Lab: Ap Review Sheets

AP Physics

CHAPTER 15 - OSCILLATION

BY RONEN PETERS

Background/Summary: This review covers oscillatory motion, with a focus on Simple Harmonic Motion (SHM). Simple harmonic motion is a form of periodic motion where the restoring force is directly proportional to displacement and acts in the opposite direction of displacement. We'll examine motion in mass-spring systems, pendulums, and circular motion as it relates to SHM. We'll also look at energy conservation in oscillating systems.

Major Topics/Definitions:

Simple Harmonic Motion (SHM): Periodic motion where acceleration/force is proportional to and opposite the displacement $x(t) = A\cos(\omega t + \phi)$

Amplitude (A): Maximum displacement from equilibrium.

Period: Time for one complete oscillation. $T = \frac{2\pi}{\omega}$

Frequency: The number of cycles per second. $f = \frac{1}{T}$

Phase Constant (ϕ): Sets the initial angle (starting point) in the oscillation.

Angular frequency: $\omega = 2\pi f = \frac{2\pi}{T}$

Important things to remember:

In mass-spring systems, **max speed occurs at the equilibrium position (x = 0).** $v_{max} = -\omega A$

Acceleration is zero at x = 0 and **maximal at the turning points (x = ±A).**

In pendulums, **SHM only applies accurately for small angles (θ < 15°).**

Use angular quantities (ω, φ) to describe linear oscillations.

SHM can be derived from circular motion: **tracking the x- or y-component of a point moving around a circle results in sinusoidal motion.**

MAJOR FORMULAE FOR OSCILATION:

Position/Velocity/Acceleration derivatives:

$x(t) = A\cos(\omega t + \phi)$ $v(t) = -\omega A\sin(\omega t + \phi)$ $a(t) = -\omega^2 A\cos(\omega t + \phi)$

$a = -\omega^2 x$

For a Mass spring system:

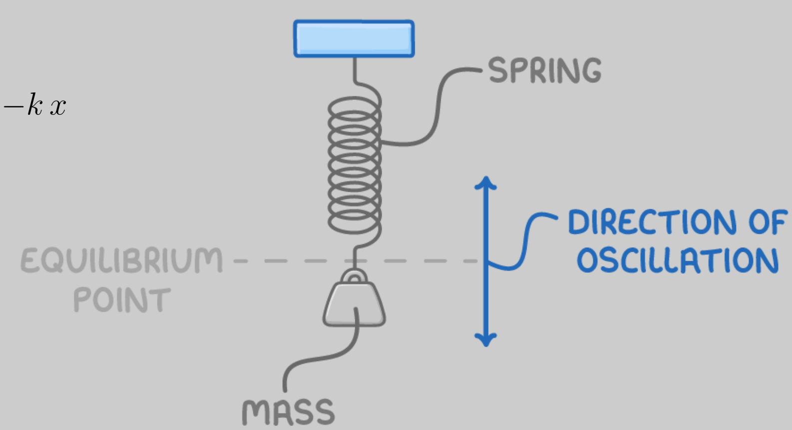
Restoring force (hooke's law): $F = -k\,x$

Equation of motion: $\omega = \sqrt{\frac{k}{m}}$

Period: $T = 2\pi\sqrt{\frac{m}{k}}$

Energy in mass-spring-system:

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}mv_{\max}^2$$



Small-Angle Pendulum Ananysis:

Small angle-approxamation: Allows you to treat the non-linear pendulum motion as linear when θ < 15°, this is because at a small value for θ the difference is negligible allowing us to use basic SHM formulas.

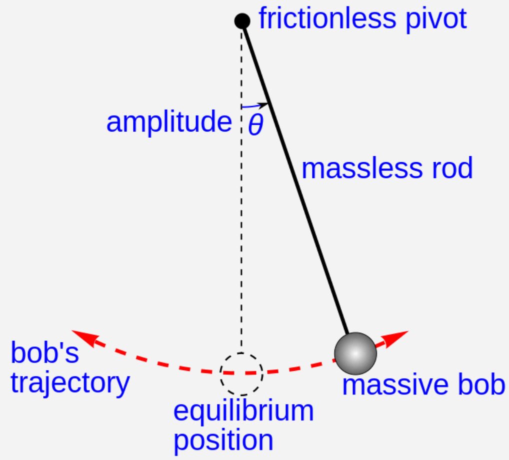
Angular Frequency: $\omega = \sqrt{\frac{g}{L}}$

Period: $T = 2\pi\sqrt{\frac{L}{g}}$

Pendulum Ananysis:

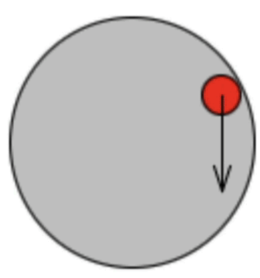
Angular Frequency: $\omega = \sqrt{\frac{mgd}{I}}$

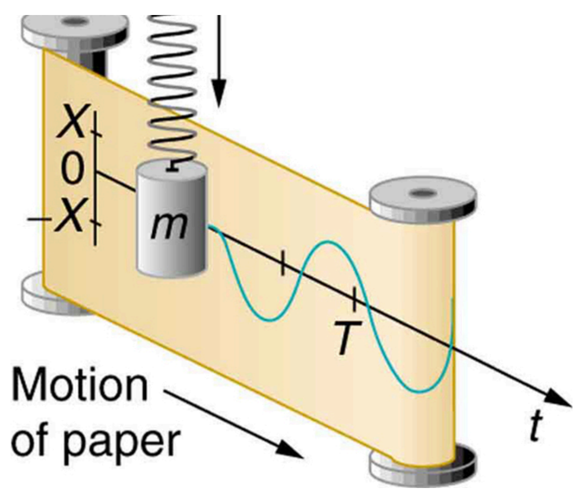
Period: $T = 2\pi\sqrt{\frac{I}{mgd}}$



Something Kinda Cool:

If you track one side of a coin as it rolls, it mimics the graphical motion of a SHM device. Here's a visualization of that graphing.





Problem 1 (Easy): Mass-Spring System

A 0.50 kg block is attached to a spring with a spring constant of 100 N/m. The block is set into oscillation on a frictionless surface.
(a) What is the period of the oscillation?
(b) What is the angular frequency?

Problem 2 (Medium): Velocity and Energy in SHM

A 2.0 kg mass on a spring with spring constant k=50 N/m oscillates with amplitude A=0.10m.
(a) Calculate the total mechanical energy of the system.
(b) What is the speed of the mass when it is 0.05 m from equilibrium?

Problem 3 (Hard): Climber is climbing around
The length of nylon rope from which a mountain climber is suspended has an effective force constant of 104N/m.
(a) What is the frequency at which he bounces, given his mass plus and the mass of his equipment are 90.0 kg?
(b) How much would this rope stretch to break the climber's fall if he free-falls 2.00 m before the rope runs out of slack? (*Hint:* Use conservation of energy.)
(c) Repeat both parts of this problem in the situation where twice this length of nylon rope is used.

Problem 1:

(a) Use the period formula for a mass-spring system:
$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.50}{100}} = 2\pi\sqrt{0.005} \approx 0.445\,\mathrm{s}$$
(b) Angular frequency:
$$\omega = \frac{2\pi}{T} \approx \frac{2\pi}{0.445} \approx 14.1\,\mathrm{rad/s}$$

Problem 2:

(a) Total mechanical energy:

$$E = \frac{1}{2}kA^2 = \frac{1}{2}(50)(0.10)^2 = \frac{1}{2}(50)(0.01) = 0.25\,\mathrm{J}$$

(b) Use conservation of energy to find speed at $(x = 0.05\,\mathrm{m})$:

$$v = \sqrt{\frac{2}{m}\left(E - \frac{1}{2}kx^2\right)}$$

$$= \sqrt{\frac{2}{2.0}(0.25 - \frac{1}{2}(50)(0.05)^2)}$$

$$= \sqrt{1(0.25 - 0.0625)} = \sqrt{0.1875} \approx 0.433\,\mathrm{m/s}$$

Problem 3:

(a)

$$\text{Given: } k = 1.40 \times 10^4\,\mathrm{N/m}, \quad m = 90.0\,\mathrm{kg}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1.40 \times 10^4}{90.0}}$$

$$= \frac{1}{2\pi}\sqrt{155.56} \approx \frac{12.47}{2\pi} \approx 1.98\,\mathrm{Hz}$$

(b)

$$\text{Use conservation of energy: } mgh = \frac{1}{2}kx^2 \Rightarrow x = \sqrt{\frac{2mgh}{k}}$$

$$x = \sqrt{\frac{2(90.0)(9.8)(2.00)}{1.40 \times 10^4}} = \sqrt{\frac{3528}{14000}} = \sqrt{0.252} \approx 0.502\,\mathrm{m}$$

(c)

$$\text{New spring constant: } k' = \frac{k}{2} = 7000\,\mathrm{N/m}$$

$$New frequency : f' = \frac{1}{2\pi}\sqrt{\frac{k'}{m}} = \frac{1}{2\pi}\sqrt{\frac{7000}{90}} = \frac{1}{2\pi}\cdot\sqrt{77.78} \approx 1.40\,\mathrm{Hz}$$

$$\text{New rope stretch:} x' = \sqrt{\frac{2mgh}{k'}} = \sqrt{\frac{3528}{7000}} = \sqrt{0.504} \approx 0.710\,\mathrm{m}$$