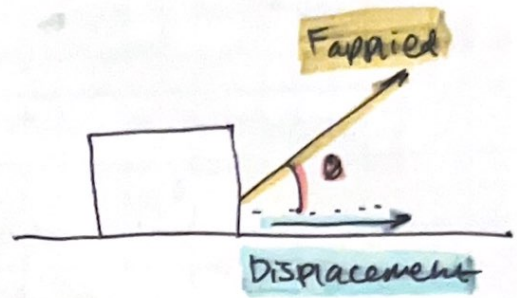
WORK: Product of force applied to an object and the distance an object moves

EQUATIONS FOR WORK:

* when F_a is constant

$$W = F x \cos\theta$$

$$W = F \cdot x$$



* QUICK TIP! Drawing FBD's for solving work problems is super helpful!

Remember, work is scalar!!!
Meaning, it doesn't have a direction.
If you see a positive or negative work value, that is refering to the energy added or removed from the object.

WORK & ENERGY: when work is done, energy is transferred!

DOT PRODUCTS: You can also use dot-products to calculate work done.

$$W = \vec{F} \cdot \vec{x}$$

Example: A force of $(6i + 2j)$ N acts on an object, displacing it $(4i + 5j)$ meters. How much work is done?

$\vec{F} = 6i + 2j \quad \vec{x} = 4i + 5j$

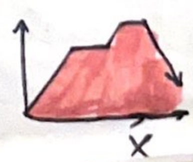
$$W = \vec{F} \cdot \vec{x} = (6i)(4i) + (2j)(5j)$$

$$= 24 + 10 = \boxed{34\text{ J}}$$

WORK DONE BY VARYING FORCE: How can we solve for work done by a varying force?

① $$W = \int_{x_i}^{x_f} \vec{F} \cdot d\vec{x}$$

② Find the area under the curve of a Force vs. disp. graph

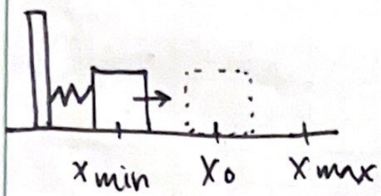


Springs are a good example where a varying force is applied...

HOOKE'S LAW: $F_{spring} = \boxed{-kx}$ ("k" is the spring constant)

Hooke's law basically tells us that the force applied to a spring is proportional to how much it stretches or compresses!

UNDERSTANDING MASS-SPRING SYSTEMS WITH HOOKE'S:



$$W = \int_{x_0}^{x_f} F_s \cdot dx = \int_{-x}^{0} (-kx) \cdot dx$$

$$= -\frac{kx^2}{2}\Big|_{-x}^{0} = 0 - \left(\frac{-k(-x)^2}{2}\right) = \boxed{\frac{1}{2}kx^2}$$

WORK-ENERGY THEOREM: Essentially tells us that work produces a change in kinetic energy.

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = K_f - K_i = \Delta K$$

POWER: A force doing work over a period of time.

$$P_{avg.} = \frac{\Delta W}{\Delta t} \qquad P_{inst.} = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

PRACTICE PROBLEMS:

EASY #1: A person pushes a box 1.2 meters across a table with a constant horizontal force of 4.00N. Assuming the the box moves at constant speed and the force is applied horizontally, how much work does the person do on the box?

MEDIUM #2: A person pushes a 10.0kg box along a horizontal floor by applying a force of 40.0N at an angle of 30.0° below the horizontal. The box is pushed 2.50 meters and the coefficient of kinetic friction between the floor and box is 0.20.

a) How much work does the person do on the box?

b) How much work does friction do on the box?

c) What is the net work done on the box?

ADVANCED #3: A particle of mass $m = 2.00kg$ moves along the x-axis under the influence of a varying force given by the function:

$$f(x) = 6x^2 - 4x$$

the particle moves from position $x = 0.00m$ to $x = 3.00m$. Also, assume the particle starts from rest at $x = 0\,m$ and no other forces are acting on it (no friction).

ANSWER KEY! :

#1

$$W = F \cdot d \cos\theta = (4N)(1.2m)\cos(0) = \boxed{4.8\,J}$$

#2 a) First find horizontal component of Fapp.

$$\hookrightarrow F_x = F\cos\theta = 40\cos(30^\circ) = 34.64N$$

$$W = F_x \cdot d = (34.64)(2.5) = \boxed{86.6J}$$

b) Find vertical component of Fapp.

$$F_y = F\sin(30^\circ) = 40(0.5) = 20N$$

$$\Sigma F = ma$$

$$F_N - F_y = mg$$

$$F_N = mg + F_y = (10)(9.8) + 20 = 98 + 20$$

$$= 118N$$

Find F_f

$$\hookrightarrow F_f = \mu_k F_N = 0.2(118)$$

$$= 23.6N$$

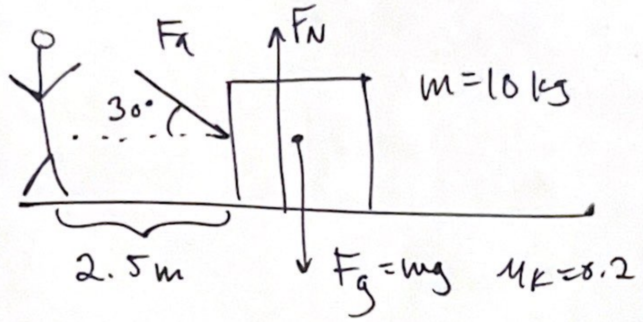
negative because in opposite direction

$$W_{friction} = -F_f \cdot d$$

$$= -23.6(2.5) = \boxed{-59\,J}$$

c) $W_{net} = W_{person} + W_{friction} = 86.6 + (-59) = \boxed{27.6\,J}$

<u>graph/diagram</u>



#3

a) $$W = \int_{x_f}^{x_i} F \cdot dx = \int_0^3 (6x^2 - 4x)\,dx$$

$$= 2x^3 - 2x^2 \Big|_0^3 = \left(2(27) - 2(9)\right) - 0 = \boxed{36\,J}$$

b) Finding speed at $x = 3\,m$

work energy theorem

$$W = \Delta K = \frac{1}{2}mv^2 - 0 = 36$$

$$\frac{1}{2}mv^2 = 36$$

$$\frac{1}{2}(2)v^2 = 36$$

$$\boxed{v = 6.00\,m/s}$$