<u>Chapter 13: Universal Gravitation</u>

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Context: Besides U_g = mgh and F_g = mg, we can express gravity and gravitational potential energy in general terms using Universal gravitational constants. This unit also includes momentum, energy, and force analysis.

Key Equations	Key Terms

$F_g = -G\frac{m_1 m_2}{r^2}$
$W_{gravity} = U_i - U_f = -\Delta U$
$\Delta U = -W_{gravity} = \int_{xi}^{xf} F_g \cdot dx$
$U = \frac{-GMm}{r}$
 	
$v_{esc} = \sqrt{\frac{2GM}{r}}$
 	
$U_{total} = G\frac{m_1 m_2}{r_{1,2}} + G\frac{m_2 m_3}{r_{2,3}} + G\frac{m_1 m_2}{r_{1,3}}$
	- Newton's Law of Universal Gravitation: Every particle exerts force between them.
	$F_g = -G\frac{m_1 m_2}{r^2}$
	Universal Gravitation Constant (G)
	G = 6.674 * 10^{-11} N * $\frac{m^2}{kg^2}$
	
	- Gravitational Potential Energy
	
	- Escape Velocity
	  - The minimum velocity necessary to escape an orbit
	  - We can find this using energy analysis (See ch. 7 & 8)
	
	- Circular orbit: In bound orbits, centripetal force keeps the satellite orbiting and is provided by gravity
	
	- Elliptical orbit: Elliptical orbits have two different radii diametrically opposed. We can use conservation of angular momentum L_i = L_f to find properties of satellites at these points, like velocity, mass, and radius
	  - Aphelion: r_a and v_a
	  - Perihelion: r_p and v_p

Remember,

➢ The gravity force ≠ the gravity field! A mass creates a gravity field, and other masses experience a force *because* of that field

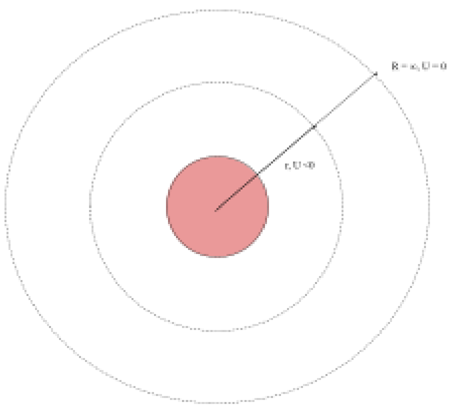
- ➢ $-\ \Delta U$ represents work done by gravity, causing a decrease in potential energy (but maybe an increase in kinetic energy!)
- ➢ As r approaches ∞, the potential energy in the system decreases to 0. That energy is converted to some other form of energy. This is key for escape velocity questions
- ➢ The energy of a planet-satellite system $E_{total} = U + K$

Determining gravitational potential energy

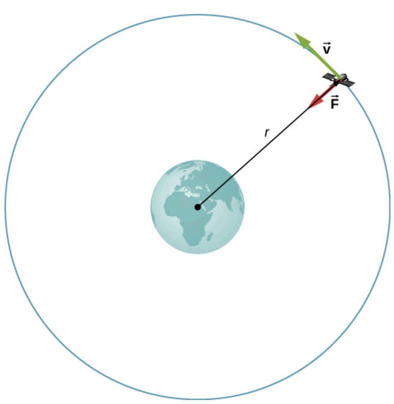
$$U_f - U_f = \ -\int_{ri}^{rf} F_g \cdot dr = -\ GMm\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

Let $U_i = 0$ at $r_i = \infty$

$$U\ = -\ \frac{GMm}{r}$$



As radius r increases, U decreases



$$\frac{GmM_E}{r^2} = ma_c = \frac{mv_{orbit}^2}{r}.$$

Questions

1. [Easy] A mass weighing 3.0E6 kg moves in an elliptical orbit around a large planet. At the aphelion, the asteroid is 5.0E5 km away, traveling at 20 km/s. At perihelion, the asteroid is 2.5E5 km away. What is the velocity of the asteroid at perihelion?

2. [Medium] A satellite orbits an asteroid with a velocity of 1500 m/s and radius of 2.0 km. What is the mass of the asteroid?

3. [Hard] A satellite with mass m is placed into a circular orbit r = R above the earth's surface, which has a mass of M. What is the total mechanical energy of the satellite at that altitude?

Solutions

1) Use conservation of momentum!

$$L_i = L_f$$
$$R_a \times mv_a = R_b \times mv_b$$
$$(5.0E5\ km)(20\ km/s) = (5.0E5\ km)(v_b)$$
$$(5.0E5\ km)(20\ km/s)/(2.5E5\ km) = (v_b)$$
$$v_b = 40 km/s$$

2)
$$F_g = F_c$$
$$\frac{GMm}{r^2} = m\frac{v^2}{r}$$
$$\mathrm{M} = \frac{v^2 r}{G}$$
$$\mathrm{M} = \frac{(1500\ m/s)^2 (2*10^3 m)}{6.67*10^{-11}} = 4.50*10^{16}\ \mathrm{kg}$$

3)

$$F_g = F_c$$
$$\frac{GMm}{r^2} = m\frac{v^2}{r}$$
$$\mathrm{v} = \sqrt{\frac{GM}{r}}$$
$$\mathrm{E} = \mathrm{K} + \mathrm{U}$$
$$\mathrm{E} = \frac{1}{2}mv^2 + -\mathrm{G}\frac{Mm}{r}$$