LINEAR MOMENTUM - DEFINITIONS

QUICK EQUATIONS

Momentum (p): $p = mv$
Impulse (J): $J = F_{\text{avg}} \cdot \Delta t$
Impulse-Momentum Theorem: $J = \Delta p = p_f - p_i$
Impulse (Variable Force): $J = \int F(t)\,dt$
Conservation of Momentum: $\Sigma p_{\text{initial}} = \Sigma p_{\text{final}}$
1D Momentum Conservation (Two Objects): $m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$
Elastic Collision: $\Sigma p_{\text{initial}} = \Sigma p_{\text{final}}$, $\Sigma KE_{\text{initial}} = \Sigma KE_{\text{final}}$
Perfectly Inelastic Collision: $m_1 v_{1,i} + m_2 v_{2,i} = (m_1 + m_2) v_f$
Explosions: $0 = m_1 v_{1,f} + m_2 v_{2,f}$
Center of Mass (Position): $r_{\text{cm}} = \frac{\Sigma m_i r_i}{M_{\text{total}}}$

LINEAR MOMENTUM - DEFINITIONS

- Definition: Measure of "inertia in motion". Vector quantity.
- Formula: $\vec{p} = m\vec{v}$
- Units: kgm/s (or Ns)
- Direction: Same as velocity $\vec{v}$.
- Change: $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = m(\vec{v}_f - \vec{v}_i)$ (Vector subtraction!)

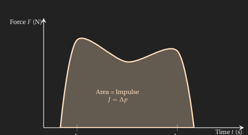
IMPULSE

- Definition: Change in momentum due to a net force acting over time. Vector quantity.
- Formula (Const Force): $\vec{J} = \vec{F}_{\text{avg}}\Delta t$
- Formula (Var Force): $\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\,dt$
- Units: Ns (or kgm/s)
- Direction: Same as average net force $\vec{F}_{\text{avg}}$.
- Intuition: Imagine you are stepping on the gas for a car. To achieve a given impulse, you can either apply a small force over a long time, or a large force over a short time. Both will get you the same momemtum change (from a standstill thats final speed).

IMPULSE-MOMENTUM THEOREM

- Statement: Impulse equals the change in momentum.
- Formula: $\vec{J} = \Delta\vec{p}$
- Combined: $\vec{F}_{\text{avg}}\Delta t = m\vec{v}_f - m\vec{v}_i$
- Newton's 2nd Law (General): $\Sigma\vec{F}_{\text{ext}} = \frac{d\vec{p}}{dt}$

GRAPHICAL INTERPRETATION OF IMPULSE



Impulse $\vec{J}$ is the area under the $\vec{F}$ vs. t graph. Impulse varies depending on how hard you push (the height of the function) and how long you push (the region you are integrating over)

BUILDING INTUITION: MOMENTUM IMPULSE

WHAT IS MOMENTUM?

Think of momentum ($\vec{p}$) as "oomph" or "inertia in motion." It's how hard it is to stop something that's moving. It depends on two things:

- **Mass (m):** A heavy truck has more "oomph" than a light bicycle moving at the same speed. More mass = more momentum.
- **Velocity ($\vec{v}$):** A fast baseball has more "oomph" than a slow one. More speed = more momentum.

Formula: $\vec{p} = m\vec{v}$. **Key Idea:** Momentum is a *vector* A head-on collision is different from a glancing blow.

WHAT IS IMPULSE?

Think of impulse ($\vec{J}$) as the "kick" or "shove" you give an object to change its motion (its momentum). It depends on two things:

- **Force ($\vec{F}$):** How hard you push or pull. A strong kick gives more impulse than a gentle nudge (in the same time).
- **Time (Δt):** How long you push or pull. Pushing gently for a long time can give the same impulse as pushing hard for a short time.

Formula (Avg. Force): $\vec{J} = \vec{F}_{avg}\Delta t$. Impulse is the *cause* of a change in momentum.

THE CONNECTION: IMPULSE-MOMENTUM THEOREM

REMEMBER THIS: **Impulse equals the change in momentum.**

$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

$$\vec{F}_{avg}\Delta t = m\vec{v}_f - m\vec{v}_i$$

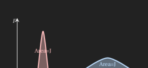
Intuition: The "kick" you give ($\vec{J}$) results directly in the change in the object's "oomph" ($\Delta\vec{p}$).

REAL-WORLD EXAMPLES

- **Airbags/Catching Egg:** Increase the collision time ($\Delta t \uparrow$) to decrease the average force ($F_{avg} \downarrow$) for the same momentum change (Δp). Less force = less injury/breakage.
- **Baseball Bat/Golf Club:** Apply a large force ($F_{avg} \uparrow$) over a short time ($\Delta t \downarrow$) to cause a large change in momentum ($\Delta p \uparrow$).
- **Rocket Launch:** Continuous force over time provides ongoing impulse, steadily increasing the rocket's momentum.

VISUALIZING IMPULSE

Impulse is the area under the Force vs. Time graph. A quick, sharp force (tall, narrow area) can give the same impulse as a weaker, longer force (short, wide area) if the areas are equal.



Different forces/times can yield the same impulse (area).

CONSERVATION OF LINEAR MOMENTUM

WHAT IS LINEAR MOMENTUM?

Linear momentum ($\vec{p}$) is a vector quantity representing an object's "quantity of motion." It depends on both mass (m) and velocity ($\vec{v}$).

$$\vec{p} = m\vec{v}$$

The unit of momentum is kg·m/s. Newton's Second Law can be expressed as the rate of change of momentum: $\vec{F}_{net} = \frac{d\vec{p}}{dt}$.

THE LAW OF CONSERVATION

The **Law of Conservation of Linear Momentum** states: *"If the net external force acting on a system of objects is zero, the total linear momentum of the system remains constant."*
An **isolated system** is one where the vector sum of the external forces is zero ($\sum\vec{F}_{ext} = 0$). Internal forces (forces objects within the system exert on each other) do not change the system's total momentum because they always occur in equal and opposite pairs (Newton's 3rd Law), causing internal impulses that cancel out.

MATHEMATICAL FORMULATION

For an isolated system involving multiple objects (1, 2, ...), the total initial momentum equals the total final momentum:

$$\sum\vec{p}_{initial} = \sum\vec{p}_{final}$$

$$\vec{p}_{1i} + \vec{p}_{2i} + ... = \vec{p}_{1f} + \vec{p}_{2f} + ...$$

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} + ... = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} + ...$$

Since momentum is a vector, this conservation applies independently to each component (x, y, z).

WHY IT'S USEFUL

- **Collisions:** Essential for analyzing all types of collisions (elastic, inelastic, perfectly inelastic), especially when collision forces are complex or unknown.
- **Explosions/Separations:** Used to analyze situations where parts of a system move apart due to internal forces (e.g., recoil, rockets).
- **System Analysis:** Allows tracking the motion of a system's center of mass ($\vec{p}_{total} = M_{total}\vec{v}_{cm}$). In an isolated system, $\vec{v}_{cm}$ is constant.
- **Fundamental Principle:** Alongside energy conservation, it's a cornerstone of mechanics.

KEY POINTS TO REMEMBER

- Momentum is a **vector**. Direction matters!
- Conservation applies only when the **net external force** is zero.
- Total momentum is conserved even if kinetic energy is not (e.g., inelastic collisions).
- Analyze components separately in 2D or 3D problems.

COLLISIONS

COLLISION TYPES

1. Elastic Collision

Mom.:	$\vec{p}_i = \vec{p}_f$ (Conserved)
KE:	$KE_i = KE_f$ (Conserved)
Coeff:	$e = 1$ (see below for what this means)
Desc:	Objects rebound, no energy loss.

1D Equations: Better to just use the general equations setting up conservation of momemtum and conservation of energy equations to slov for this. But if you wan't the shorthand for 1d equations here is what I found.

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

2. Inelastic Collision

Mom.:	$\vec{p}_i = \vec{p}_f$ (Conserved)
KE:	$KE_i > KE_f$ (Lost)
Coeff.:	$0 < e < 1$
Desc:	Objects rebound, some energy loss.

1D Equations (this really only helps in some specific cases): Better to just use the general equations setting up conservation of momentum and conservation of energy equations to slow for this. But if you wan't the shorthand for 1d equations here is what I found. (2D strat below)

$$v_{1f} = v_{1i} - \frac{m_2(1+e)}{m_1 + m_2}(v_{1i} - v_{2i})$$
$$v_{2f} = v_{2i} + \frac{m_1(1+e)}{m_1 + m_2}(v_{1i} - v_{2i})$$

3. Totally Inelastic Collision

Mom.:	$\vec{p}_i = \vec{p}_f$ (Conserved)
KE:	$KE_i \gg KE_f$ (Max loss)
Coeff.:	$e = 0$
Desc:	Objects stick together.

1D Equation:

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

IMPULSE-MOMENTUM ANALYSIS

For a Collision: The impulse J delivered during a collision equals the change in momentum of each object:

$$\vec{J} = m_1(\vec{v}_{1f} - \vec{v}_{1i}) = -m_2(\vec{v}_{2f} - \vec{v}_{2i})$$

COEFFICIENT OF RESTITUTION

Definition: $e = \frac{|\vec{v}_{2f} - \vec{v}_{1f}|}{|\vec{v}_{1i} - \vec{v}_{2i}|}$ = ratio of relative velocity after collision to before collision
Usage: For any collision, use momentum conservation and coefficient of restitution:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \quad \text{(Conservation of Momentum)}$$
$$\vec{v}_{2f} - \vec{v}_{1f} = -e(\vec{v}_{1i} - \vec{v}_{2i}) \quad \text{(Restitution Equation)}$$

These two equations allow you to solve for final velocities $\vec{v}_{1f}$ and $\vec{v}_{2f}$ in any collision when e is known.

Classification: $e = 1$ (elastic), $0 < e < 1$ (inelastic), $e = 0$ (totally inelastic)
Notice how we only really know e in perfectly elastic and inelastic equations. In an inelastic equation we don't have 2 things to solve for. But with an elastic collision we do, and we can use the restitution equation with e = 1 to solve it. (2D strat below)

MOMENTUM PROBLEM-SOLVING STRATEGIES

KEY APPROACHES BY PROBLEM TYPE

Problem Type	Strategic Approach
Impulse Problems	1. Identify given info (mass, velocity changes, time). 2. Calculate impulse using one method: • Change in momentum: $J = m\Delta v$. • Avg. force × time: $J = F_{avg}\Delta t$. • Area under F-t graph.
Elastic Collisions	1. Conserve momentum: $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$. 2. Conserve kinetic energy: $KE_i = KE_f$. 3. Note: Identical masses often scatter at 90° in 2D glancing collisions.
Inelastic Collisions	1. Conserve momentum. 2. KE is lost (converted to heat/deformation). 3. Perfectly inelastic: objects stick ($v_{1f} = v_{2f}$). 4. Solve momentum eq. (plus $v_{1f} = v_{2f}$ if perfectly inelastic).
Explosions/ Separations	1. Initial momentum often known (e.g., zero if starting from rest). 2. Conserve momentum for the system. 3. Solve for unknown velocities. 4. Think "reverse inelastic collision".
Multi-dimension Problems	1. Decompose vectors into components (x, y). 2. Conserve momentum separately for each component ($\sum p_{ix} = \sum p_{fx}$, etc.). 3. Solve component equations. 4. Recombine components for final vector answer if needed.
Center of Mass Motion	1. Find CM position: $\vec{r}_{cm} = (\sum m_i \vec{r}_i)/M_{tot}$. 2. Isolated system: CM velocity $\vec{v}_{cm}$ is constant. 3. Total momentum $\vec{p}_{tot} = M_{tot}\vec{v}_{cm}$. 4. Track overall system motion.

FINAL NOTES

General Approach for All Momentum Problems:

1. **Diagram:** Draw 'before' and 'after', label knowns/unknowns.
2. **Coordinates:** Choose a system and positive direction(s).
3. **System:** Clearly define the boundaries of the system.
4. **External Forces:** Check if net external force is zero (is momentum conserved?).
5. **Conservation Eq(s):** Write $\sum \vec{p}_i = \sum \vec{p}_f$. Use components for 2D/3D.
6. **Collision Type:** (If applicable) Elastic, inelastic, perfectly inelastic? Use KE or $v_{1f} = v_{2f}$ accordingly.
7. **Solve:** Algebraically solve the system of equations.
8. **Check:** Does the answer make physical sense? Check units.

LASTLY

- Momentum is conserved in isolated systems (no net external force).
- Kinetic Energy (KE) is conserved ONLY in elastic collisions.
- In perfectly inelastic collisions, objects stick together ($v_{1f} = v_{2f}$). Max KE loss.
- For 2D/3D, conserve momentum components independently along perpendicular axes.
- An isolated system's Center of Mass (CM) moves with constant velocity.

CENTER OF MASS (CM)

CONCEPT AND DEFINITION

The Center of Mass (CM) is the unique point representing the mean position of the matter in a body or system. Intuitively, it's the "balance point."
Formally, it's the **weighted average position** of the system's mass. A force applied directly at the CM will cause linear acceleration without rotation. The CM location depends on the distribution of mass; it's often closer to regions of larger mass density.

CALCULATING THE CENTER OF MASS

1. Discrete Masses: For a system of point masses $m_1, m_2, ...$ at positions $\vec{r}_1, \vec{r}_2, ...$, the CM position $\vec{r}_{cm} = (x_{cm}, y_{cm}, z_{cm})$ is found by:

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{m_1 x_1 + m_2 x_2 + ...}{M_{total}}$$

$$y_{cm} = \frac{\sum m_i y_i}{\sum m_i} = \frac{m_1 y_1 + m_2 y_2 + ...}{M_{total}}$$

$$z_{cm} = \frac{\sum m_i z_i}{\sum m_i} = \frac{m_1 z_1 + m_2 z_2 + ...}{M_{total}}$$

In vector form:

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{M_{total}}$$

where $M_{total} = \sum m_i$ is the total mass.

2. Continuous Mass Distributions: For extended objects, we integrate over infinitesimal mass elements dm at position $\vec{r}$:

$$x_{cm} = \frac{1}{M_{total}} \int x\, dm$$

$$y_{cm} = \frac{1}{M_{total}} \int y\, dm$$

$$z_{cm} = \frac{1}{M_{total}} \int z\, dm$$

In vector form:

$$\vec{r}_{cm} = \frac{1}{M_{total}} \int \vec{r}\, dm$$

To solve these integrals, relate dm to position using density:

- Linear density λ: $dm = \lambda\, dl$ (for rods, wires)
- Surface density σ: $dm = \sigma\, dA$ (for plates, shells)
- Volume density ρ: $dm = \rho\, dV$ (for solid objects)

Density can be uniform or variable (e.g., $\lambda = \alpha x$).

CENTER OF MASS MOTION

The velocity of the center of mass $\vec{v}_{cm} = d\vec{r}_{cm}/dt$ relates to the total momentum $\vec{p}_{total}$ of the system:

$$\vec{p}_{total} = \sum m_i \vec{v}_i = M_{total}\vec{v}_{cm}$$

The total momentum of a system is equal to the momentum of its center of mass (as if all mass were concentrated there).

NEWTON'S SECOND LAW FOR SYSTEMS

Taking the time derivative of the momentum equation yields Newton's Second Law for a system of particles:

$$\sum \vec{F}_{ext} = \frac{d\vec{p}_{total}}{dt} = M_{total}\frac{d\vec{v}_{cm}}{dt} = M_{total}\vec{a}_{cm}$$

The net *external* force on a system determines the acceleration of its center of mass, regardless of internal forces or motion. If $\sum \vec{F}_{ext} = 0$ (isolated system), then $\vec{a}_{cm} = 0$ and $\vec{v}_{cm}$ is constant.

SIGNIFICANCE

- Simplifies analysis of complex system motion.
- Represents the translational motion of the system as a whole.
- Crucial for understanding balance and stability.
- Essential link between forces and system momentum changes.

SPECIAL CASES TO REMEMBER

CENTER OF MASS - UNIFORM OBJECTS

- Uniform rod: CM at the middle ($x_{cm} = L/2$)
- Uniform rectangular plate: CM at intersection of diagonals
- Uniform triangular plate: CM at intersection of medians ($\frac{1}{3}$ distance from any base to opposite vertex)

COLLISION SHORTCUTS

- Equal masses 1D elastic collision: objects exchange velocities ($v_{1f} = v_{2i}$ and $v_{2f} = v_{1i}$)
- Very heavy object hits very light object elastically: light object rebounds with 2× the heavy object's velocity
- Very light object hits stationary heavy object elastically: light object rebounds with nearly same speed in opposite direction
- Identical masses in 2D elastic collision: final velocity vectors are perpendicular if initially one was stationary
- Perfectly inelastic collision: final velocity $v_f = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$
- Maximum KE loss: occurs in perfectly inelastic collisions
- Maximum energy transfer: occurs when masses are equal

MOMENTUM MOTION PRINCIPLES

- Isolated system: center of mass moves with constant velocity regardless of internal changes
- Explosion from rest: center of mass remains stationary
- Rocket propulsion: $v_{final} = v_{initial} + v_{exhaust} \ln \frac{m_{initial}}{m_{final}}$
- Recoil velocity ratio: $\frac{v_1}{v_2} = -\frac{m_2}{m_1}$ (from rest)
- Two-body orbits: CM follows simple Keplerian orbit while bodies orbit around CM
- CM velocity in completely inelastic collision: unchanged by collision
- Walking on ice: CM moves opposite to foot motion during step

MATHEMATICAL SHORTCUTS

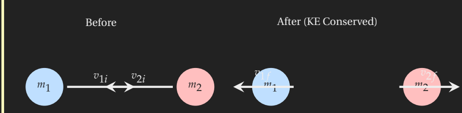
- System of n equal masses: $\vec{r}_{cm} = \frac{1}{n}\sum \vec{r}_i$ (simple average of positions)
- CM of composite object = weighted average of CMs of components
- Symmetrical objects: CM lies on axis/plane of symmetry
- Objects with a symmetry point: CM is at that point
- Uniform density objects with a hole: treat as solid minus missing piece
- Energy lost in inelastic collision: $\Delta E = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(v_{1i} - v_{2i})^2(1 - e^2)$

VISUAL CLARIFICATIONS

VISUALIZING COLLISION TYPES (1D)

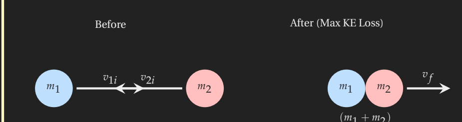
Diagrams illustrate the key difference between elastic, inelastic, and perfectly inelastic collisions for two masses (m_1, m_2) initially moving towards each other. KE = Kinetic Energy.

1. Elastic Collision ($e = 1$)



Momentum conserved, Kinetic Energy conserved.

2. Perfectly Inelastic Collision ($e = 0$)

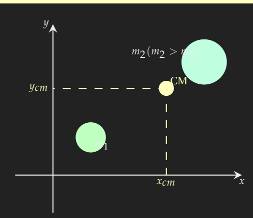


Momentum conserved, Objects stick, Max KE loss.

CENTER OF MASS (CM) VISUALIZATION

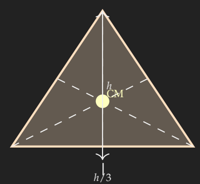
The CM is the weighted average position of mass.

1. Discrete System



CM is closer to the larger mass m_2.

2. Continuous Object (Uniform Triangle)



CM at the intersection of medians (centroid), 1/3 height from base.

SOME MOMENTUM EXAMPLES

PROBLEM 1: BASIC MOMENTUM

Problem: A blue bowling ball with mass $m = 6.0$ kg rolls down the lane at $\vec{v} = +3.0$ m/s. Calculate its momentum.

Solution:

$$\vec{p} = m\vec{v}$$

$$\vec{p} = (6.0\,\text{kg})(+3.0\,\text{m/s})$$

$$\vec{p} = +18.0\,\text{kg}\cdot\text{m/s}$$

PROBLEM 2: MOMENTUM CHANGE

Problem: A red car ($m = 1000$ kg) slows from $\vec{v}_i = +25$ m/s to $\vec{v}_f = +10$ m/s. Find the change in momentum.

Solution:

$$\Delta\vec{p} = m\vec{v}_f - m\vec{v}_i$$

$$\Delta\vec{p} = m(\vec{v}_f - \vec{v}_i)$$

$$\Delta\vec{p} = (1000\,\text{kg})(+10\,\text{m/s} - (+25\,\text{m/s}))$$

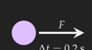
$$\Delta\vec{p} = (1000\,\text{kg})(-15\,\text{m/s})$$

$$\Delta\vec{p} = -15000\,\text{kg}\cdot\text{m/s}$$

(Note: Negative sign indicates direction opposite to initial velocity.)

PROBLEM 3: IMPULSE FROM FORCE-TIME

Problem: A constant force $\vec{F} = +50$ N acts on an object for $\Delta t = 0.2$ s. What is the impulse?



Solution:

$$\vec{J} = \vec{F}_{avg}\Delta t$$

$$\vec{J} = (+50\,\text{N})(0.2\,\text{s})$$

$$\vec{J} = +10\,\text{N}\cdot\text{s}$$

PROBLEM 4: IMPULSE FINAL VELOCITY

Problem: A yellow puck ($m = 0.2$ kg) is initially at rest. An impulse $\vec{J} = +4.0$ Ns acts on it. Find its final velocity.

Solution:

$$\vec{J} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i$$

Since $\vec{v}_i = 0$:

$$\vec{J} = m\vec{v}_f$$

$$+4.0\,\text{N}\cdot\text{s} = (0.2\,\text{kg})\vec{v}_f$$

$$\vec{v}_f = \frac{+4.0\,\text{N}\cdot\text{s}}{0.2\,\text{kg}}$$

$$\vec{v}_f = +20\,\text{m/s}$$

PROBLEM 5: AVERAGE FORCE DURING IMPACT

Problem: A green golf ball ($m = 0.045$ kg) leaves the tee with speed $v_f = 60$ m/s. If the club was in contact for $\Delta t = 0.0005$ s (assume $v_i = 0$), find the average force.

Solution: First find impulse:

$$\vec{J} = \Delta\vec{p} = m(\vec{v}_f - \vec{v}_i)$$

$$\vec{J} = (0.045\,\text{kg})(60\,\text{m/s} - 0)$$

$$\vec{J} = +2.7\,\text{N}\cdot\text{s}$$

Then find average force:

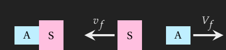
$$\vec{F}_{avg} = \frac{\vec{J}}{\Delta t}$$

$$\vec{F}_{avg} = \frac{+2.7\,\text{N}\cdot\text{s}}{0.0005\,\text{s}}$$

$$\vec{F}_{avg} = +5400\,\text{N}$$

PROBLEM 6: CONSERVATION - RECOIL

Problem: A blue astronaut ($M = 80$ kg) pushes off a red satellite ($m = 200$ kg) in space. The astronaut moves away at $\vec{V}_f = +1.5$ m/s. What is the satellite's velocity $\vec{v}_f$? (Assume initially at rest).



Solution: Total initial momentum $\vec{p}_i = 0$. Use conservation of momentum: $\vec{p}_i = \vec{p}_f$.

$$0 = M\vec{V}_f + m\vec{v}_f$$

$$0 = (80\,\text{kg})(+1.5\,\text{m/s}) + (200\,\text{kg})\vec{v}_f$$

$$0 = +120\,\text{kg}\cdot\text{m/s} + (200\,\text{kg})\vec{v}_f$$

$$(200\,\text{kg})\vec{v}_f = -120\,\text{kg}\cdot\text{m/s}$$

$$\vec{v}_f = \frac{-120\,\text{kg}\cdot\text{m/s}}{200\,\text{kg}}$$

$$\vec{v}_f = -0.6\,\text{m/s}$$

PROBLEM 7: PERFECTLY INELASTIC COLLISION

Problem: A violet blob ($m_1 = 3$ kg) moving at $v_{1i} = +2$ m/s collides and sticks to a green blob ($m_2 = 2$ kg) moving at $v_{2i} = -4$ m/s. Find their final velocity v_f.

Solution: Use conservation of momentum. Since they stick, $v_{1f} = v_{2f} = v_f$.

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2)v_f$$

$$(3\,\text{kg})(+2\,\text{m/s}) + (2\,\text{kg})(-4\,\text{m/s}) = (3\,\text{kg} + 2\,\text{kg})v_f$$

$$+6\,\text{kg m/s} - 8\,\text{kg m/s} = (5\,\text{kg})v_f$$

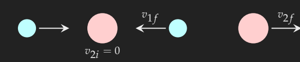
$$-2\,\text{kg m/s} = (5\,\text{kg})v_f$$

$$v_f = \frac{-2\,\text{kg m/s}}{5\,\text{kg}}$$

$$v_f = -0.4\,\text{m/s}$$

PROBLEM 8: ELASTIC COLLISION (UNEQUAL MASS)

Problem: A cyan ball ($m_1 = 1$ kg, $v_{1i} = +5$ m/s) hits a stationary rose ball ($m_2 = 3$ kg) elastically. Find final velocities v_{1f}, v_{2f}.



Solution: For a 1D elastic collision with $v_{2i} = 0$, use the standard formulas:
Final velocity of ball 1:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i}$$

$$v_{1f} = \left(\frac{1\,\text{kg} - 3\,\text{kg}}{1\,\text{kg} + 3\,\text{kg}}\right)(+5\,\text{m/s})$$

$$v_{1f} = \left(\frac{-2}{4}\right)(+5\,\text{m/s})$$

$$v_{1f} = -2.5\,\text{m/s}$$

Final velocity of ball 2:

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i}$$

$$v_{2f} = \left(\frac{2(1\,\text{kg})}{1\,\text{kg} + 3\,\text{kg}}\right)(+5\,\text{m/s})$$

$$v_{2f} = \left(\frac{2}{4}\right)(+5\,\text{m/s})$$

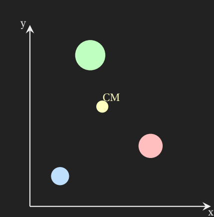
$$v_{2f} = +2.5\,\text{m/s}$$

$$v_{1f} = -2.5\,\text{m/s}, \quad v_{2f} = +2.5\,\text{m/s}$$

(*Verification: Momentum* $1(5) + 0 = 1(-2.5) + 3(2.5) \implies 5 = -2.5 + 7.5 = 5$. *KE* $\frac{1}{2}(1)(5^2) + 0 = \frac{1}{2}(1)(-2.5)^2 + \frac{1}{2}(3)(2.5)^2 \implies 12.5 = 3.125 + 9.375 = 12.5$.)

PROBLEM 9: CENTER OF MASS (2D DISCRETE)

Problem: Find CM of: m1=2kg at (1,1), m2=3kg at (4,2), m3=5kg at (2,5).



Solution: Total mass $M_{tot} = m_1 + m_2 + m_3 = 2 + 3 + 5 = 10$ kg.
X-coordinate of CM:

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{M_{tot}}$$

$$x_{cm} = \frac{(2)(1) + (3)(4) + (5)(2)}{10}$$

$$x_{cm} = \frac{2 + 12 + 10}{10} = \frac{24}{10} = 2.4$$

Y-coordinate of CM:

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{M_{tot}}$$

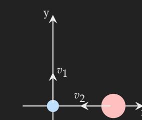
$$y_{cm} = \frac{(2)(1) + (3)(2) + (5)(5)}{10}$$

$$y_{cm} = \frac{2 + 6 + 25}{10} = \frac{33}{10} = 3.3$$

CM is at (2.4, 3.3)

PROBLEM 10: CM VELOCITY

Problem: A 1kg ball at (0,0) moves at $\vec{v}_1 = +2\hat{j}$ m/s. A 3kg ball at (2,0) moves at $\vec{v}_2 = -1\hat{i}$ m/s. Find the velocity of the CM.



Solution: Total mass $M_{tot} = m_1 + m_2 = 1 + 3 = 4$ kg. Total momentum $\vec{p}_{tot} = m_1\vec{v}_1 + m_2\vec{v}_2$

$$\vec{p}_{tot} = (1\,\text{kg})(+2\hat{j}\,\text{m/s}) + (3\,\text{kg})(-1\hat{i}\,\text{m/s})$$

$$\vec{p}_{tot} = +2\hat{j}\,\text{kg m/s} - 3\hat{i}\,\text{kg m/s}$$

$$\vec{p}_{tot} = -3\hat{i} + 2\hat{j}\,(\text{kg}\cdot\text{m/s})$$

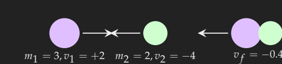
CM velocity $\vec{v}_{cm} = \frac{\vec{p}_{tot}}{M_{tot}}$

$$\vec{v}_{cm} = \frac{-3\hat{i} + 2\hat{j}\,\text{kg m/s}}{4\,\text{kg}}$$

$$\vec{v}_{cm} = -0.75\hat{i} + 0.5\hat{j}\,\text{m/s}$$

PROBLEM 11: CM MOTION IN ISOLATED SYSTEM

Problem: The two blobs from Problem 7 collide. What is the velocity of their center of mass before and after the collision?



Solution: The collision is an internal process. If the system is isolated (no net external force), the velocity of the center of mass remains constant throughout the process. Calculate $\vec{v}_{cm}$ using the initial state: Total mass $M_{tot} = m_1 + m_2 = 3 + 2 = 5$ kg. Total initial momentum $\vec{p}_{initial} = m_1 v_{1i} + m_2 v_{2i}$

$$\vec{p}_{initial} = (3)(+2) + (2)(-4) = 6 - 8 = -2\,\text{kg}\cdot\text{m/s}$$

CM velocity:

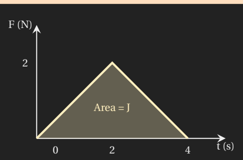
$$\vec{v}_{cm} = \frac{\vec{p}_{initial}}{M_{tot}} = \frac{-2\,\text{kg m/s}}{5\,\text{kg}} = -0.4\,\text{m/s}$$

Since momentum is conserved for the isolated system, $\vec{p}_{final} = \vec{p}_{initial}$, and thus $\vec{v}_{cm}$ is constant.

$\vec{v}_{cm} = -0.4$ m/s (both before and after collision)

PROBLEM 12: IMPULSE FROM GRAPH

Problem: The force on an object varies as shown in the F-t graph. Find the impulse delivered between $t = 0$ and $t = 4$ s.



Solution: Impulse J is the area under the Force-time graph. The shape is a triangle with base $b = 4$ s and height $h = 2$ N.

$$J = \text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$J = \frac{1}{2} \times (4\,\text{s}) \times (2\,\text{N})$$

$$J = 4\,\text{N} \cdot \text{s}$$

PROBLEM 13: COEFFICIENT OF RESTITUTION

Problem: A ball drops onto a floor. Before impact $v_{1i} = -5$ m/s. After impact $v_{1f} = +4$ m/s. The floor (m_2) is stationary ($v_{2i} = v_{2f} = 0$). Find e.

Solution: The coefficient of restitution e is the ratio of relative speed after collision to relative speed before collision.

$$e = \frac{|\vec{v}_{2f} - \vec{v}_{1f}|}{|\vec{v}_{1i} - \vec{v}_{2i}|}$$

Substitute the given values (taking upwards as positive):

$$e = \frac{|0 - (+4)|}{|(-5) - 0|}$$

$$e = \frac{|-4|}{|-5|}$$

$$e = \frac{4}{5}$$

$$e = 0.8$$

ENCOURAGEMENT

You got this! You know more than you think! Good luck on the test!

INSTRUCTOR DISCLOSURE

Full disclosure this was a Tex Note template I found off of reddit. However this still took a long time to figure out all the formatting.