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Lab: AP Review Sheets
Chapter 6: Circular Motion

Summary:
Circular motion continues Newton's second law of motion, $F_{net} = ma$, interacting with the centripetal force that objects experience when they move in curved paths. This section also covers resistive forces like friction and drag forces that impede an object's motion.

Key Topics:	**Key Terms:**
- Centripetal Force - Circular Motion - Friction - Drag	- Kinetic friction - the friction force that opposes the motion of a sliding/moving object on a surface - Static friction - the friction force that opposes the motion of a stationary object on a surface - Coefficient of friction (μ) - a constant that describes how much friction opposes the motion of an object - Drag - the resistance when an object moves through any fluid

Important Equations:

$$a_c = \frac{v^2}{r}$$

$$\mu = \frac{F_{friction}}{F_{normal}}$$

$R = -bv$ (when drag is proportional to velocity)

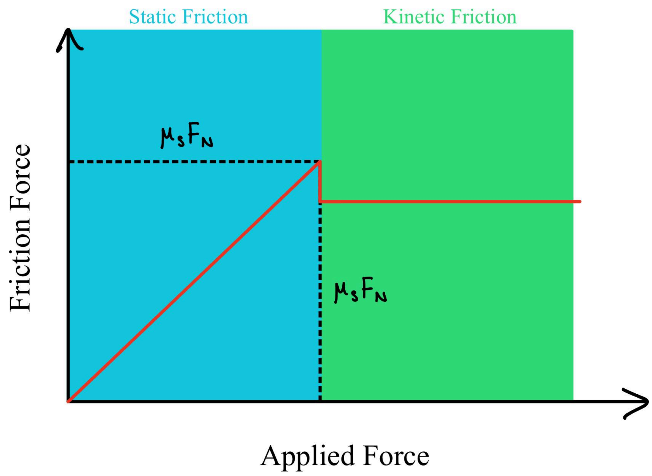
$R = \frac{1}{2} D\rho A v^2$ (when s is proportional to velocity *squared*)

Remember that centripetal force points towards the center of the circular path!

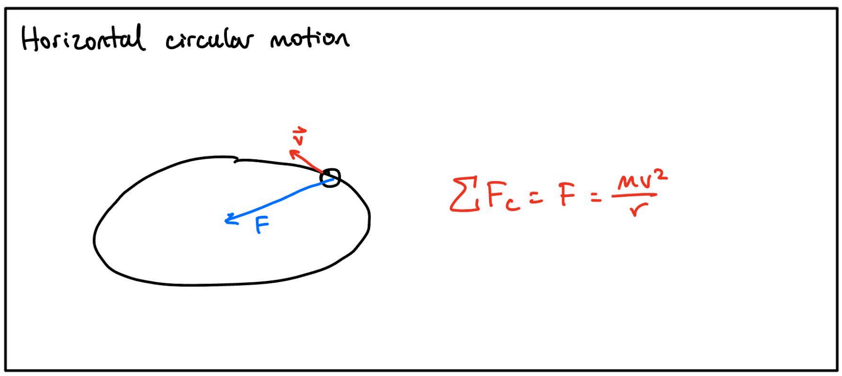
Important features of friction:

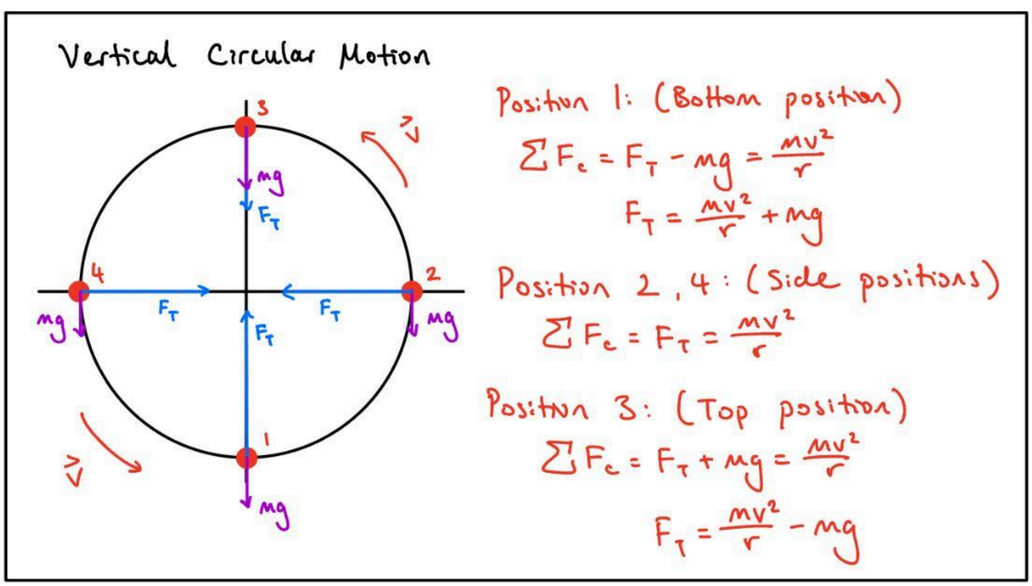
- Bonds between a surface and an object impede the object's sliding motion. When an object is at rest, there is more time to form bonds with the surface that "sticks" the object to the surface
- Static friction force changes to match applied force:
    - The force of static friction on an object increases as the force applied (F_{app}) increases until $F_{app} > \mu_s F_N$. Then, the object starts moving.
- Kinetic friction is constant and less than static friction:
    - When an object is moved from rest, kinetic friction impedes motion instead of static friction.
    - Because the object has already overcome the bonds that hold it in place on a surface once it's moving, kinetic friction is less than static friction. There is also less time for the bonds to form as the object is in motion.

- The diagram below demonstrates these features.



Types of Motion Problems:





When solving circular motion problems, draw out the forces and separate components (if needed) just like any other force problem before solving through!

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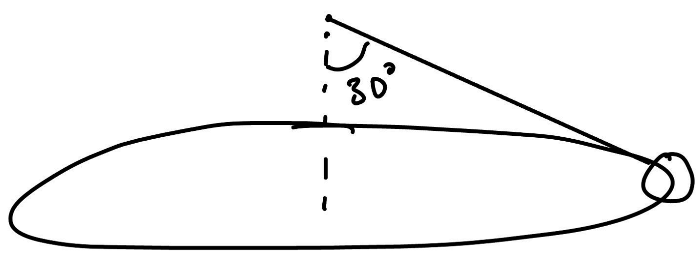
Problems:

Problem 1 [EASY]
A car with a mass of 1000 kg moves at a constant speed around a circular track that has a radius of 50 meters. The coefficient of static friction between the tires and road is 0.6.

What is the maximum speed the car can drive without slipping?

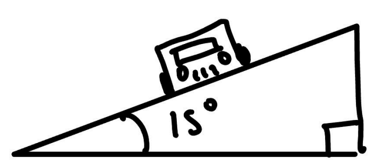
Problem 2 [MEDIUM]
A 0.5 kg ball is attached to a 1.2-meter string that swings the ball in a horizontal circle that makes a 30˚ angle with the vertical.

How fast does the ball have to travel for it to stay at a 30˚ angle?



Problem 3 [HARD]
A highway curve has a radius of 100 meters and is banked at an angle of 15˚. The same car from problem 1, equipped with an extra 200 kg of feathers (so the mass is 1200 kg), is moving around the curve at 25 m/s. The coefficient of static friction between the tires and the road is 0.4.

Will the car stay on the road? If not, will it slide up or down the banked curve?



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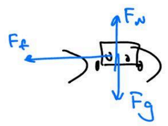
Solutions:

Problem 1



x-component:

$\sum F_c = F_f = \frac{mv^2}{r}$

because $F_f = \mu_s F_N$

$\mu_s F_N = \frac{mv^2}{r}$

y-component:

$F_{net} = F_N - F_g = 0$

$F_N = mg$

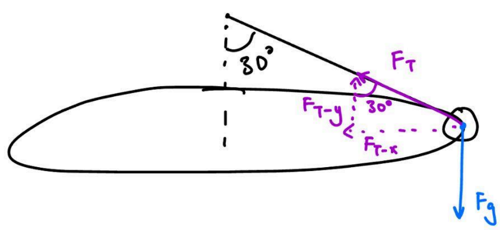
substitute

$\mu_s \cancel{m} g = \frac{\cancel{m} v^2}{r}$

$v = \sqrt{\mu_s r g}$

$= \sqrt{0.6 \cdot 9.8 \cdot 50} = \boxed{17.15 \text{ m/s}}$

Problem 2



x-axis:

$\sum F_c = F_{T-x} = \frac{mv^2}{r}$

$F_T \sin\theta = \frac{mv^2}{r}$

$F_T = \frac{mv^2}{r \sin\theta}$

y-axis:

$F_{net} = F_{T-y} - F_g = 0$

$F_T \cos\theta - mg = 0$

$F_T = \frac{mg}{\cos\theta}$

set equations equal to each other

$$\frac{mv^2}{r\sin\theta} = \frac{mg}{\cos\theta}$$

$$\frac{\cancel{m}v^2}{r\sin\theta} = \frac{\cancel{m}g}{\cos\theta}$$

$$v^2 = \frac{rg\sin\theta}{\cos\theta} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$v = \sqrt{rg\tan\theta}$$

$$v = \sqrt{0.6 \cdot 9.8 \cdot \tan(30°)} = \boxed{1.84 \text{ m/s}}$$

Problem 3

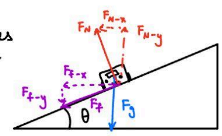
First find ideal speed to determine direction of friction

$V_{ideal} = \sqrt{rg\tan\theta}$ (same process a problem 2)

$= \sqrt{100 \cdot 9.8 \cdot \tan(15°)} = 16.2 \text{ m/s}$

The car goes at 25 m/s, which is greater than 16.2 m/s, causing the car to go uphill. To counteract this, friction points downhill.

We set the axes this way because a_c points ←



Calculate maximum velocity and see if given velocity exceeds it.

x-axis:

$\sum F_c = F_{N-x} + F_{f-x} = \frac{mv^2}{r}$

$F_N \sin\theta + F_f \cos\theta = \frac{mv^2}{r}$

y-axis:

$F_{net} = F_{N-y} - F_g - F_{f-y} = 0$

$F_N \cos\theta - mg - F_f \sin\theta = 0$

substitute $F_f = \mu_s F_N$ for both components

$F_N \sin\theta + F_N \mu_s \cos\theta = \frac{mv^2}{r}$

$F_N(\sin\theta + \mu_s \cos\theta) = \frac{mv^2}{r}$

$F_N \cos\theta - mg - \mu_s F_N \sin\theta = 0$

$F_N(\cos\theta - \mu_s \sin\theta) = mg$

$F_N = \frac{mg}{\cos\theta - \mu_s \sin\theta}$

substitute this into x-component

$$\left(\frac{\cancel{m}g}{\cos\theta - \mu_s \sin\theta}\right)(\sin\theta + \mu_s\cos\theta) = \frac{\cancel{m}v^2}{r}$$

$$V_{max} = \sqrt{\frac{rg(\sin\theta + \mu_s\cos\theta)}{\cos\theta + \mu_s\sin\theta}}$$

$$= \sqrt{\frac{100(9.8)(\sin(15°) + 0.4\cos(15°))}{\cos(15°) + 0.4\sin(15°)}}$$

$= 27$ m/s, greater then the given speed.

Thus, $\boxed{\text{the car stays on.}}$