Chapter 4: Motion in Two Dimensions

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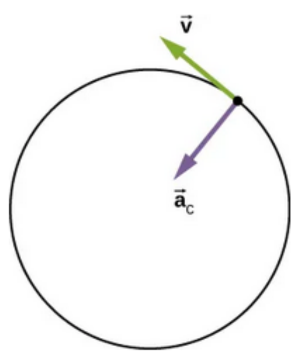
Background/Summary: Two-dimensional motion describes an object moving in multiple dimensions, not just on one line. This unit mostly uses the fundamentals from one-dimensional motion, incorporating vector addition and introducing circular motion as well.

Major Topics: Projectile motion, uniform circular motion, and relative motion.

Key Terms/Vocab:
- Vector addition: adding the components of two vectors together to find a resultant vector; graphically, we use the tip-to-tail method
- Types of acceleration: radial/centripetal/normal (along the radius/into the center during uniform circular motion), tangential (along the tangent line)
- Frame of reference: the point from which motion is measured

Key equations/formulae:
- In this unit, we use the basic kinematics equations from the one-dimensional motion unit extensively; here, we often will use them in the x-direction and y-direction separately. The most relevant equation for projectile motion will be $\Delta y = v_{0y} + \frac{1}{2}a_y t^2$, which we will often use to solve for t, plugging into $\Delta x = v_x t$ to find horizontal displacement.
- For projectile motion, we also often use $v_x = v cos(\theta)$ and $v_y = v sin(\theta)$
- In cases of uniform circular motion, $a_c = \frac{v^2}{r}$, where a_c is the centripetal acceleration

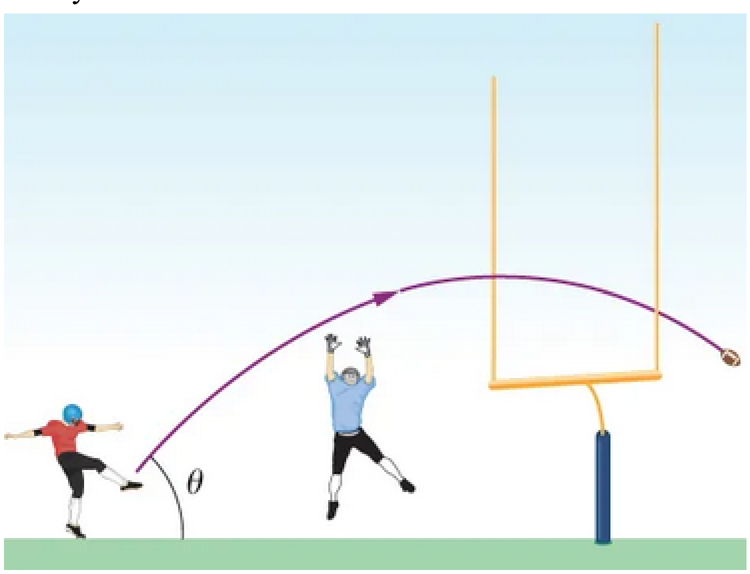


(from OpenStax textbook)

- For relative motion, we use the equation $\vec{v}_{a-b} + \vec{v}_{b-c} = \vec{v}_{a-c}$, where the - denotes relativity between two points; this equation really means that the vector of a relative to b plus the vector of b relative to c is the vector of a relative to c.

Problems: (All from OpenStax textbook)

- Problem 1: The velocity of a particle in reference frame *A* is $(2.0\hat{i} + 3.0\hat{j})$ m/s. The velocity of reference frame *A* with respect to reference frame *B* is $4.0\hat{k}$ m/s, and the velocity of reference frame *B* with respect to *C* is $2.0\hat{j}$ m/s. What is the velocity of the particle in reference frame *C*?
- Problem 2: A runner taking part in the 200-m dash must run around the end of a track that has a circular arc with a radius of curvature of 30.0 m. The runner starts the race at a constant speed. If she completes the 200-m dash in 23.2 s and runs at constant speed throughout the race, what is her centripetal acceleration as she runs the curved portion of the track?
- Problem 3: When a field goal kicker kicks a football as hard as he can at $45°$ to the horizontal, the ball just clears the 3-m-high crossbar of the goalposts 45.7 m away. (a) What is the maximum speed the kicker can impart to the football? (b) In addition to clearing the crossbar, the football must be high enough in the air early during its flight to clear the reach of the onrushing defensive lineman. If the lineman is 4.6 m away and has a vertical reach of 2.5 m, can he block the 45.7-m field goal attempt? (c) What if the lineman is 1.0 m away?

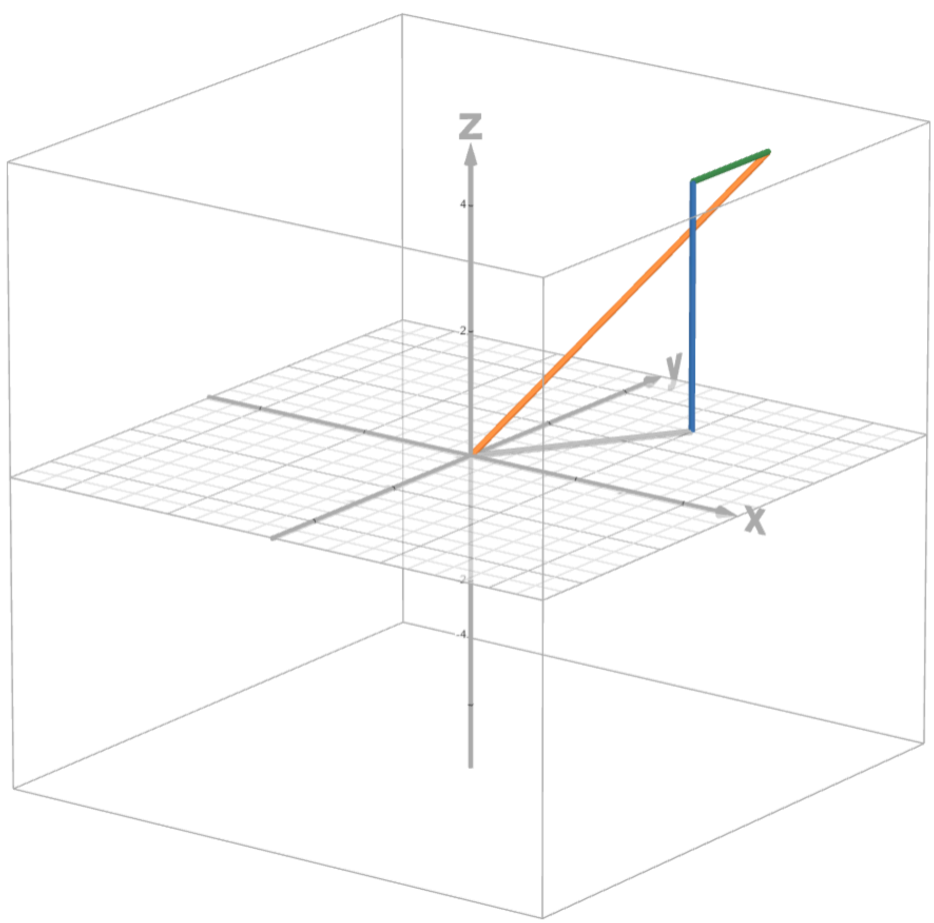


Solutions:

- Problem 1:

- To find the velocity of the particle in reference frame C, we have to find the relation between reference frame A and reference frame C; luckily, we have an equation to do this from earlier: $\vec{v}_{a-b} + \vec{v}_{b-c} = \vec{v}_{a-c}$
- $\vec{v}_{p-C} = \vec{v}_{p-A} + \vec{v}_{A-C}$
- $\vec{v}_{p-C} = \vec{v}_{p-A} + \vec{v}_{a-b} + \vec{v}_{b-c}$
- $\vec{v}_{p-A} = (2.0\hat{i} + 3.0\hat{j})$
- $\vec{v}_{a-b} - 4.0\hat{k}$
- $\vec{v}_{b-c} = 2.0\hat{j}$
- $\vec{v}_{p-C} = (2.0\hat{i} + 3.0\hat{j}) + 4.0\hat{k} + 2.0\hat{j}$
- $\vec{v}_{p-C} = (2.0\hat{i} + 5.0\hat{j} + 4.0\hat{k})$ m/s

Graph of vectors + the resultant using 3D Desmos:



- Problem 2:
  - We know that the equation for centripetal acceleration is $a_c = \frac{v^2}{r}$, so we need to convert our given values to fit into the equation
    - $v = \frac{d}{t} \rightarrow v = \frac{200\ m}{23.2\ s} = 8.62\ m/s$
  - Now we have radius and velocity, which makes calculating centripetal acceleration simple:
    - $a_c = \frac{v^2}{r} \rightarrow a_c = \frac{(8.62\ m/s)^2}{30.0m} = 2.48\ m/s^2$
- Problem 3:
  - a) To find the velocity, we have to create an equation that combines the motion in the y and x directions, since we don't know anything about velocity in either direction, only that the acceleration is -9.8 m/s^2 in the y-direction, that the distance we have to cover in the x-direction is $45.7\ m$, and that we have to gain $3.0\ m$ in height. Using our known equations:
    - $\Delta x = v cos(\theta) t$
    - $\Delta y = v sin(\theta) t + \frac{1}{2}(-9.8)t^2$
  - From this, we can derive that:
    - $t = \frac{\Delta x}{v cos(\theta)} \rightarrow \Delta y = v sin(\theta)(\frac{\Delta x}{v cos(\theta)}) + \frac{1}{2}(-9.8)(\frac{\Delta x}{v cos(\theta)})^2$
    - $\Delta y = \Delta x \cdot tan(\theta) - 4.9(\frac{\Delta x^2}{v^2 cos^2(\theta)})$
    - Since $\theta = 45$, $tan(45) = 1$, and $cos^2(45) = 1/2$:
      - $\Delta y = \Delta x - 9.8(\frac{\Delta x^2}{v^2})$
  - Plugging in values: $3 = 45.7 - 9.8(\frac{45.7^2}{v^2}) \rightarrow v = \sqrt{\frac{(9.8)(45.7)^2}{42.7}} = 21.9\ m/s$
  - b) Since the lineman is $4.6\ m$ away, we use that as the Δx, keeping the same v from last time. If our result is greater than the lineman's reach, which is $2.5\ m$, then he won't be able to block it. If it's less than his reach, then he will:
    - $\Delta y = 4.6 - 9.8(\frac{4.6^2}{21.9^2}) = 4.17m > 2.5m$, so he won't reach it.
  - c) With the lineman being $1\ m$ away, we follow the same process as above:
    - $\Delta y = 1 - 9.8(\frac{1^2}{21.9^2}) = 0.98\ m < 2.5m$, so the lineman will block it.