Lab: AP Review Sheets **Jake Huang**

Chapter 10: Rotational Motion

Summary:

Rotational motion translates concepts of Newtonian motion, force, and energy from previous chapters into a rotational context. While the concepts such as torque, moment of inertia, and angular acceleration differ slightly from the concepts we've learned before, they are applied similarly to their linear counterparts, with very similar equations.

Key Topics:	Key Terms:
- Angular Kinematics - Moment of Inertia - Torque - Torque and Energy	- Moment of inertia - how much an object resists rotational motion; analagous to mass in linear motion. - Torque - rotational force

Important Equations:

$a = r\alpha$

$v = r\omega$

$x = r\theta$

All the kinematic equations (just replace v with ω, a with α, and x with θ)

$I = \Sigma m_i r_i^2$ (point moment of inertia)

$I = \int r^2 dm$ (continuous moment of inertia)

$$dm = \lambda\ dx$$
$$dm = \sigma\ dA$$
$$dm = \rho\ dV$$

$I = I_{cm} + MD^2$ (parallel-axis theorem)

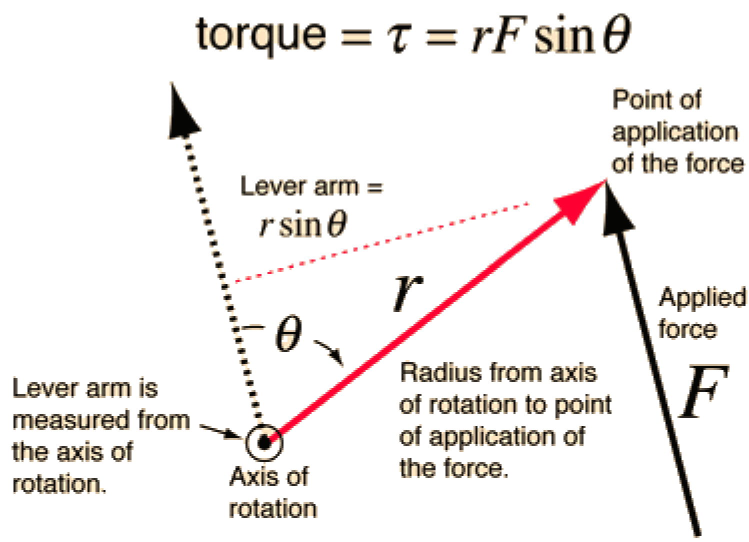
$\tau = r \times F = rF\,sin(\theta)$

$\tau = I\alpha$

$K_{rot} = \frac{1}{2}I\omega^2$

$W = \int \tau\, d\theta$

Explaining Torque a little more:



Taking the cross product:

$$\vec{a} = [a_1 \quad a_2 \quad a_3]$$

$$\vec{b} = [b_1 \quad b_2 \quad b_3]$$

Cross product is determinant of this matrix:

$$\vec{a} \times \vec{b} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{matrix}$$

To find that determinant, first setup equation of 2x2 matrices using given pattern above

$$= \begin{bmatrix} a_2 & a_3 \\ b_2 & b_3 \end{bmatrix} \hat{i} + \begin{bmatrix} a_3 & a_1 \\ b_3 & b_1 \end{bmatrix} \hat{j} + \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} \hat{k}$$

Then find determinant of each 2x2 matrix using formula: determinant of $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$

$$= (a_2 b_3 - a_3 b_2)\hat{i} + (a_3 b_1 - a_1 b_3)\hat{j} + (a_1 b_2 + a_2 b_1)\hat{k}$$

Problems

Problem 1 [EASY]

1. A wheel 1.0 m in radius rotates with an angular acceleration of 4.0 rad/s^2.
    a. If the wheel's initial angular velocity is 2.0 rad/s, what is its angular velocity after 10s?
    b. Through what angle does it rotate in the 10s interval?
    c. What are the tangential speed and acceleration of a point on the rim of the wheel at the end of the 10s interval?

Problem 2 [MEDIUM]

2. Two flywheels of negligible mass and different radii are bonded together and rotate about a common axis. The smaller flywheel of radius 30 cm has a cord that has a pulling force of 50 N on it. What pulling force needs to be applied to the cord connecting the larger flywheel of radius 50 cm such that the combination does not rotate?

Problem 3 [HARD]

3.

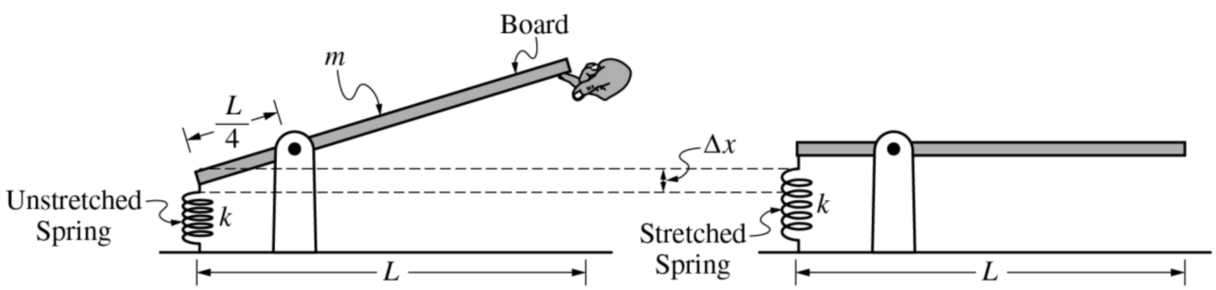


Figure 1 Figure 2

A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The right end of the board is initially held by a student so that the spring is unstretched, as shown in Figure 1. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring stretched, as shown in Figure 2. The rotational inertia of the board about the pivot is I.

Derive an expression for the distance the spring stretches, Δx, when the board is in equilibrium. Express your answer in terms of k, L, m, and physical constants, as appropriate.

Answers

1.

$\alpha = 4\ rad/s^2$

a. $\omega_i = 2\ rad/s$ $\quad t = 10s$

$\omega_f = \omega_i + \alpha t$

$\omega_f = 2 + 4(10)$

$\boxed{= 42\ rad/s}$

b. $\theta_f = \omega_i t + \frac{1}{2}\alpha t^2$

$\theta_f = 2(10) + \frac{1}{2}(4)(10)^2$

$\boxed{= 220\ rad}$

c. ω_f after 10 s $= 42\ rad/s$

$v = r\omega$

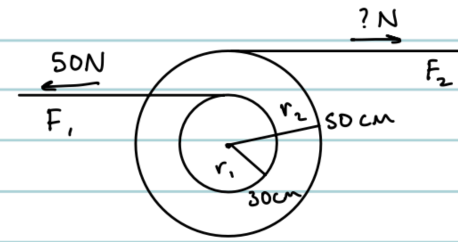
$v_f = 1(42) = \boxed{42\ m/s}$

α after 10s $= 4\ rad/s^2$ (same)

$a = r\alpha$

$a = 1(4) = \boxed{4\ m/s^2}$

2.



$\tau_{net} = \tau_1 - \tau_2 = 0$

$r_1 F_1 \sin\theta - r_2 F_2 \sin\theta = 0$

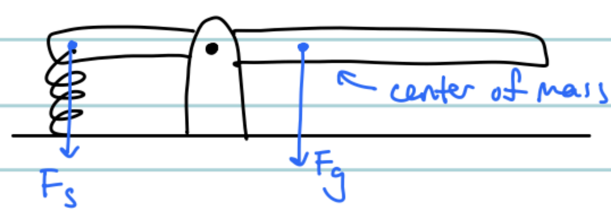
$\theta = 90°$ because the force is applied perpendicularly

$r_1 F_1 = r_2 F_2$

$0.3(50) = 0.5 F_2$

$F_2 = 30N$

3.



$\tau_{net} = \tau_s - \tau_g = 0$ ← because it's static

$\tau_s = \tau_g$

$rF_s = rF_g$

$\cancel{\frac{L}{4}} k\Delta x = \cancel{\frac{L}{4}} mg$

$\Delta x = \frac{mg}{k}$