CHAPTER 8 - POTENTIAL ENERGY

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Background/Summary: This review packet will cover the different types of potential energy and how we can apply them to conservation of mechanical energy problems. This will enable us to analyze complex systems where many things are happening simultaneously or in series.

Major Topics/Definitions:	Important things to remember:
Potential Energy: Stored energy associated with the position of an object. **Conservative Force:** A force where the work done is path-independent (e.g., gravity, spring force) **Non-conservative Force:** Path-dependent forces (ex., friction). **Elastic Potential Energy** (ex., ideal spring): $U = \frac{1}{2}kx^2$ **Gravitational Potential Energy** (near Earth's surface): $U = mgh$ **Stable Equilibrium:** A point where a small displacement results in a restoring force back toward the equilibrium position. **(minimums of a potential energy graph)** **Unstable Equilibrium:** A point where a small displacement results in a force pushing the object further away from equilibrium. **(maximums of a potential energy graph)**	**Only changes in potential energy matter.** You can start by defining U=0 wherever it makes solving the problem easiest. If U(x) is given to you in a problem, **remember to use** $F(x) = -dU/dx$ **to find force.** Remember that **the energy in a spring is always positive and quadratic** in x. If "down" is negative, then h can be negative too. **Be consistent with your coordinate system.** In **a non-isolated system** where energy can enter or leave the system, **total energy is always conserved!**

ALL FORMULAE FOR POTENTIAL ENERGY:

Difference in potential energy: $\Delta U_{AB} = U_B - U_A = -W_{AB}$

Potential Energy with Respect to Zero at r0 = $\Delta U = U(\vec{r}) - U(\vec{r}_0)$

Gravitational Potential Energy = $U = mgh$

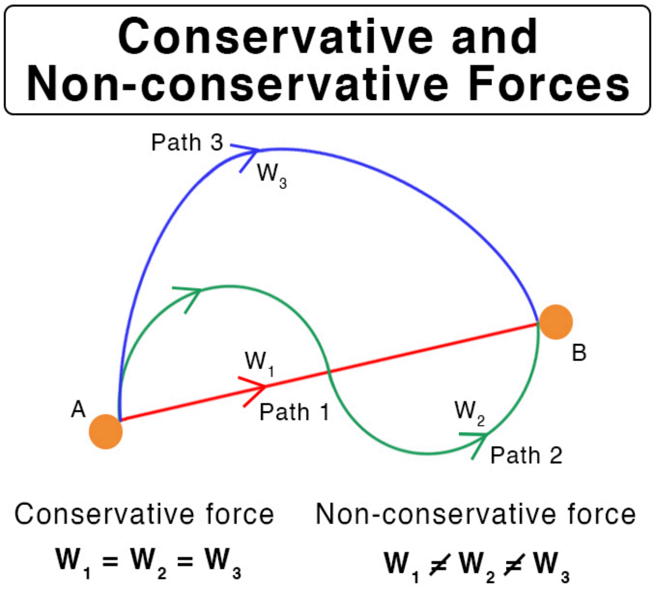
Potential Energy for an Ideal Spring = $U(x) = \frac{1}{2}kx^2$

Work Done by Conservative Force Over a Closed Path = $W_{\text{closed path}} = \oint \vec{F}_{\text{cons}} \cdot d\vec{r} = 0$

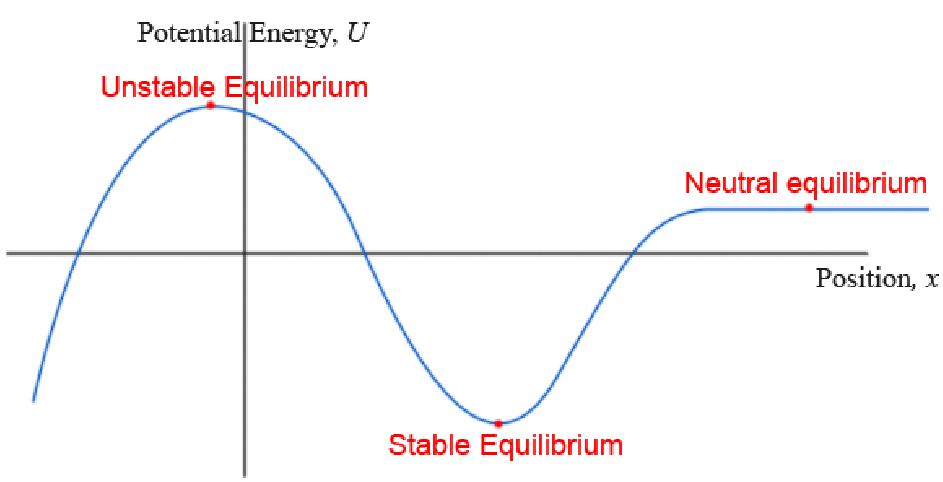
Conservative Force definition = $F_l = -\frac{dU}{dl}$

Conservation of Energy = $K_i + U_i + W_{\text{nc}} = K_f + U_f$

Conservation of Energy with No Non-Conservative Forces = $0 = W_{\text{nc},AB} = \Delta(K+U)_{AB} = \Delta E_{AB}$



If a force can be applied over very different paths and result in the same total work done, it is a conservative force. If the work done varies based on the path it takes, the force is not conservative. Imagine a ball falling in a zig-zag down, and the same ball falling straight down, the work done by gravity is the same, thereby gravity is a conservative force.



This graph of potential energy, U(x) shows points of stable, unstable, and neutral equilibrium. **At a stable equilibrium**, the graph reaches a minimum. From here, if the object is slightly displaced, the force derived from the slope of U(x) will push it back to equilibrium. **At an unstable equilibrium**, the graph has a maximum, any displacement will result in a force that pushes the object further away, like a ball on top of a hill. **A neutral equilibrium appears** at the end of the graph where the slope of is flat or zero. At a neutral equilibrium, a small displacement doesn't result in any force to return or move away from equilibrium, so the object will remain wherever it's moved.

Problem 1 (Easy): Spring Compression

A 2.0 kg block compresses a spring with spring constant k=300 N/m by 0.1 m. How much potential energy is stored in the spring?

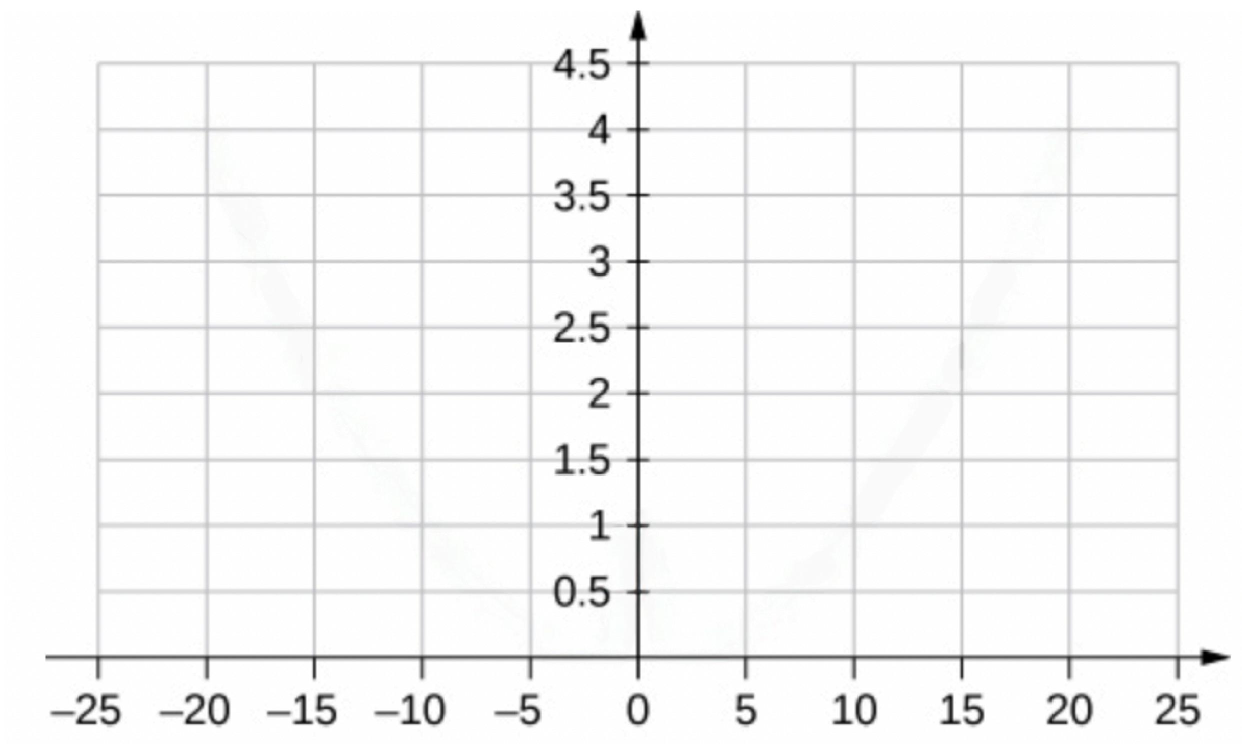
Problem 2 (Medium): Finding Force from U(x)

Given a potential energy function U(x)=3x2−2x3 (in joules, x in meters), find the force as a function of x. Where is the equilibrium, and is it stable?

Problem 3 (Hard): Wacky potential energy function

(a) Sketch a graph of the potential energy function $U(x) = kx^2/2 + Ae^{-\alpha x^2}$, where $k, A,$ and α are constants. (b) What is the force corresponding to this potential energy? (c) Suppose a particle of mass *m* moving with this potential energy has a velocity v_a when its position is $x = a$. Show that the particle does not pass through the origin unless

$$A \leq \frac{m{v_a}^2 + ka^2}{2\left(1 - e^{-\alpha a^2}\right)}.$$



PROBLEM 1:

A 2.0 kg block compresses a spring with spring constant $k = 300\,\text{N/m}$ by $x = 0.1$ m. How much potential energy is stored?

$$U = \frac{1}{2}kx^2$$
$$U = \frac{1}{2}(300)(0.1)^2 = \frac{1}{2}(300)(0.01)$$
$$U = 1.5\,\text{J}$$

The spring stores 1.5 J of potential energy, which could convert to kinetic energy if released.

PROBLEM 2:

Given $U(x) = 3x^2 - 2x^3$, find the force $F(x)$, the equilibrium points, and their stability.

$$F(x) = -\frac{dU}{dx} = -\left(6x - 6x^2\right) = -6x + 6x^2$$

Set $F(x) = 0$:

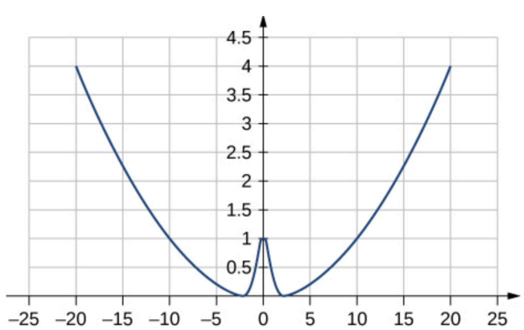
$$-6x + 6x^2 = 0 \Rightarrow 6x(-1 + x) = 0 \Rightarrow x = 0,\ 1$$

To determine stability, use the second derivative:

$\frac{d^2U}{dx^2} = 6 - 12x$ At $x = 0$: $\frac{d^2U}{dx^2} = 6 > 0 \Rightarrow$ Stable equilibrium

At $x = 1$: $\frac{d^2U}{dx^2} = -6 < 0 \Rightarrow$ Unstable equilibrium

Problem 3:



b)

$$U(x) = \frac{1}{2}kx^2 + Ae^{-\alpha x^2}$$
$$\frac{dU}{dx} = \frac{d}{dx}\left(\frac{1}{2}kx^2\right) + \frac{d}{dx}\left(Ae^{-\alpha x^2}\right)$$
$$\frac{dU}{dx} = kx - 2\alpha Axe^{-\alpha x^2}$$
$$F(x) = -\frac{dU}{dx} \qquad F(x) = -kx + 2\alpha Axe^{-\alpha x^2}$$

c)

$$E = \frac{1}{2}mv_a^2 + U(a) \qquad E = \frac{1}{2}mv_a^2 + \frac{1}{2}ka^2 + Ae^{-\alpha a^2}$$

In order for the particle to reach x=0, we need E ≥ U(0) = A . Thus

$$\frac{1}{2}mv_a^2 + \frac{1}{2}ka^2 + Ae^{-\alpha a^2} \geq A$$
$$\frac{1}{2}mv_a^2 + \frac{1}{2}ka^2 \geq A\left(1 - e^{-\alpha a^2}\right)$$
$$A \leq \frac{mv_a^2 + ka^2}{2\left(1 - e^{-\alpha a^2}\right)}$$