AP Review Sheets - Theodore Kratter
With assistance from the notes on OpenStax Physics Textbook and crashwhite.com

MOTION IN TWO DIMENSIONS:

This unit covers motion in two dimensions. Some notable examples include projectile motion, centrifugal force, uniform circular motion, and relative velocity. This unit is really helpful for understanding and using equations to describe the motion of objects. It also describes how objects will move relative to one another, in different reference frames.

Major Topics: Projectile Motion, Relative Velocity, Tangential and Radial Acceleration, Parabolic Movement, Circular Movement

Key Vocab: Vector Components, Reference Frame, Tangential and Centripetal Acceleration, Projectile, Trajectory

$$x = vt \qquad y = vt - \frac{1}{2}at^2$$

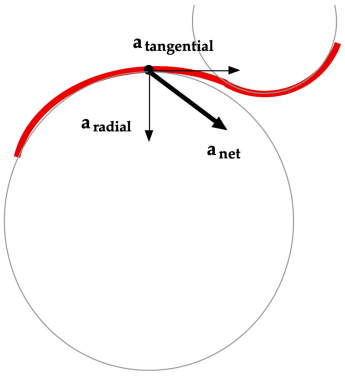
(sub a = g when dealing with the y component analysis. These equations are especially helpful for projectile motion)

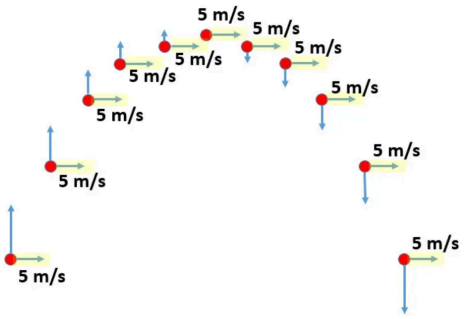
$$a_c = \frac{v^2}{r} \qquad F_c = \frac{mv^2}{r} \qquad a_t = \frac{dv}{dt}$$

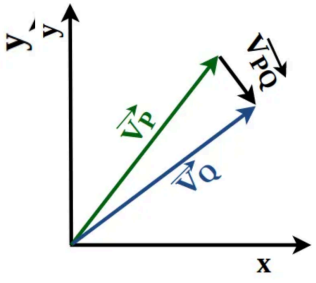
(these equations are helpful for circular motion)

$$\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$$

(VELOCITY OF A RELATIVE TO B – use this equation for relative motion. Remember to split it up into components if necessary)



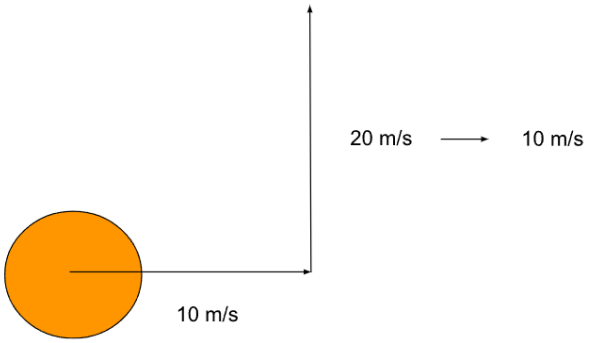




(Craswhite.com) (stickmanphysics.com) (geeksforgeeks.com)

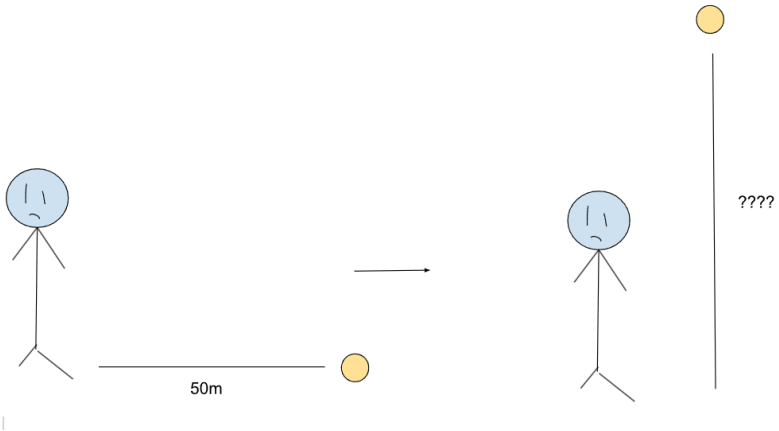
ALL PROBLEM CREDITS GO TO OPENSTAX PHYSICS TEXTBOOK-
(https://openstax.org/details/books/college-physics-2e)

PROBLEM #1: EASY



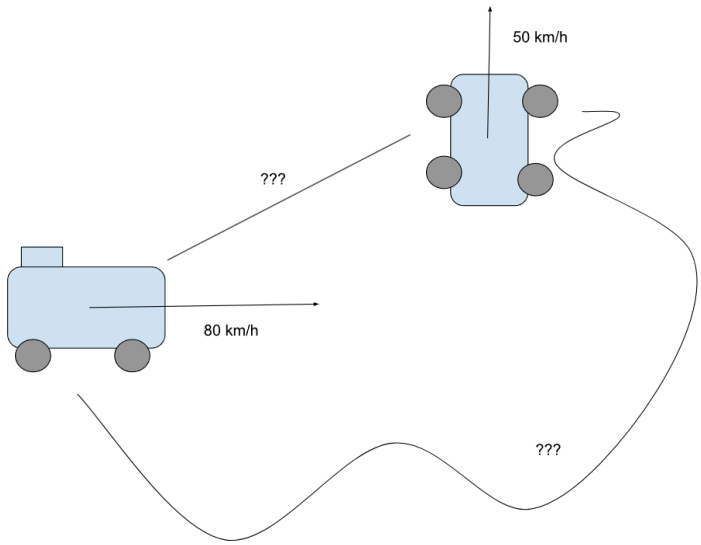
The acceleration of a particle is a constant. At t = 0 the velocity of the particle is $(10\hat{i}+20\hat{j})$m/s. At t = 4 s the velocity is $10\hat{j}$m/s. (a) What is the particle's acceleration? (b) How do the position and velocity vary with time? Assume the particle is initially at the origin.

PROBLEM #2: MEDIUM



The maximum horizontal distance a boy can throw a ball is 50 m. Assume he can throw with the same initial speed at all angles. How high does he throw the ball when he throws it straight upward?

PROBLEM #3: EXPERT MODE ACTIVATED 😎



A truck is traveling east at 80 km/h. At an intersection 32 km ahead, a car is traveling north at 50 km/h. (a) How long after this moment will the vehicles be closest to each other? (b) How far apart will they be at that point?

PARTICLE ACCELERATION SOLUTIONS

$V_0 = (10\hat{i} + 20\hat{j})$ m/s

$V(4) = (0\hat{i} + 10\hat{j})$ m/s

a) $v(t) = V_0 + at$ — using a plugging into equation. solve for a ($\hat{i}$ & $\hat{j}$) directio

$V(4) = V_0 + 4a$

$(10\hat{i} + 20\hat{j}) + 4a = (0\hat{i} + 10\hat{j})$

$\hat{i}$: $0 = 10 + 4a_x = \frac{-10}{4} = -2.5$ m/s

$\hat{j}$: $10 = 20 + 4a_y = \frac{-10}{4} = -2.5$ m/s

$\vec{a} = (-2.5\hat{i} - 2.5\hat{j})$ m/s²

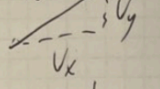
b) $v(t) = V_0 + at \Rightarrow (10 - 2.5t)\hat{i} + (20 - 2.5)\hat{j}$

position $(t) = d_0 + V_0 t + \frac{1}{2}at^2$

position $(t) = (10t - \frac{1}{2} \cdot 2.5t^2)\hat{i} + (20t - \frac{1}{2} \cdot 2.5t^2)\hat{j}$

position $(t) = (10t - 1.25t^2)\hat{i} + (20t - 1.25t^2)\hat{j}$

BOX SLIDE PROBLEM SOLUTIONS



so.. 45, 45 is max. Assuming 45° for initial to maximize.

$V_x = V_y = \frac{V_0}{\sqrt{2}}$

$d = V_i t \rightarrow \frac{V_0}{\sqrt{2}} \cdot t_{total} = 50$ m

$t_{total} = \frac{50\sqrt{2}}{V_0}$

$a = \frac{\Delta V}{\Delta t} \rightarrow t = \frac{V_y}{g} \rightarrow \frac{\frac{V_0}{\sqrt{2}}}{g} = t_{top}$ ← time to reach peak

total Time $= 2 \cdot t_{pp}$ ← so x2 for total

$= \frac{\sqrt{2}\, V_0}{g}$

Set horizontal & vertical elements equal

$$\frac{50\sqrt{2}}{V_0} = \frac{\sqrt{2}}{g} V_0$$

$$50\sqrt{2} = \frac{\sqrt{2} V_0^2}{g}$$

$$V_0^2 = 50 \cdot 9.8 = 490 \rightarrow V_0 = \sqrt{490} \rightarrow \boxed{22.14 \text{ m/s}}$$

$V_f^2 = V_i^2 + 2a\Delta x$

$\Delta y = \frac{V_0^2}{2g}$

$\Delta y = \frac{490}{19.6}$

$\rightarrow \boxed{25 \text{ m}}$

TRUCK SOLUTIONS THEODORE KRATTEN

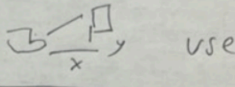
a) Truck: 32 km W → velo 80 km/h

Car: 0 km to intersection → velo 50 km/h

So,

$\boxed{x(t) = 32 - 80t}$, $\boxed{y(t) = 50t}$

b) Distance?



use phythag

$$d = \sqrt{(32-80t)^2 + (50t)^2}$$

↳ as a function of time

$$d^2 = 1024 - 5120t + 6400t^2 + 2500t^2$$

$$d^2 = 1024 - 5120t + 8900t^2$$

$$\frac{1}{2} d^2 = -5120 + 17800t$$

minimize by setting derivative to zero

$$-5120 + 17800t = 0$$

equations from before

$$t = \frac{5120}{17800} \rightarrow \boxed{0.288 \text{ h}}$$

$x(0.288) = 9.0$

$y(0.288) = 14.38$

→ phythag: $\sqrt{9^2 + 14.38^2}$

↳ $\boxed{16.96 \text{ km}}$