Luciana Piro

Chapter 11: Angular Momentum

Angular Momentum *- the rotational analog of linear momentum*

Use when analyzing a point mass

$$\vec{L} = \vec{r} \times m\vec{v}$$

Use when analyzing a rotating rigid body (ex: a rod pivoting about an axis, a spinning disk, etc.)

$$L = I\omega$$

Torque *- the rotational analog of Force*

$\vec{\tau} = \frac{d\vec{L}}{dt}$ *or "Newton's Second Law for rotation"* $\sum \tau = I\alpha$

Kinetic Energy of a Rotating Object *- combine the translational KE and rotational KE*

$$K_{total} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Important Relationships of Motion for Rolling Objects

$x = r\theta$

$v = r\omega$

$a = r\alpha$

Conservation of Angular Momentum *- when external torque equals zero, total initial momentum is equal to total final momentum (common example problem: figure skater pulling in their arms, see figure below from [University Physics Volume I](#))*

$$\sum L_i = \sum L_f$$

.. in rotational collision problems where a mass sticks to a rotating object, angular momentum is conserved but mechanical energy may not be conserved!

Common Moments of Inertia:

*(*good to be able to remember/recognize these)*

Solid Sphere = $\frac{2}{5}MR^2$

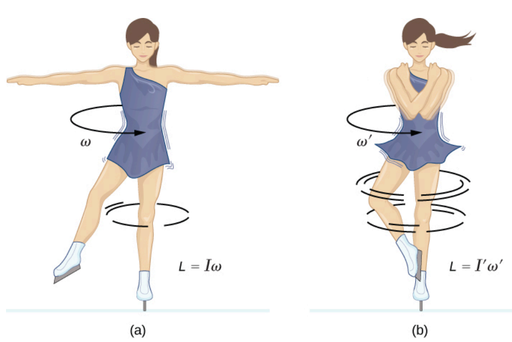
Hollow Sphere = $\frac{2}{3}MR^2$

Thin Hoop = MR^2

Cylinder/Disk = $\frac{1}{2}MR^2$

Rod about an end = $\frac{1}{3}ML^2$

Rod about center = $\frac{1}{12}ML^2$



My special tips!

- *It's good to familiarize yourself with the common problem types under this topic such as rolling objects, falling rods, spinning disks and collisions, deriving the moment of inertia, etc.*
- *When finding moment of inertia, make sure to check if the axis of rotation is moving or fixed*
- *For conservation of angular momentum problems, make sure to check that there is no external torque present*

Problems: *(all problems are from the University Physics Volume I online textbook)

[Easy] A DVD is rotating at 500 rpm. What is the angular momentum of the DVD if has a radius of 6.0 cm and mass 20.0 g?

[Medium] A solid cylinder of mass 2.0 kg and radius 20 cm is rotating counterclockwise around a vertical axis through its center at 600 rev/min. A second solid cylinder of the same mass and radius is rotating clockwise around the same vertical axis at 900 rev/min. If the cylinders couple so that they rotate about the same vertical axis, what is the angular velocity of the combination?

[Hard] A bowling ball of radius 8.5 cm is tossed onto a bowling lane with speed 9.0 m/s. The direction of the toss is to the left, as viewed by the observer, so the bowling ball starts to rotate counterclockwise when in contact with the floor. The coefficient of kinetic friction on the lane is 0.3. (a) What is the time required for the ball to come to the point where it is not slipping? What is the distance d to the point where the ball is rolling without slipping?

Solutions:

[EASY]

$$m = \frac{20.0\,\cancel{g}}{} \Big| \frac{1\,kg}{1000\,\cancel{g}} = 0.02\,kg$$

$$r = \frac{6.0\,\cancel{cm}}{} \Big| \frac{1\,m}{100\,\cancel{cm}} = 0.06\,m$$

$$\omega = \frac{500\,\cancel{rev}}{\cancel{min}} \Big| \frac{1\,\cancel{min}}{60\,sec} \Big| \frac{2\pi\,rad}{1\,\cancel{rev}} = 8.33 \times 2\pi = 52.36\,rad/s$$

Moment of Inertia for a solid disk $= I = \frac{1}{2}MR^2 = \frac{1}{2}(0.02)(0.06)^2$

$$I = 3.6e{-5}\ kg \cdot m^2$$

$$L = I\omega = (3.6e{-5})(52.36) = \boxed{1.88e{-3}\ kg \cdot m^2/s}$$

[MEDIUM]

we have 2 solid wheels rotating around the same axis...

$M = 2\,kg$ $\quad \omega_1 = 600\,rev/min$ (CCW)

$r = 0.2\,m$ $\quad \omega_2 = 900\,rev/min$ (CW)

$\omega_f = ?$

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(2)(0.2)^2 = 0.04\,kg \cdot m^2$$

$$\omega_1 = \frac{600\,\cancel{rev}}{\cancel{min}} \left| \frac{2\pi\,rad}{1\,\cancel{rev}} \right| \frac{1\,\cancel{min}}{60\,s} = 62.83\,rad/s$$

$$\omega_2 = \frac{-900\,\cancel{rev}}{\cancel{min}} \left| \frac{2\pi\,rad}{1\,\cancel{rev}} \right| \frac{1\,\cancel{min}}{60\,s} = -94.25\,rad/s$$

(−) for CW

$$\Sigma L_i = \Sigma L_f$$

$$\cancel{I}(\omega_1 + \omega_2) = 2\cancel{I}\omega_f$$

$$\frac{(62.83) + (-94.25)}{2} = \omega_f$$

$$\boxed{\omega_f = -15.71\,rad/s}$$

combined system rotates clockwise!

[HARD]

$f_k = ma \to a = \frac{f_k}{m} = \frac{\mu_k mg}{m} = -\mu_k g$

$f_k = \mu_k F_N = \mu_k mg$

slows down

$\tau = r \times F = r \cdot f_k = r(\mu_k mg) = \mu_k mgR$

moment of inertia for solid sphere

$I = \frac{2}{5} MR^2$

$\tau = I\alpha \to \alpha = \frac{\tau}{I} = \frac{\mu_k mgR}{\frac{2}{5} mR^2} = \frac{5\mu_k g}{2R}$

$v = R\omega$

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a) $v = v_0 + at = v_0 - \mu_k g t = R\left(\frac{5\mu_k g}{2R}\right)t$

$\omega = 0 + \alpha t = \frac{5\mu_k g}{2R} t$

$v_0 = \left(\mu_k g + \frac{5\mu_k g}{2}\right)t = \frac{7\mu_k g}{2} t$

$t = \frac{2v_0}{7\mu_k g} = \frac{2(9)}{7(0.3)(9.8)} = \boxed{0.875\,s}$

b) $x = v_0 t + \frac{1}{2} a t^2 = 9(0.875) + \frac{1}{2}(-0.3)(9.8)(0.875)^2$

$\boxed{x = 6.75\,m}$