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# Chapter 4 Motion in Two Dimensions

**Background**: This chapter explores two types of motion, precisely, projectile motion and circular motion. It also explores relative motion.

**Major Topics**: Projectile motion, uniform circular motion, relatie motion in one and two dimensions

**Vocabulary(from the textbook)**:

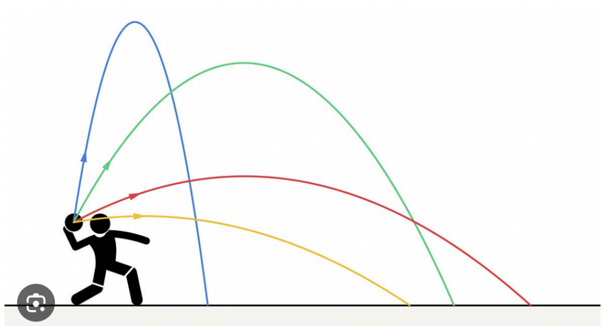
Centripetal acceleration: component of acceleration of an object moving in a circle that is directed radially inward toward the center of the circle

Projectile motion: motion of an object subject only to the acceleration of gravity

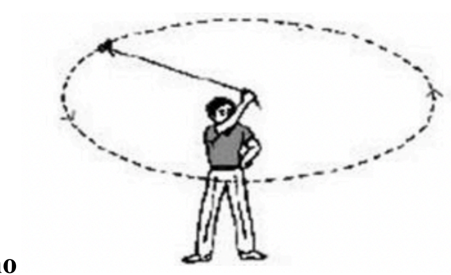
Tangential acceleration: magnitude of which is the time rate of change of speed. Its direction is tangent to the circle.

Relative velocity: velocity of an object as observed from a particular reference frame, or the velocity of one reference frame with respect to another reference frame

Projectile Motion:



Uniform Circular Motion:



**Formulas You Should Know**

| | |
|---|---|
| $a_c = \frac{V^2}{C}$ | $V_x = V_{x0} + a_x t$ |
| $x = x_0 + V_{x0} t + \frac{1}{2} a_x t^2$ | |
| $\Delta x = \int V_x(t)\,dt$ | $V_x^2 = V_{x0}^2 + 2a_x(x - x_0)$ |

Problems

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1. (Easy) A Formula One race car is traveling at 89.0 m/s along a straight track enters a turn on the race track with radius of curvature of 200.0 m. What centripetal acceleration must the car have to stay on the track?
2. (Medium) A rock is thrown off a cliff at an angle of 53 degrees with respect to the horizontal. The cliff is 100 m high. The initial speed of the rock is 30 m/s. (a) How high above the edge of the cliff does the rock rise? (b) How far has it moved horizontally when it is at maximum altitude? (c) How long after the release does it hit the ground? (d) What is the range of the rock? (e) What are the horizontal and vertical positions of the rock relative to the edge of the cliff at t = 2.0 s, t = 4.0 s, and t = 6.0 s?
3. (Hard) World's Longest Par 3. The tee of the world's longest par 3 sits atop South Africa's Hanglip Mountain at 400.0 m above the green and can only be reached by helicopter. The horizontal distance to the green is 359.0 m. Neglect air resistance and answer the following questions. (a) If a golfer launches a shot that is with respect to the horizontal, what initial velocity must she give the ball? (b) What is the time to reach the green?

Solutions

1. (easy)

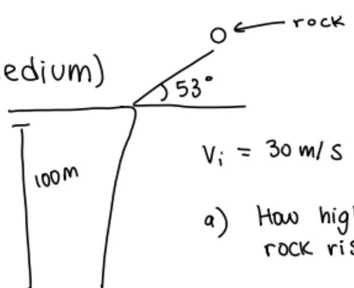
Race car velocity = 89.0 m/s

radius of curvature of 200.0 m

$a_c = ?$

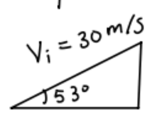
$a_c = \frac{v^2}{r} = \frac{(89)^2}{200} = \boxed{39.605\,m/s^2}$

2. (medium)



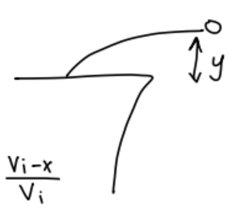

$V_i = 30\,m/s$

a) How high above cliff does rock rise?




$V_{i-y} =$

$V_{i-x} =$




$\sin 53 = \frac{V_{i-y}}{V_i}$ $\qquad$ $\cos 53 = \frac{V_{i-x}}{V_i}$

$V_i \sin 53 = V_{i-y}$ $\qquad$ $V_i \cos 53 = V_{i-x}$

$(30)(\sin 53) = V_{i-y}$ $\qquad$ $(30)(\cos 53) = 18.05\,m/s$

$V_{i-y} = 23.96\,m/s$ $\qquad$ $V_{i-x} = 18.05\,m/s$

$V_{f-y}^2 = V_{i-y}^2 + 2a(\Delta y)$

$0^2 = (23.96)^2 + 2(-9.8)(\Delta y)$

$\boxed{\Delta y = 29.29\,m}$

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b) How far has it moved horizontally when it is at maximum altitude?

$$V_{f\text{-}y} = V_{i\text{-}y} + at$$

$$0 = 23.96 + (-9.8)(t)$$

$$t = 2.44\ s$$

$$X = X_0 + V_{i\text{-}x}(t) + \cancel{\tfrac{1}{2}at^2}$$

no acceleration horizontally

$$X - X_0 = V_{i\text{-}x}(t)$$

$$\Delta x = (18.05)(2.44)$$

$$\boxed{\Delta X = 44.04\ m}$$

c) When does the rock hit the ground?

$$y = y_0 + V_{i\text{-}y}(t) + \tfrac{1}{2}at^2$$

$y_f = 0\ m$

$y_i = 100\ m$

$V_{i\text{-}y} = 23.96\ m/s$

$$0 = 100 + 23.96(t) + \tfrac{1}{2}(-9.8)(t)^2$$

use quadratic equation

$$t = \frac{-23.96 \pm \sqrt{(23.96)^2 + 2(9.8)(100)}}{-9.8}$$

$$\boxed{t = 7.58\ s}$$

d)

$$V_x = \frac{d}{t}$$

$$V_x(t) = d$$

$$(18.05)(7.58) = \boxed{136.78\ m}$$

e) Position at $t = 2s, 4s, 6s$

$$x = V_{0x}(t)$$

$$y = 100 + V_{i\text{-}y}(t) + \tfrac{1}{2}(a)(t)^2$$

$t = 2\ s$

$$x = (18.05)(2) = \boxed{36.11\ m}$$

$$y = 100 + 23.96(2) + \tfrac{1}{2}(-9.8)(2)^2 = \boxed{128.30\ m}$$

$t = 4\ s$

Perform same calculations

$$\boxed{x = 72.22\ m}$$

$$\boxed{y = 117.36\ m}$$

$t = 6\ s$

$$\boxed{x = 108.33\ m} \quad \boxed{y = 67.17\ m}$$

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3. (hard)

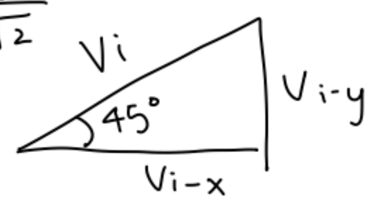
$y_0 =$ 400m above the green

$x = 359$m horizontally

a) initial speed $V_0$, given that launch angle is 45°

b) time to reach green

$V_{i-x} = V_i \cos(45) = \frac{V_i}{\sqrt{2}}$




$V_{i-y} = V_i \sin(45)$

$= \frac{V_i}{\sqrt{2}}$

$y = y_i + V_{i-y} t + \frac{1}{2}(a)(t^2)$

$y_f = 0$

$0 = 400 + \frac{V_i}{\sqrt{2}}(t) + \frac{1}{2}(-9.8)(t)^2$

$y_i = 400$

plugin

$V_{i-y} = \frac{V_i}{\sqrt{2}}$

$x = V_{i-x} t = \frac{V_i}{\sqrt{2}} t = 359$

$V_i = \frac{359\sqrt{2}}{t}$

$x = 359$m

$0 = 400 + \frac{\frac{359\sqrt{2}}{t}}{\sqrt{2}}(t) + \frac{1}{2}(-9.8)(t)^2$

quadratic equation

$\boxed{t = 12.44s}$

time to reach green

$V_i = ?$

$V_i = \frac{359\sqrt{2}}{t} = \frac{359\sqrt{2}}{12.44} = \boxed{40.8 m/s}$