AP Review Sheets - Theodore Kratter
With assistance from the notes on OpenStax Physics Textbook and crashwhite.com

UNIVERSAL GRAVITATION:

I've always wondered why the earth stays in orbit around the sun. Have you? Well no fear! Universal Gravitation is here! Universal gravitation describes how masses attract one another with a force proportional to their mass, and inversely proportional to the square of their distance apart. I think it's a very interesting bridge between smaller scale force analysis physics (astrophysics) on a bigger scale.

Major Topics: Gravitational Potential Energy, Newton's Law of Universal Gravitation, Elliptical Orbits, Energy of a Circular Orbit, Gravity and Radius, Kepler's Laws

Key Vocab: Centripetal Force (produced by gravity), Gravitational Fields, Escape Velocity, Inverse-Square Law, Gravitational constant (but it's big G), Period of Rotation

$$F = G\frac{m_1 m_2}{r^2}$$

(use Newton's Universal Gravitation law when having to describe gravitational attraction between planets, objects in space, separated by some distance, r)

$$G = 6.674 \times e{-11}\,\mathrm{N}\frac{\mathrm{m}^2}{\mathrm{kg}^2}$$

(use this value of G, for universal gravitation, not to be confused with little g, = 9.81m/s^2)

$$U = \frac{-GM_m}{r}$$

(M represents one mass, "m" represents a different mass. This equation gives gravitational potential energy of a mass at r away from a different mass). Dividing out the little "m" from both sides, gives the strength of the gravitational field (g). This is a similar process to what we did in class that let us go from Coulomb's law to Electrical field strength.

$$U_{\text{total}} = \sum U_i$$

(important to note that you can sum potential energies)

$$F_g = mg$$

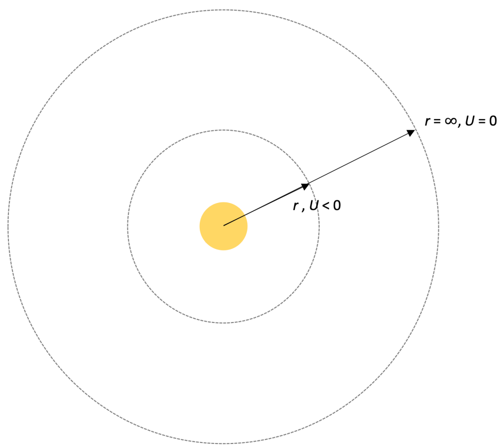
(don't forget this basic equation either! We know a lot more about what little g is now, though. It is 9.81 at the surface of the earth…, but it could be other values based on masses, distance between the masses, etc.)

$$\Delta U = -W_{\text{gravity}} = -\int_{x_i}^{x_f} F_g \cdot dx$$

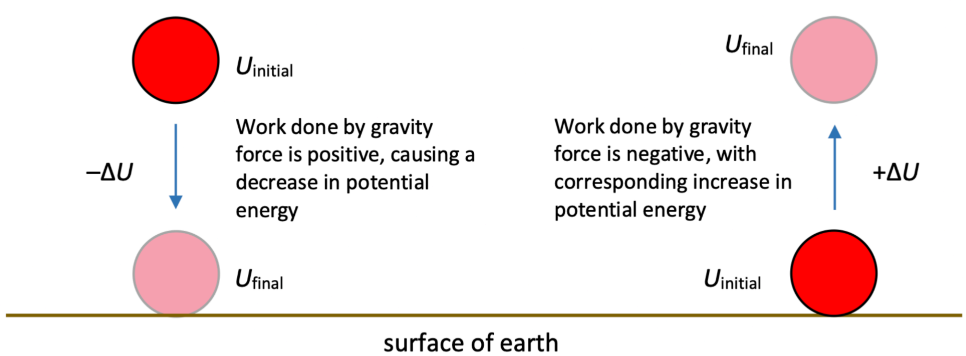
(This connects potential energy to work done by the gravitational force, very helpful for analyzing how energy changes when a mass moves in a g field)

$$v = \sqrt{\frac{GM}{r}} \quad v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$$

(Orbital velocity and escape velocity respectively. Pretty self explanatory)



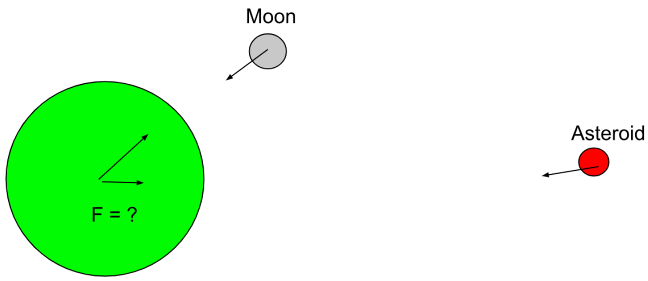
Visualizing how potential energy changes at varying “r” from a center point. Infinitely far away it has zero potential E and it loses potential E as it goes closer to the center, so the potential has a negative value. *Diagram credit to crashwhite.com and analysis of diagram inspired by crashwhite.com’s description of the diagram.*



All diagram credit to crashwhite.com

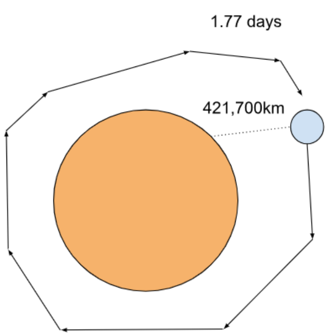
ALL PROBLEM CREDITS GO TO OPENSTAX PHYSICS TEXTBOOK-
(https://openstax.org/details/books/college-physics-2e)

PROBLEM #1: EASY



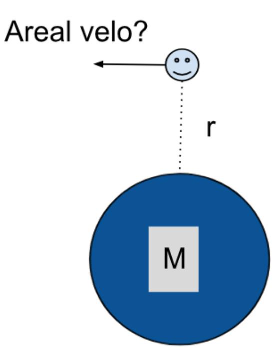
Asteroid Toutatis passed near Earth in 2006 at four times the distance to our Moon. This was the closest approach we will have until 2060. If it has a mass of 5.0*10^13 kg, what force did it exert on earth at its closest approach?

PROBLEM #2: MEDIUM



Find the mass of Jupiter based on the fact that Io, its innermost moon, has an average orbital radius of 421,700 km and a period of 1.77 days.

PROBLEM #3: EXPERT MODE ACTIVATED 😎 A.K.A “DEWIER MODE ACTIVATED” (Jake Huang will understand that minecraft reference)



Show that the areal velocity for a circular orbit of radius, r, about a mass, M, is $\frac{\Delta A}{\Delta t} = \frac{1}{2}\sqrt{GMr}$

Does your expression give the correct value for Earth’s areal velocity about the Sun?

EASY:

ANSWERS: ASTEROID TOUTATIS

$F = G\frac{m_1 m_2}{r^2}$ online distance btwn Earth & moon is 3.84×10^8 m

mass of E = 5.97×10^{24}

mass of asteroid is given

$4 \cdot D_{E-M} = D_{E-TOUTATIS} \rightarrow 3.84 \times 10^8 \cdot 4 = 1.54 \times 10^9$ m

plug into Newtons law $\rightarrow F = \frac{(6.674 \times 10^{-11})(5.0 \times 10^{13})(5.972 \times 10^{24})}{(1.54 \times 10^9)^2}$

$F = 8.44 \times 10^9$ N

MEDIUM:

JUPITERS MOON SOLUTION

$1.77\,\text{day}\left(\frac{86400\,\text{s}}{\text{day}}\right) = 1.53 \times 10^5$ s

$F_{gravity} = F_{centripetal}$ for orbiting moon

$\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow GM = rv^2 \Rightarrow$ knowing that $v = \frac{2\pi r}{T}$

$GM = r\left(\frac{2\pi r}{T}\right)^2 \Rightarrow \frac{4\pi^2 r^3}{T^2} = GM$ so... plug in $\frac{4\pi^2(4.217 \times 10^8\,\text{m})^3}{(1.529 \times 10^5)^2} = (6.67 \times 10^{-11})M$

$M = 1.90 \times 10^{27}$ kg

HARD:

SUPER EPIC DEWIER GAMER MODE PROBLEM SOLUTION

This problem might be a bit out of AP level. Its cool regardless.

$\frac{\Delta A}{\Delta t} = \frac{1}{2}\sqrt{GMr}$ $\left(\frac{dA}{dt}\right)$... $\Rightarrow$ areal velo defined by $\frac{dA}{dt} = \frac{1}{2}rv$ ← the rate at which area is like moved across by radius

Equation given $\rightarrow v = \sqrt{\frac{GM}{r}}$

plugging in... $\frac{dA}{dt} = \frac{1}{2}r\left(\sqrt{\frac{GM}{r}}\right) \Rightarrow \frac{1}{2}\sqrt{GMr}$ which matches!

Double checking with Earth & Sun, plugging in to solved equation gives google values of mass.

$\frac{\Delta A}{\Delta t} = \frac{1}{2}\sqrt{(6.67 \times 10^{-11})(1.496 \times 10^{11})(1.989 \times 10^{30})}$

$\frac{\Delta A}{\Delta t} = 2.2 \times 10^{15}\,\text{m}^2/\text{s}$