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Lab: AP Physics C Mechanics Review Sheets
05/07/2025

Chapter 13: Universal Gravitation

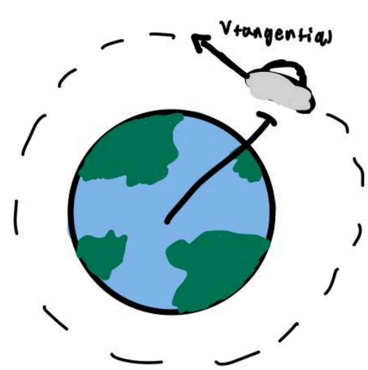
Background/Summary:
This chapter is about what happens to gravity and gravitational force when we're no longer looking at Earth's surface but much higher, into space, and sometimes between two celestial bodies. It also deals with potential energy and orbits in general.

Major Topics Covered:
- Newton's Law of Universal Gravitation
- Gravitational Potential Energy
- Orbits and Energy
- Gravity Force as a Function of Radius

Vocabulary:
- **Newton's Law of Universal Gravitation:** every particle has its own attractive force on other particles that is a product of the objects' masses and inversely proportional to the square of the distance between the objects.
- **Universal Gravitation Constant:** the G value that can relate the force of gravity to Newton's Law of Universal Gravitation.
- **Geosynchronous Orbit:** an orbit that matches Earth's rotation.
- **Escape Velocity:** velocity needed to escape an object's gravity field.
- **Elliptical Orbit:** an oval-shaped orbit
    a. **Perihelion**: point at which orbiting object is closest to what it's orbiting around.
    b. **Aphelion**: point at which orbiting object is furthest to what it's orbiting around.
- **Gravity Force:** the force that a mass feels due to the presence of another mass nearby.
- **Gravity Field:** a field created around a mass that will produce a force on any object in the field.
- **Gravitational Potential Energy:** energy that is created due to the force of gravity. This is calculated with different equations based on if an object is near the surface of earth or not.
- **Satellite:** any body that is in orbit around another.

Key Equations:

Equation	**Description/*When to Use***
$F_g = \frac{Gm_1m_2}{r^2}$	Newton's Law of Universal Gravitation *Use: To calculate gravitational force between two point masses*

$W_{gravity} = U_i - U_f = -\ \Delta U$ $\Delta U = U_f - U_i = -W_{gravity} = -\int_{xi}^{xf} F_g \cdot dx$	Gravitational Potential Energy *Use: To calculate the energy change as an object moves between two radial distances (e.g., between two orbits, or falling toward a planet/star)*
$U_f - U_i = \int_{r_i}^{r_f} F_g \cdot dr$ $U_f - U_i = \int_{r_i}^{r_f} F_g \cdot \quad -\frac{GM_{earth}m}{r^2} \cdot dr$ $U_f - U_i = \quad -\frac{GM_{earth}m}{r_f - r_i}$ Let $U_i = 0$ at $r_i = \infty$ $U = -\frac{GM_{earth}m}{r}$	Potential Energy for a Mass in a Gravity Field where the earth is one of the masses *Use: For potential energy at a distance r from the center of a mass M*
$U_{total} = \Sigma U_i$ For three masses: $U_{total} = (-\frac{Gm_1m_2}{r_{1,2}}) + (-\frac{Gm_2m_3}{r_{2,3}}) + (-\frac{Gm_1m_3}{r_{1,3}})$	Gravitational Potential Energy for Multiple Masses *Use: To calculate the total gravitational potential energy of a system with multiple masses interacting with each other, such as a group of planets, stars, or satellites, by summing the potential energy for each unique pair of masses.*
$E_{total} = U + K$ $E_{total} = \frac{-GMm}{r} + \frac{1}{2}mv^2$	Energy of a Planet-Satellite System *Use: To calculate the total mechanical energy of a satellite in orbit and determine if it is bound, changing orbits, or escaping.*
$F_c = F_g$ $\frac{mv^2}{r} = \frac{GMm}{r^2}$ $(\frac{r}{2})(\frac{mv^2}{r}) = (\frac{GMm}{r^2})(\frac{r}{2})$ $\frac{1}{2}mv^2 = (\frac{1}{2})(\frac{GMm}{r})$ $K = -\frac{1}{2}U$ $E_{total} = U + K$ $E_{total} = \frac{1}{2}mv^2 - \frac{GMm}{r}$ $E_{total} = (\frac{1}{2})(\frac{GMm}{r}) - \frac{GMm}{r}$ $E_{total} = -\ (\frac{1}{2})(\frac{GMm}{r})$ $E_{total} = \frac{1}{2}U$, where $U = -\frac{GMm}{r}$	Energy of a bound, circular orbit *Use: To calculate the constant total energy of an object in a stable, circular orbit around a planet or star.* 

Problems in Order of Difficulty (Easy, Medium, & Hard):

1) (a) What is the acceleration due to gravity on the surface of Venus? **(easy)**

$F_{net} = ma$

$F_g = ma$

$\frac{Gm_1\cancel{m_2}}{r^2} = \cancel{m_2} a$

$a = \frac{Gm_1}{r^2}$

$|F_g| = \frac{Gm_1m_2}{r^2}$

$m_1 = M_{venus} = 4.87 \times 10^{24}$ kg

m_2 = any object with an irrelevant mass

$r_{venus} = 6050\text{ km} \times \frac{1\text{ m}}{10^{-3}\text{ km}} = 6.05 \times 10^{6}\text{ m}$

$G = 6.67 \times 10^{-11}\ \frac{N \cdot m^2}{kg^2}$

$a = \frac{(6.67e^{-11})(4.87e^{24})}{(6.05e^{6})^2} = 8.874503108 = \boxed{8.87 m/s^2}$

(b) On the surface of Jupiter? The mass of Jupiter is $1.90 \times 10^{27} kg$ and its radius is $6.99 \times 10^{7} m$. **(easy)**

$a = \frac{Gm_1}{r^2}$ ← same derivation as (a)

$m_1 = M_{jupiter} = 1.90 \times 10^{27}$ kg

$r_{jupiter} = 69{,}911\text{ km} \times \frac{1\text{ m}}{10^{-3}\text{ km}} = 6.99e^{7}\text{ m}$

$a = \frac{(6.67e^{-11})(1.90 \times 10^{27})}{(6.99e^{7})^2}$

$= 25.937319 \quad \boxed{= 25.9 m/s^2}$

2) What is the escape speed of a satellite located at the Moon's orbit around Earth? Assume the Moon is not nearby. **(medium)** ***#31 from openstax University Physics Volume 1 textbook***

$V_{escape} = ?$

Final position is infinitely far from Earth, so K_f & $U_f = 0$

$K_i + U_i = K_f + U_f$ conservation of mechanical energy

$\frac{1}{2} m_2 (V_{esc})^2 + -\frac{G m_1 m_2}{r} = 0 + 0$

$m_1 = $ Earth $= 5.97e^{24}$ kg

$m_2 = $ the satellite

$G = $ gravitation constant

$r = $ radius

$\left(\frac{1}{2} m_2 (V_{esc})^2 = \frac{G m_1 m_2}{r} \right) 2$

$\frac{\cancel{m_2} (V_{esc})^2}{\cancel{m_2}} = \frac{\frac{2 G m_1 \cancel{m_2}}{r}}{\cancel{m_2}}$

$\sqrt{(V_{esc})^2} = \sqrt{\frac{2 G m_1}{r}}$

$V_{esc} = \sqrt{\frac{2 G m_1}{r}} = \sqrt{\frac{2(6.67e^{-11})(5.97e^{24})}{(384{,}400{,}000)}} \approx \sqrt{2071795.005}$

$= 1439.373129$

$\boxed{\approx 1440 \text{ m/s}}$

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3) In order to keep a small satellite from drifting into a nearby asteroid, it is placed in orbit with a period of 4.00 hours and radius of 3.0 km. What is the mass of the asteroid? **(hard)**

$F_c = F_g$

$\frac{m_2 (v)^2}{r} = \frac{G m_1 m_2}{r^2}$

$m_2 = $ satellite

$m_1 = $ asteroid

$V = \frac{dx}{dt} = \frac{\Delta x}{\Delta t}$

period: $4.00\text{hrs} \times \frac{60\text{min}}{1\text{hr}} \times \frac{60\text{s}}{1\text{min}} = 14400\text{s}$

circumference: $2\pi r$

$= 2\pi(3000\text{m})$

$= 18849.55592$

$\approx 18900\text{ m}$

$= \frac{18900}{14400} = 1.3125$

$V = 1.31$ m/s

$\left(\frac{m_2 v^2}{r} = \frac{\frac{G m_1 \cancel{m_2}}{r^2}}{\cancel{m_2}} \right) r$

$v^2 = \frac{G m_1}{r}$

$V = \sqrt{\frac{G m_1}{r}}$

$\left(1.31 = \sqrt{\frac{(6.67e^{-11})(m_1)}{3000}} \right)^2$

$\left(1.7161 = \frac{(6.67e^{-11})(m_1)}{3000} \right) 3000$

$\frac{5148.3}{6.67e^{-11}} = \frac{\cancel{6.67e^{-11}} (m_1)}{\cancel{6.67e^{-11}}}$

$\boxed{m_1 = 7.72e^{13} \text{ kg}}$