Ashley Kim
Lab: AP Physics C Mechanics Review Sheets
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Chapter 7: Work and Energy

Background/Summary:

This unit uses the concept of force applied to a mass while it travels some displacement (distance in a direction) and focuses on how forces do work on objects, transferring energy, and causing motion. It introduces **work, kinetic energy**, and the **work-energy theorem**, and leads into the concept of **power**. This unit connects force and motion with energy analysis and lays the **groundwork** for understanding conservation of energy (which is Chapter 8).

Major Topics Covered:

- Definition of Work and the Dot Product
- Work Done by Constant and Varying Forces
    a. Hooke's Law / Work Done by a Spring
- Work-Kinetic Energy Theorem
    a. Kinetic Energy
- Definition and Calculation of Power

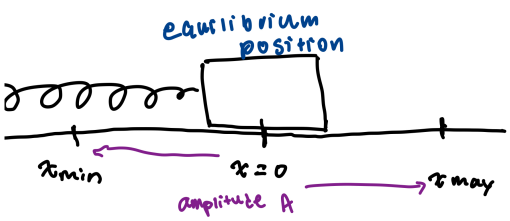
Key Terms/Vocabulary:

- **Work:** a scalar quantity; energy transferred when a force causes displacement; calculated as the dot product of force and displacement vectors **(+** when force and displacement are in the **same direction**, **increasing the object's energy; –** = occurs when force and displacement are in **opposite directions, decreasing the object's energy)**
- **Kinetic Energy:** energy of motion ($K = \frac{1}{2}mv^2$)
- **Work-Energy Theorem:** the net work done on an object **equals** its change in kinetic energy
- **Power:** the rate at which work is done or energy is transferred ($P = \frac{W}{t}$)

Key Equations:

Equation	**Description**	**When to Use**
$W = F_x x = Fx\cos\theta = F \cdot x$	Work done by a constant force at angle θ (joules)	When the force is applied at an angle to the displacement
$W = \int_{x_i}^{x_f} F \cdot dx$	The total amount of Work done by a Force that varies as a function of displacement (joules)	When **the force is variable (not constant)** and acts **along the direction of motion**; must know the force as a **function of position** or the **graph** (calculate area under the curve)
$F_{spring} = -kx$; $W_{spring} = \frac{1}{2}kx^2$	**Hooke's Law;** the behavior of certain springs within a limited	When a spring is **stretched or compressed** within its **elastic range.** When you're asked

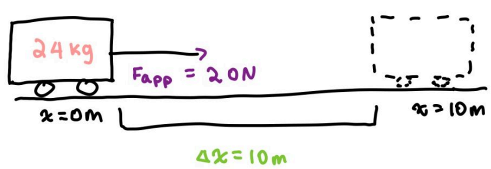
k = spring constant (N/m), a measure of the spring's stiffness	range, where the applied force is linearly proportional and opposite to the displacement.	to find **how much force**, **displacement**, or **spring constant.** When the spring is **ideal** or assumed to behave **linearly**
$K = \frac{1}{2}mv^2$	Kinetic energy of an object (joules)	When calculating energy due to motion
$W_{net} = \Delta K = K_f - K_i$ $= \frac{1}{2}m(v_f)^2 - \frac{1}{2}m(v_i)^2$	Work-Kinetic Energy Theorem (joules)	When relating net work to change in kinetic energy
$P_{avg} = \frac{W}{t}$	Average power (watts)	When time duration is known
$P_{inst} = \frac{dW}{dt} = Fv$	Instantaneous power (watts)	When force and velocity are constant and in the same direction



equilibrium position - the position x_0 at which the mass experiences no net force
x_{min} - the position at which the spring is maximally compressed, at a distance *A* from the equilibrium position
x_{max} - the position at which the spring is maximally extended, at a distance *A* from the equilibrium position

Problems in Order of Difficulty (Easy, Medium, & Hard):

1) A constant 20-N horizontal force is applied to a 24-kg cart at rest on a level floor. If friction is negligible, what is the speed of the cart when it has pushed 10.0 m? **(easy)**



work-kinetic energy theorem

$\Sigma W = \Delta K = K_f - K_i$

$= \frac{1}{2}m(v_f)^2 - \frac{1}{2}m(v_i)^2$

$W = \vec{F}\cdot\vec{x}$

$= Fx\cos\theta$

work done by a constant force

with these two work equations, let's equate them in order to find v_f

$Fx\cos\theta = \frac{1}{2}m(v_f)^2 - \frac{1}{2}m(v_i)^2$

$F_{app}\cdot x\cdot\cos\theta = \frac{1}{2}m\left((v_f)^2 - (v_i)^2\right)$

starts at rest!

$(20)(10)(\cos(0)) = \frac{1}{2}(24)\left((v_f)^2 - (0)^2\right)$

no incline

$\frac{200}{12} = \frac{\cancel{12}(v_f)^2}{\cancel{12}}$

$\sqrt{(v_f)^{\cancel{2}}} = \sqrt{16.666666...}$

$\boxed{v_f = 4.08\ m/s}$

2) (a) What is the average useful power output of a person who does 5.00×10^{5} J of useful work in 6.0 hours? (medium)

(a) $P_{avg} = \frac{\text{work}}{\Delta t}$ $\quad \Delta t = 6 \text{ hours} \times \frac{60 \text{ min}}{1 \text{ hour}} \times \frac{60 \text{ sec}}{1 \text{ min}} = 21600 \text{ sec}$

$= \frac{5.00 \times 10^{5} \text{ J}}{21600 \text{ s}}$

$= 23.148148148$

$\boxed{P_{avg} = 23.1 \text{ W}}$

(b) Working at this rate, how long will it take this person to lift 1000 kg of bricks 2.00 m to a platform? (Work done to lift his body can be omitted because it is not considered useful output here). **(medium)**

(b) work = ?

$m = 1000 \text{ kg}$

$\Delta x = 2.0 \text{ m}$

$W = \vec{F} \cdot \vec{x}$

$W = F_g \cdot x$ we'll find $\vec{F}$ using $F_g = mg$ because the $\vec{F}$ used to lift the bricks is working against the force of gravity

$W = mg\Delta x$ disregarded the $\cos\theta$ part because $\theta = 0$ since F & x are parallel; thus, $\cos(0) = 1$ and that's irrelevant.

$= (1000)(9.8)(2.0)$

$= 19600 = 1.96e^{4} \text{ J}$

$\left(P_{avg} = \frac{\text{work}}{\Delta t}\right)\Delta t$

$\frac{\Delta t \, \cancel{P_{avg}}}{\cancel{P_{avg}}} = \frac{\text{work}}{P_{avg}} = \frac{1.96e^{4} \text{ J}}{23.1 \text{ W}} = 848.484848$

$\boxed{\Delta t = 848 \text{ seconds}}$

3) Engineers desire to model the magnitude of the elastic force of a bungee cord using the equation:

$$F(x) = a\left[\frac{x+7}{7} - \left(\frac{7}{x+7}\right)^{2}\right]$$

where x is the stretch of the cord along its length and a is a constant. If it takes **15.0 kJ** of work to stretch the cord by **20.0 m**, determine the value of the constant a. **(hard)**

$$W = \int_{x_i}^{x_f} F(x) \cdot dx$$

$$15.0 \times 10^3 J = \int_0^{20} a \left[\frac{x+7}{7} - \left(\frac{7}{x+7}\right)^2 \right] dx$$

$$15.0 \times 10^3 = a \left(\int_0^{20} \frac{x+7}{7} dx - \int_0^{20} \left(\frac{7}{x+7}\right)^2 \cdot dx \right)$$

↳ this constant can be brought out & terms can be split into 2 integrals if they're added or subtracted

$$\int_0^{20} \frac{x+7}{7} \cdot dx = \frac{1}{7} \int_0^{20} x+7 \cdot dx = \frac{1}{7}\left(\int_0^{20} x \cdot dx + \int_0^{20} 7 \cdot dx \right)$$

$$= \frac{1}{7}\left(\frac{1}{2}x^2 \Big|_0^{20} + 7x \Big|_0^{20} \right)$$

$$= \frac{1}{7}\left(\left(\frac{1}{2}(20)^2 - \cancel{\frac{1}{2}(0)^2}\right) + \left(7(20) - \cancel{(0)}\right) \right)$$

$$= \frac{1}{7}(200 + 140) = \frac{1}{7}(340)$$

$$= 48.5714285714$$

$$= 48.6$$

$$\int_0^{20} \left(\frac{7}{x+7}\right)^2 \cdot dx = \int_0^{20} \frac{49}{(x+7)^2} \cdot dx = \int_0^{20} 49(x+7)^{-2} \cdot dx$$

$$= -49(x+7)^{-1} \Big|_0^{20}$$

$$= -49(20+7)^{-1} - \left(-49(0+7)^{-1}\right)$$

$$= \left(-49 \cdot \frac{1}{27}\right) - \left(-49 \cdot \frac{1}{7}\right)$$

$$= -1.814814815 - (-7)$$

$$= 5.185185185$$

$$= 5.19$$

$$15.0 \times 10^3 = a\,(48.6 - 5.19) = 43.41\,a$$

$$\frac{15.0 \times 10^3}{43.41} = \frac{\cancel{43.41}\,a}{\cancel{43.41}} \qquad a = 345.5425017$$

$$\boxed{a = 346 \text{ N}}$$
