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AP Physics Review Sheet

Unit 11: Angular Momentum

Unit 11 in AP Physics focuses on Angular Momentum when analyzing situations like rolling objects with translational and rotational motion. Similar to linear momentum, angular momentum examines rotating and spinning systems. The unit covers how to calculate angular momentum using an object's moment of inertia and angular velocity, and how the momentum changes when torque is applied.

Major Topics:

I. Angular momentum
   A. Conservation of angular momentum
II. Angular impulse
III. Torque
IV. Rotational motion
V. Moment of Inertia

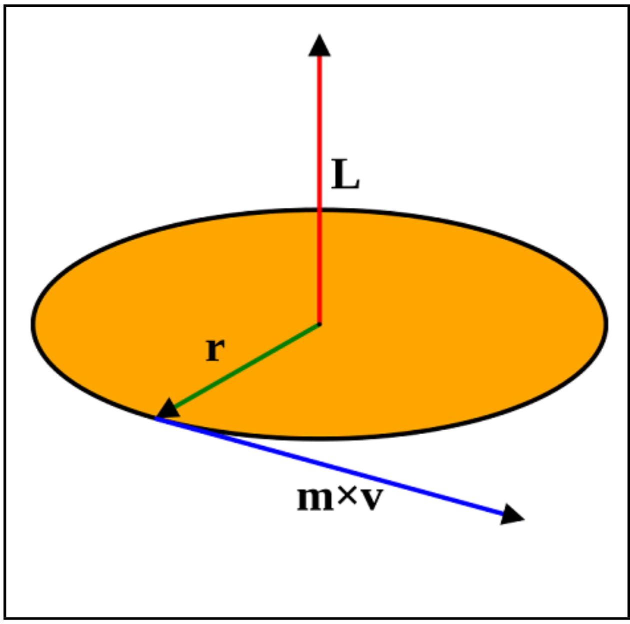
Key Vocabulary:

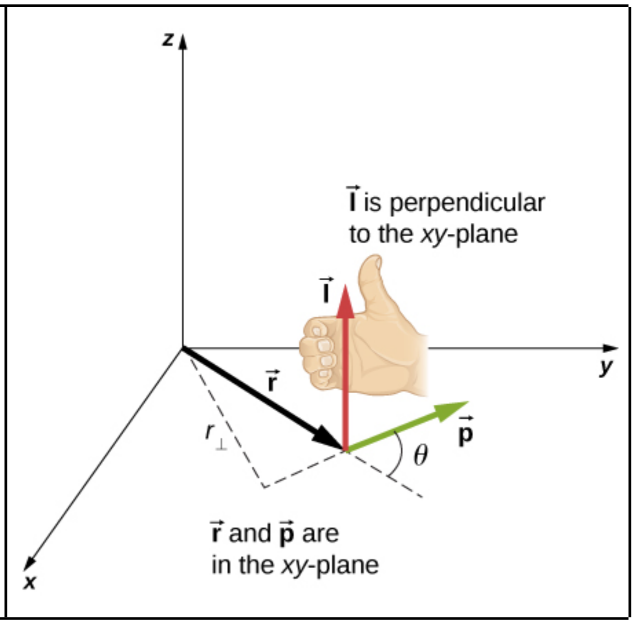
a. *Angular momentum:* the rotational equivalent to linear momentum
b. *Moment of Inertia*: the measurement of an object's resistance to rotational change in motion
c. *Angular Velocity: the* rate at an object spins, measured in radians/sec
d. *Torque:* measurement of the rotational force applied to an object
e. *Axis of rotation:* the axis line around which an object rotates
f. *Angular impulse:* product of the torque and the time in which it acts on the object

Useful Equations:

Angular Momentum: $L = I\omega$
Angular momentum (particle motion): L = r x p = rmvsin(θ)
Torque: $\tau = I\alpha$
Angular impulse: $\tau\Delta t = \Delta L$
Conservation of angular momentum (with no outside torque acting upon the object)
$L_i = L_f$
$I_i\omega_i = I_f\omega_f$
gt
Moment of Inertia formulas depending on object shape and distribution:
Solid disk/cylinder: $I = \frac{1}{2}MR^2$
Solid sphere: $I = \frac{2}{5}MR^2$

Hoop: I = MR^2





Practice problems:
(Easy):
If the boy on the bicycle in the preceding problem accelerates from rest to a speed of 10.0 m/s in 10.0 s, what is the angular acceleration of the tires?
(Medium):
A boulder of mass 20 kg and radius 20 cm rolls down a hill 15 m high from rest. What is its angular momentum when it is halfway down the hill?
(Hard):
a) Calculate the angular momentum of an ice skater spinning at 6.00 rev/s given his moment of inertia is 0.400kg$\cdot m^2$ (b) He reduces his rate of spin (his angular velocity) by extending his arms and increasing his moment of inertia. Find the value of his moment of inertia if his angular velocity decreases to 1.25 rev/s. (c) Suppose instead he keeps his arms in and allows friction of the ice to slow him to 3.00 rev/s. What average torque was exerted if this takes 15.0 s?

<u>Work:</u>

(Easy) :

Easy:

write down what we know:

$V_0 = 0$ m/s (since the bicycle starts from rest)

$V_f = 10.0$ m/s

Time = 10.0 sec

find the acceleration (angular) of the tires:

Remember

$v = r\omega$

$a = r\alpha$

$$a = \frac{V - V_0}{t} = \frac{10\,m/s - 0}{10.0s} = 1.0\,m/s^2$$

$$\alpha = \frac{a}{r} \longrightarrow \alpha = \frac{1.0\,m/s^2}{0.35m} = \boxed{2.86\,rad/s^2}$$

(Medium) :

Medium:

Knowns:

mass = 20kg

radius: R = 0.20m

height of hill = 15m

Boulder starts from rest

→ use conservation of energy, find angular velocity (ω), and compute angular momentum (L)

Initial Potential Energy = Translational KE + Rotational KE

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Solid sphere moment of Inertia:

$$I = \frac{2}{5}mR^2$$

and relationship: $\omega = \frac{v}{R}$

a) halfway point:

half of hill = 7.5m

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$mg(7.5m) = \frac{1}{2}mv^2 + \frac{1}{2}\cdot\left(\frac{2}{5}mR^2\right)\cdot\left(\frac{v}{R}\right)^2$

Simplify:

$$mg(7.5) = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \left(\frac{1}{2}+\frac{1}{5}\right)mv^2 = \frac{7}{10}mv^2$$

$$g(7.5) = \frac{7}{10}v^2 \Rightarrow v^2 = \frac{10}{7}g(7.5)$$

$$v^2 = \frac{10}{7}\cdot 9.8\cdot 7.5 = \frac{10}{7}\cdot 73.5 = 105 \Rightarrow v = \sqrt{105} \approx 10.25\,m/s$$

$$\omega = \frac{v}{R} = \frac{10.25}{0.20} = 51.25\,rad/s$$

Angular Momentum

$L = I\omega$

$I = \frac{2}{5}mR^2 = \frac{2}{5}(20)(0.2)$

$= \left(\frac{2}{5}\right)(20)(0.04) =$

0.32kg

$L = (0.32)(51.25) =$

16.4 kg-m²/sec

(Hard) :

Hard:

Known:

$\omega_1 = 6.00 rev/sec$

$I = 0.400 kg \cdot m^2$

$\omega_2 = 1.25 rev/sec \rightarrow$ part b

$\omega_3 = 3.00 rev/sec \rightarrow$ part c

$t = 15.0 sec \rightarrow$ time to slow down in part (c)

a) calc. angular momentum at 6.00 rev/s

$L = I \cdot \omega$

$\hookrightarrow$ convert to rad/s $\rightarrow \omega = 6.00 \frac{rev}{s} \times \frac{2\pi rad}{rev} = 37.7 rad/s$

$L = (0.400)(37.70) = \boxed{15.08 \ kg \cdot m^2}$

b) find new moment of inertia if $\omega = 1.25 rev/s$

no external torque, so angular momentum is conserved.

$\omega_1 = 6.00 \cdot 2\pi = 12\pi = 37.70 rad/s$ $\quad L_1 = L_2 = I_1 \cdot \omega_1 = I_2 \cdot \omega_2$

$\omega_2 = 1.25 \cdot 2\pi = 2.5\pi \approx 7.85 rad/s$

$(0.400)(37.70) = I_2 \cdot 7.85 = I_2 = \frac{15.08}{7.85} \approx \boxed{1.92 kg \cdot m^2}$

c) average torque if skater slows to 3.00 rev/s in 15.00 sec.

$\tau = I \cdot \alpha$

$\alpha = \frac{\omega_f - \omega_i}{t} \longrightarrow \alpha = \frac{18.85 - 37.70}{15.0} = \frac{-18.85}{15} \approx -1.26 rad/s^2$

$\omega_i = 37.70 rad/s$

$\omega_f = 3.00 \cdot 2\pi = 18.85 rad/s$

Find Torque:

$\tau = (0.400)(-1.26) = \boxed{0.504 \ N \cdot m}$ negative sign meant opposing motion.