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Chapter 13: Universal Gravitation

Background: This chapter explores the history of gravitation, Newton’s law of universal gravitation, and how to apply it to calculate gravitational force.

Major Topics:

1. Newton's Law of Universal Gravitation
2. Gravitation Near Earth's Surface
3. Gravitational Potential Energy and Total Energy
4. Kepler's Laws
5. Energy of a Planet-Satellite System

Vocabulary:
Newton’s law of gravitation: every mass attracts every other mass with a force proportional to the product of their masses, inversely proportional to the square of the distance between them, and with direction along the line connecting the center of mass of each
Escape velocity: initial velocity an object needs to escape the gravitational pull of another; it is more accurately defined as the velocity of an object with zero total mechanical energy
Gravitational field: vector field that surrounds the mass creating the field; the field is represented by field lines, in which the direction of the field is tangent to the lines, and the magnitude (or field strength) is inversely proportional to the spacing of the lines; other masses respond to this field
Orbital period: time required for a satellite to complete one orbit
Orbital speed: speed of a satellite in a circular orbit; it can be also be used for the instantaneous speed for noncircular orbits in which the speed is not constant
Aphelion: farthest point from the Sun of an orbiting body; the corresponding term for the Moon’s farthest point from Earth is the apogee
Perihelion: point of closest approach to the Sun of an orbiting body; the corresponding term for the Moon’s closest approach to Earth is the perigee

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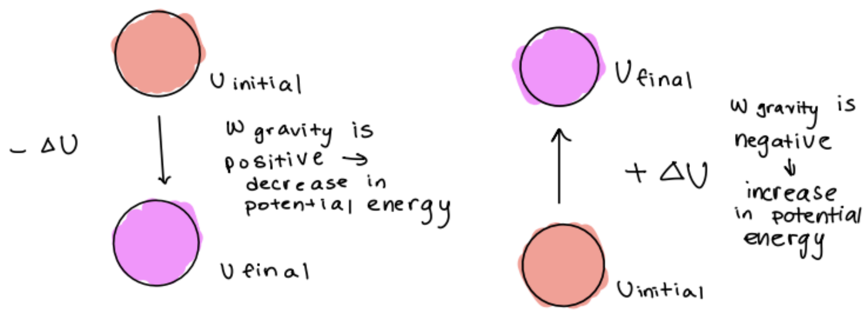
Potential Energy and Work done by the force of gravity

$$W_{gravity} = U_i - U_f = -\Delta U$$

$$\Delta U = U_f - U_i = -W_{grav.} = -\int_{x_i}^{x_f} F_g \cdot dx$$



Formulas:

Newton's Law of gravitation	$F = G\frac{m_1 m_2}{r^2}$
Acceleration due to gravity at Earth's surface	$g = G\frac{M_{Earth}}{r^2}$
Escape velocity	$V_{esc} = \sqrt{\frac{2GM}{R}}$
Total mechanical energy for a planet-satellite system	$E_{total} = U + K = \frac{-GMm}{r} + \frac{1}{2}mv^2$

Problems:

1. Estimate the gravitational force between two sumo wrestlers, with masses 220 kg and 240 kg, when they are embraced and their centers are 1.2 m apart.
2. Find the escape speed of a projectile from the surface of Jupiter.
3. Show that the areal velocity for a circular orbit of radius r about a mass M is

   $$\frac{\Delta A}{\Delta t} = \frac{1}{2}\sqrt{GMr}.$$

   Does your expression give the correct value for Earth's areal velocity about the Sun?

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Solutions

1. (easy)

Gravitational force between 2 sumo wrestlers

$m_1 = 220\,kg \quad m_2 = 240\,kg$

$r = 1.2\,m$

Newton's Law of Universal Gravitation

$$F = G\frac{m_1 m_2}{r^2}$$

$$G = 6.674 \times 10^{-11}\ N \cdot m^2/kg^2$$

$$F = (6.674\,e{-11})\frac{(220)(240)}{(1.2)^2}$$

$$= \boxed{2.45 \times 10^{-6}\,N}$$

2. (medium) Find escape speed of a projectile from the surface of Jupiter

$$V_{esc} = \sqrt{\frac{2GM}{R}}$$

$G = 6.674\,e{-11}\ Nm^2/kg^2$

$M = 1.898\,e27\,kg$ (mass of Jupiter found from the internet)

$R = 6.9911\,e7\,m$ (radius of Jupiter)

$$V_{esc} = \sqrt{\frac{2(6.674\,e{-11})(1.898\,e27)}{6.9911\,e7}}$$

$$= 60{,}230\,m/s = \boxed{60.2\,km/s}$$

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3. (hard)

Areal velocity: rate at which area is swept out by the position vector of an orbiting body (from internet)

Areal velocity $= \frac{1}{2} rv$

r = orbital radius

V = orbital speed

Orbital speed

use $\frac{GMm}{r} = \frac{mv^2}{r}$ isolate $\underline{v}$

gravitational force — centripetal force

$V = \sqrt{\frac{GM}{r}}$ — plug back in areal velocity formula

Areal V. $= \frac{1}{2} r \cdot V$

$= \frac{1}{2} r \sqrt{\frac{GM}{r}} = \frac{1}{2}\sqrt{GMr}$

Areal velocity $= \frac{1}{2}\sqrt{GMr}$

use this to calculate earth's areal velocity

areal velocity $= \frac{1}{2}\sqrt{(6.674e{-11})(1.989e30)(1.496e11)} = \boxed{2.23\times10^{15}\ m^2/s}$

$M = 1.989e30$ kg (mass of Sun)

$r = 1.496e11$ (distance between Earth & Sun)

$G = 6.674e{-11}$ Nm^2/kg^2

Doest it match Earth's real areal velocity?

Area of orbit $= \pi r^2 = \pi(1.496e11)^2 = 7.03e22 m^2$

$t = 3.156\times10^7 s$ (time it takes for Earth to sweep out a full orbital area)

Areal velocity: $\frac{Area}{Time} = \frac{7.03\times10^{22}}{3.156\times10^7} = \boxed{2.23\times10^{15}\ m^2/s}$

$r = 1.496e11 m$ (radius of earth)