ANGULAR MOMENTUM - DEFINITIONS

QUICK EQUATIONS

Angular Momentum ($\vec{L}$): $\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$
Magnitude: $L = rmv \sin\theta$ where θ is angle between $\vec{r}$ and $\vec{v}$
Rotational Inertia: $I = \sum m_i r_i^2$ (point masses) or $I = \int r^2 dm$ (continuous)
Angular Momentum (Rotating Object): $\vec{L} = I\vec{\omega}$
Torque: $\vec{\tau} = \vec{r} \times \vec{F}$
Magnitude of Torque: $\tau = rF\sin\theta$ or $\tau = r_\perp F$
Angular Impulse: $\vec{J}_{\text{ang}} = \vec{\tau}_{\text{avg}} \cdot \Delta t$
Angular Impulse-Momentum: $\vec{J}_{\text{ang}} = \Delta\vec{L} = \vec{L}_f - \vec{L}_i$
Conservation of Angular Momentum: $\Sigma\vec{L}_{\text{initial}} = \Sigma\vec{L}_{\text{final}}$
Angular Acceleration: $\vec{\alpha} = \vec{\tau}/I$

ANGULAR MOMENTUM - DEFINITIONS

- Definition: Measure of rotational inertia in motion. Vector quantity.
- Formula (Point Mass): $\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$
- Formula (Rotating Object): $\vec{L} = I\vec{\omega}$
- Units: kgm²/s (or Js)
- Direction: Determined by right-hand rule perpendicular to both $\vec{r}$ and $\vec{v}$
- Change: $\Delta\vec{L} = \vec{L}_f - \vec{L}_i$

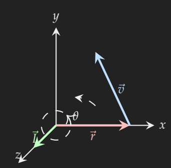
TORQUE AND ANGULAR IMPULSE

- Definition: Torque is the rotational analog of force; causes angular acceleration.
- Formula: $\vec{\tau} = \vec{r} \times \vec{F}$
- Magnitude: $\tau = rF\sin\theta$ or $\tau = r_\perp F$ where $r_\perp$ is perpendicular distance.
- Units: Nm
- Direction: Perpendicular to both $\vec{r}$ and $\vec{F}$ (right-hand rule).
- Angular Impulse: $\vec{J}_{\text{ang}} = \vec{\tau}_{\text{avg}}\Delta t$
- Impulse-Momentum: $\vec{\tau}_{\text{avg}}\Delta t = \Delta\vec{L}$

ROTATIONAL INERTIA (MOMENT OF INERTIA)

- Definition: Measure of an object's resistance to changes in rotational motion.
- Formula (Point Masses): $I = \sum m_i r_i^2$
- Formula (Continuous): $I = \int r^2 dm$
- Units: kgm²
- Depends on: Mass distribution relative to axis of rotation.
- Parallel Axis Theorem: $I = I_{CM} + Md^2$ where d is distance from CM.
- Perpendicular Axis Theorem: $I_z = I_x + I_y$ (for planar objects).

· RIGHT-HAND RULE FOR ANGULAR MOMENTUM ·



$\vec{L} = \vec{r} \times \vec{p}$ points in direction given by right-hand rule when curling fingers from $\vec{r}$ to $\vec{p}$ (or $\vec{v}$). Magnitude depends on $\sin\theta$ between vectors.

BUILDING INTUITION: ANGULAR MOMENTUM

WHAT IS ANGULAR MOMENTUM?

Angular momentum ($\vec{L}$) is the rotational version of linear momentum. It measures how much rotational motion an object has and how hard it is to stop that rotation. Angular momentum depends on:

- **Mass/Mass Distribution:** More mass or mass farther from the axis creates more angular momentum.
- **spinning Speed:** Faster rotation means more angular momentum.
- **Axis:** Where the object rotates around affects the value.

For a single particle: $\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$
For a rotating object: $\vec{L} = I\vec{\omega}$

WHAT IS TORQUE?

Torque ($\vec{\tau}$) as the rotational version of force. It causes changes in angular momentum. Torque depends on:

- **Force magnitude:** Stronger forces create more torque.
- **Lever arm:** Distance from rotation axis to line of force action.
- **Angle of applied force:** Most effective when perpendicular to radius.

Formula: $\vec{\tau} = \vec{r} \times \vec{F}$
Magnitude: $\tau = rF\sin\theta$ or $\tau = r_\perp F$
Torque is the *cause* of change in angular momentum.

THE CONNECTION: ANGULAR IMPULSE-MOMENTUM T

REMEMBER THIS: **Angular impulse equals the change in angular momentum.**

$$\vec{\tau}_{avg}\Delta t = \Delta\vec{L} = \vec{L}_f - \vec{L}_i$$

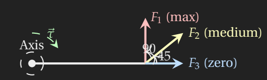
Intuition: The rotational kick you give ($\vec{\tau}\Delta t$) results directly in the change in the object's rotational speed ($\Delta\vec{L}$).

REAL-WORLD EXAMPLES

- **Ice Skater Spin:** Arms in (I↓) = spin faster (↑). Angular momentum stays constant. (note work is done by the skater here in pulling in her arms)
- **Gyroscope/Bicycle Wheel:** Resists changes in orientation due to angular momentum.
- **Helicopter Rotor Reaction:** Body rotates opposite to blades due to conservation of angular momentum.

VISUALIZING TORQUE

Torque is maximized when force is applied perpendicular to the lever arm.



F_1 (perpendicular) creates maximum torque. F_3 (parallel) creates zero torque.

CONSERVATION OF ANGULAR MOMENTUM

THE LAW OF CONSERVATION

The **Law of Conservation of Angular Momentum** saus: *If the net external torque acting on a system is zero, the total angular momentum of the system remains constant.*
An **isolated system** for angular momentum is one where the vector sum of the external torques is zero ($\sum\vec{\tau}_{ext} = 0$).

MATHEMATICAL FORMULATION

For an isolated system:

$$\sum\vec{L}_{initial} = \sum\vec{L}_{final}$$

For a collection of objects (1, 2, ...):

$$\vec{L}_{1i} + \vec{L}_{2i} + ... = \vec{L}_{1f} + \vec{L}_{2f} + ...$$

Since angular momentum is a vector, conservation applies separately to each component.
For a single object changing its configuration (e.g., ice skater pulling in arms):

$$I_i\vec{\omega}_i = I_f\vec{\omega}_f$$

If the direction of rotation doesn't change: $I_i\omega_i = I_f\omega_f$

COMMON APPLICATIONS

- **Rotational Collisions:** Two rotating objects colliding (e.g., gear interactions)
- **Shape Changes:** Objects changing their mass distribution (e.g., diver, gymnast, ice skater)
- **Precession:** Motion of spinning tops, gyroscopes
- **Orbital Motion:** Planets speeding up when closer to the sun (Kepler's 2nd Law)
- **Combined Translation-Rotation:** Objects that both move and spin

CONSERVATION EQUATION FORMS

Depending on the situation, we can write conservation of angular momentum in different forms:
1. Multiple Objects (vector form):

$$\sum\vec{L}_i = \sum\vec{L}_f$$

2. Single object with changing rotation (scalar form, same axis):

$$I_i\omega_i = I_f\omega_f$$

3. Multiple rotating parts:

$$I_1\omega_1 + I_2\omega_2 + ... = (I_1 + I_2 + ...)\omega_f$$

4. Orbiting objects (central force):

$$mr^2\omega = \text{constant} \quad \text{or} \quad r^2\frac{d\theta}{dt} = \text{constant}$$

REMEMBER ME

- Angular momentum is a **vector**. Direction matters!
- Conservation applies only when **net external torque** is zero.
- When moment of inertia (I) changes, angular velocity (ω) must change inversely to maintain constant L.
- The axis of rotation may change if the direction of $\vec{L}$ is preserved.
- For fixed-axis rotation, we can use scalar equations. For general motion, vector analysis is needed.

SYSTEMS WITH VARIABLE MOMENT OF INERTIA

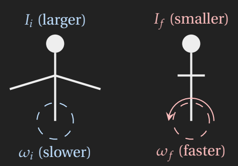
SHAPE AND MASS DISTRIBUTION CHANGES

Analyzing Systems with Changing I

- If no external torque acts on a system, when I changes, ω must be different to maintain constant L:
$$I_i\omega_i = I_f\omega_f$$
- When I decreases (mass moves closer to axis), ω increases.
- When I increases (mass moves away from axis), ω decreases.

Common Examples:

1. **Ice Skater Spin:** Starting with arms extended (I_i large) and bringing them close to body (I_f small) increases spin rate: $\omega_f = \omega_i \cdot \frac{I_i}{I_f}$
2. **Diver's Tuck:** Opening from straight position to tucked position increases rotation speed.
3. **Satellite with Extending Panels:** Deploying solar panels increases I, decreasing rotation rate.



ENERGY CONSIDERATIONS

Rotational Kinetic Energy: $KE_{rot} = \frac{1}{2}I\omega^2$
When angular momentum is conserved ($I_i\omega_i = I_f\omega_f$) but I changes:

$$KE_{rot,f} = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}I_f\left(\frac{I_i\omega_i}{I_f}\right)^2 = \frac{1}{2}I_i^2\omega_i^2\frac{1}{I_f}$$

Energy comparison:

$$\frac{KE_{rot,f}}{KE_{rot,i}} = \frac{I_i}{I_f}$$

Key Insights:

- When I decreases ($I_f < I_i$), rotational KE increases.
- This additional energy typically comes from internal work done by the system (e.g., muscles contracting).
- For objects wit no external torque, increasing I requires work against centripetal force.

ROTATIONAL COLLISION ANALYSIS

General ideas:

- In the absence of external torques, total angular momentum is conserved in collisions.
- May involve translation-to-rotation conversion (e.g., stick striking ball).
- For objects rotating about fixed axes, use: $\sum I_i\omega_i = \sum I_f\omega_f$

Types of Rotational Collisions:

Elastic: Angular momentum and rotational KE both conserved

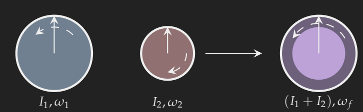
Inelastic: Angular momentum conserved, some rotational KE lost

Coupling: Objects stick together, continue rotating with common angular velocity

Coupling Example (Rotational Analog of Perfectly Inelastic):

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_f$$

where ω_f is common final angular velocity.



MOMENT OF INERTIA (ROTATIONAL INERTIA)

CONCEPT AND DEFINITION

Moment of Inertia (I) quantifies an object's resistance to changes in rotational motion. It depends on both the mass and its distribution relative to the axis of rotation.
The farther mass is distributed from the rotation axis, the greater the moment of inertia. A given torque will produce less angular acceleration for a system with larger moment of inertia ($\alpha = \tau/I$).

CALCULATING MOMENT OF INERTIA

1. For Discrete Point Masses: For a system of point masses $m_1, m_2, \ldots$ at distances $r_1, r_2, \ldots$ from the axis of rotation:

$$I = \sum m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + \ldots$$

2. For Continuous Mass Distributions: For extended objects, we integrate over the entire mass:

$$I = \int r^2\, dm$$

Where r is the perpendicular distance from the axis to the mass element dm.

COMMON MOMENTS OF INERTIA

Object	Axis	Moment of Inertia
Solid sphere	Through center	$\frac{2}{5}MR^2$
Hollow sphere	Through center	$\frac{2}{3}MR^2$
Solid cylinder	Long axis	$\frac{1}{2}MR^2$
Solid cylinder	Diameter	$\frac{1}{4}MR^2 + \frac{1}{12}ML^2$
Thin rod	Perpendicular to rod at end	$\frac{1}{3}ML^2$
Thin rod	Perpendicular to rod at center	$\frac{1}{12}ML^2$
Thin hoop	Through center, to plane	MR^2
Thin rectangular plate	through center	$\frac{1}{12}M(a^2 + b^2)$

IMPORTANT THEOREMS

1. Parallel Axis Theorem: If you know moment of inertia I_{CM} about an axis through the center of mass, the moment of inertia about a parallel axis at distance d is:

$$I = I_{CM} + Md^2$$

where M is the total mass.

2. Perpendicular Axis Theorem: For any planar object (all mass in x-y plane), the moment of inertia about the z-axis (perpendicular to the plane) equals the sum of the moments of inertia about any two perpendicular axes in the plane that intersect at the same point on the z-axis:

$$I_z = I_x + I_y$$

Note: Only valid for planar objects!

APPLICATIONS

- Connects torque to angular acceleration: $\tau = I\alpha$
- Determines rotational kinetic energy: $KE_{rot} = \frac{1}{2}I\omega^2$
- Key to angular momentum: $L = I\omega$ (for rigid bodies)
- Essential for predicting rotational dynamics
- Practical engineering applications (flywheels, gyroscopes, etc.)

SPECIAL ANGULAR MOMENTUM CASES

ROLLING WITHOUT SLIPPING

For an object rolling without slipping (e.g., wheel on ground), two key conditions must be met:

- The contact point is instantaneously at rest.
- The center of mass velocity v_{CM} and angular velocity ω are related by: $v_{CM} = R\omega$

Special properties of rolling motion:

- Total kinetic energy: $KE_{total} = KE_{trans} + KE_{rot} = \frac{1}{2}mv^2 + \frac{1}{2}I_{CM}\omega^2$
- For a uniform solid sphere: $KE_{total} = \frac{1}{2}mv^2 + \frac{1}{2}(\frac{2}{5}mR^2)(\frac{v}{R})^2 = \frac{7}{10}mv^2$
- Angular momentum about contact point: $L = I_{CM}\omega + mRv_{CM} = I_{CM}\omega + mR^2\omega = (I_{CM} + mR^2)\omega$

ORBITAL ANGULAR MOMENTUM

- For objects in orbit: $L = mr^2\omega = mvr$ (where v is tangential velocity)
- Kepler's Second Law (equal areas in equal time) is a direct consequence of angular momentum conservation.
- For elliptical orbits: $L = mvb$ where b is the semi-minor axis.
- If orbit changes shape but L remains constant, r and v must adjust to maintain $r^2\omega = constant$.
- For central forces (like gravity), angular momentum about the force center is always conserved.

ANGULAR MOMENTUM AND QUANTUM MECHANICS

- Angular momentum is quantized in quantum mechanics: $L = \sqrt{\ell(\ell+1)}\hbar$ where ℓ is angular momentum quantum number.
- Electron spin is an intrinsic angular momentum: $S = \sqrt{s(s+1)}\hbar$ where $s = 1/2$ for electrons.
- Component of angular momentum along z-axis: $L_z = m_\ell\hbar$ where $m_\ell = -\ell, -\ell+1, ..., \ell-1, \ell$
- Selection rules for atomic transitions are based on conservation of angular momentum.

TOP ANGULAR MOMENTUM SHORTCUTS

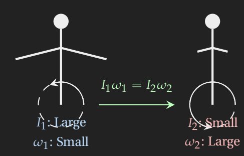
- For particle moving in straight line: $L = mvd$ where d is perpendicular distance to axis
- Maximum torque occurs when force is perpendicular to lever arm
- For combined rotation-translation: $L_{total} = L_{orbital} + L_{spin}$
- For an explosion from rest: total angular momentum remains zero
- For satellite in circular orbit: $L = mr^2\omega = m\sqrt{GMr}$ (G = gravitational constant, M = central mass)
- Angular momentum transport: $\frac{dL}{dt} = \dot{m}(r \times v)$ (for mass flow rate $\dot{m}$)

VISUAL CLARIFICATIONS

CONSERVATION EXAMPLES

Visualization of angular momentum conservation:

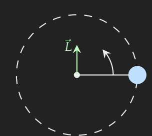
1. Ice Skater Spin



ANGULAR MOMENTUM EXAMPLES

PROBLEM 1: BASIC ANGULAR MOMENTUM

Problem: A ball (m = 0.5 kg) on a 2.0 m string rotates in a horizontal circle at 3.0 rad/s. Find its angular momentum.



Solution: For a mass in circular motion:

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} = rmv\sin(90) = rmv$$

With $v = r\omega$:

$$L = mr^2\omega$$

$$L = (0.5\,\text{kg})(2.0\,\text{m})^2(3.0\,\text{rad/s})$$

$$L = 6.0\,\text{kg}\cdot\text{m}^2/\text{s}$$

The direction is perpendicular to the plane of rotation, upward by right-hand rule.

PROBLEM 2: TORQUE CALCULATION

Problem: A force $\vec{F} = 20\,\text{N}$ acts at a 30° angle to a 0.5 m rod, applied at the end. Find the torque about the rod's pivot.

Solution:

$$\tau = rF\sin\theta$$

$$\tau = (0.5\,\text{m})(20\,\text{N})\sin(30)$$

$$\tau = (0.5\,\text{m})(20\,\text{N})(0.5)$$

$$\tau = 5.0\,\text{N}\cdot\text{m}$$

Alternative method using perpendicular component:

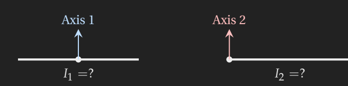
$$F_\perp = F\sin\theta = (20\,\text{N})\sin(30) = 10\,\text{N}$$

$$\tau = rF_\perp = (0.5\,\text{m})(10\,\text{N}) = 5.0\,\text{N}\cdot\text{m}$$

The torque direction is into the page (by right-hand rule).

PROBLEM 3: MOMENT OF INERTIA

Problem: Calculate the moment of inertia of a uniform rod (length L = 1.0 m, mass M = 2.0 kg) about an axis (1) through its center, perpendicular to the rod, and (2) through one end, perpendicular to the rod.



Solution: Case 1: Through center

$$I_1 = \frac{1}{12}ML^2$$

$$I_1 = \frac{1}{12}(2.0\,\text{kg})(1.0\,\text{m})^2$$

$$I_1 = \frac{1}{12}(2.0\,\text{kg}\,\text{m}^2) = \frac{2.0}{12}\,\text{kg}\,\text{m}^2$$

$$I_1 = 0.167\,\text{kg}\cdot\text{m}^2$$

Case 2: Through end Using the parallel axis theorem:

$$I_2 = I_1 + Md^2$$

where $d = L/2 = 0.5$ m (distance from center to end)

$$I_2 = 0.167\,\text{kg}\,\text{m}^2 + (2.0\,\text{kg})(0.5\,\text{m})^2$$

$$I_2 = 0.167\,\text{kg}\,\text{m}^2 + (2.0\,\text{kg})(0.25\,\text{m}^2)$$

$$I_2 = 0.167\,\text{kg}\,\text{m}^2 + 0.5\,\text{kg}\,\text{m}^2$$

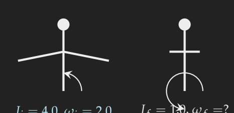
$$I_2 = 0.667\,\text{kg}\cdot\text{m}^2$$

Alternatively, use the direct formula for a rod about its end:

$$I_2 = \frac{1}{3}ML^2 = \frac{1}{3}(2.0\,\text{kg})(1.0\,\text{m})^2 = 0.667\,\text{kg}\cdot\text{m}^2$$

PROBLEM 4: CONSERVATION IN SHAPE CHANGE

Problem: An ice skater spins with her arms extended at 2.0 rad/s. Her moment of inertia is initially 4.0 kg·m². She pulls her arms in, reducing her moment of inertia to 1.0 kg·m². What is her new angular velocity?



Solution: With no external torque, angular momentum is conserved:

$$L_i = L_f$$

$$I_i\omega_i = I_f\omega_f$$

$$(4.0\,\text{kg}\,\text{m}^2)(2.0\,\text{rad/s}) = (1.0\,\text{kg}\,\text{m}^2)\omega_f$$

$$8.0\,\text{kg}\,\text{m}^2\text{rad/s} = (1.0\,\text{kg}\,\text{m}^2)\omega_f$$

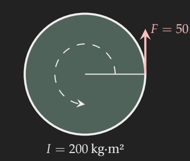
$$\omega_f = \frac{8.0\,\text{kg}\,\text{m}^2\text{rad/s}}{1.0\,\text{kg}\,\text{m}^2}$$

$$\omega_f = 8.0\,\text{rad/s}$$

The skater spins 4 times faster after pulling in her arms.

PROBLEM 5: ANGULAR IMPULSE

Problem: A merry-go-round with moment of inertia 200 kg·m² is initially at rest. A person pushes tangentially with a force of 50 N at radius 2.0 m for 10 seconds. What is the final angular velocity?



Solution: Step 1: Calculate the torque:

$$\tau = rF\sin(90) = rF$$

$$\tau = (2.0\,\text{m})(50\,\text{N}) = 100\,\text{N}\cdot\text{m}$$

Step 2: Calculate the angular impulse:

$$J_{ang} = \tau\Delta t$$

$$J_{ang} = (100\,\text{N}\,\text{m})(10\,\text{s}) = 1000\,\text{N}\,\text{m}\,\text{s}$$

Step 3: Apply the angular impulse-momentum theorem:

$$J_{ang} = \Delta L = L_f - L_i$$

Since $L_i = 0$ (initially at rest): $J_{ang} = L_f = I\omega_f$

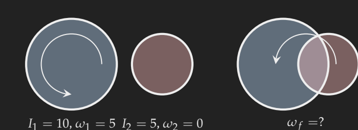
$$1000\,\text{N}\,\text{m}\,\text{s} = (200\,\text{kg}\,\text{m}^2)\omega_f$$

$$\omega_f = \frac{1000\,\text{N}\,\text{m}\,\text{s}}{200\,\text{kg}\,\text{m}^2}$$

$$\omega_f = 5.0\,\text{rad/s}$$

PROBLEM 6: ROTATIONAL COLLISION

Problem: A spinning disk (moment of inertia $I_1 = 10$ kg·m², initial angular velocity $\omega_1 = 5$ rad/s) makes contact with a stationary disk ($I_2 = 5$ kg·m²). If they couple together, find the final angular velocity.



Solution: When the disks couple, angular momentum is conserved:

$$L_i = L_f$$

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_f$$

$$(10\,\text{kg}\,\text{m}^2)(5\,\text{rad/s}) + (5\,\text{kg}\,\text{m}^2)(0\,\text{rad/s}) = (10\,\text{kg}\,\text{m}^2 + 5\,\text{kg}\,\text{m}^2)\omega_f$$

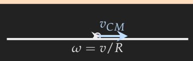
$$50\,\text{kg}\,\text{m}^2\text{rad/s} = (15\,\text{kg}\,\text{m}^2)\omega_f$$

$$\omega_f = \frac{50\,\text{kg}\,\text{m}^2\text{rad/s}}{15\,\text{kg}\,\text{m}^2}$$

$$\omega_f = 3.33\,\text{rad/s}$$

PROBLEM 7: ROLLING WITHOUT SLIPPING

Problem: A solid sphere (mass M = 2.0 kg, radius R = 0.1 m) rolls without slipping at velocity v = 3.0 m/s. Calculate its total kinetic energy.



Solution: For rolling without slipping, $\omega = v/R$
Total kinetic energy = translational KE + rotational KE:

$$KE_{total} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

For a solid sphere, $I = \frac{2}{5}MR^2$:

$$KE_{total} = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2$$

$$KE_{total} = \frac{1}{2}Mv^2 + \frac{1}{2}\frac{2}{5}M\frac{v^2}{R^2}R^2$$

$$KE_{total} = \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2$$

$$KE_{total} = \frac{1}{2}Mv^2\left(1+\frac{2}{5}\right) = \frac{1}{2}Mv^2\left(\frac{7}{5}\right) = \frac{7}{10}Mv^2$$

Substituting values:

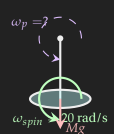
$$KE_{total} = \frac{7}{10}(2.0\,\mathrm{kg})(3.0\,\mathrm{m/s})^2$$

$$KE_{total} = \frac{7}{10}(2.0\,\mathrm{kg})(9.0\,\mathrm{m^2/s^2})$$

$$KE_{total} = 12.6\,\mathrm{J}$$

PROBLEM 8: GYROSCOPIC PRECESSION

Problem: A gyroscope spins at 20 rad/s. Its rotor has mass 0.5 kg and radius 0.05 m (treat as a thin ring). The center of mass is 0.1 m from the pivot. Calculate the precession rate.



Solution: Step 1: Calculate the moment of inertia of the rotor (thin ring):

$$I = MR^2 = (0.5\,\mathrm{kg})(0.05\,\mathrm{m})^2 = 0.00125\,\mathrm{kg\,m^2}$$

Step 2: Calculate the angular momentum of the spinning rotor:

$$L = I\omega_{spin} = (0.00125\,\mathrm{kg\,m^2})(20\,\mathrm{rad/s}) = 0.025\,\mathrm{kg\,m^2/s}$$

Step 3: Calculate the torque due to gravity:

$$\tau = mgd = (0.5\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(0.1\,\mathrm{m}) = 0.49\,\mathrm{N\,m}$$

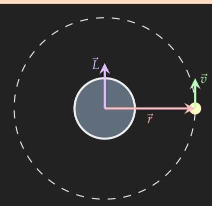
Step 4: Calculate the precession rate:

$$\omega_p = \frac{\tau}{L} = \frac{0.49\,\mathrm{N\,m}}{0.025\,\mathrm{kg\,m^2/s}}$$

$$\omega_p = 19.6\,\mathrm{rad/s}$$

PROBLEM 9: ORBITAL ANGULAR MOMENTUM

Problem: A satellite (mass = 300 kg) orbits Earth in a circular orbit with radius 8000 km at 7.5 km/s. Calculate its angular momentum.



Solution: For orbital motion, angular momentum is:

$$L = mvr\sin(90) = mvr$$

Converting radius to meters: $r = 8000\,\mathrm{km} = 8\times10^6\,\mathrm{m}$

$$L = (300\,\mathrm{kg})(7.5\times10^3\,\mathrm{m/s})(8\times10^6\,\mathrm{m})$$

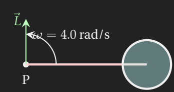
$$L = 300\times7.5\times8\times10^9\,\mathrm{kg\,m^2/s}$$

$$L = 1.8\times10^{13}\,\mathrm{kg\cdot m^2/s}$$

The direction is perpendicular to the orbital plane (by right-hand rule).

PROBLEM 10: ANGULAR MOMENTUM OF COMBINED SYSTEMS

Problem: A rod (length 1.0 m, mass 2.0 kg) and a disk (radius 0.2 m, mass 3.0 kg) are attached in the plane as shown. The system rotates at 4.0 rad/s about an axis through point P. Find the total angular momentum.



Solution: Step 1: Calculate moment of inertia of the rod about point P.

$$I_{rod} = \frac{1}{3}ML^2 = \frac{1}{3}(2.0\,\mathrm{kg})(1.0\,\mathrm{m})^2 = \frac{2.0}{3}\,\mathrm{kg\,m^2}$$

Step 2: Calculate moment of inertia of the disk about its center.

$$I_{disk,CM} = \frac{1}{2}MR^2 = \frac{1}{2}(3.0\,\mathrm{kg})(0.2\,\mathrm{m})^2 = 0.06\,\mathrm{kg\,m^2}$$

Step 3: Use the parallel axis theorem to find moment of inertia of the disk about point P.

$$I_{disk,P} = I_{disk,CM} + Md^2 = 0.06\,\mathrm{kg\,m^2} + (3.0\,\mathrm{kg})(1.0\,\mathrm{m})^2$$

$$I_{disk,P} = 0.06\,\mathrm{kg\,m^2} + 3.0\,\mathrm{kg\,m^2} = 3.06\,\mathrm{kg\,m^2}$$

Step 4: Find the total moment of inertia about point P.

$$I_{total} = I_{rod} + I_{disk,P} = \frac{2.0}{3}\,\mathrm{kg\,m^2} + 3.06\,\mathrm{kg\,m^2}$$

$$I_{total} = 0.667\,\mathrm{kg\,m^2} + 3.06\,\mathrm{kg\,m^2} = 3.727\,\mathrm{kg\,m^2}$$

Step 5: Calculate the total angular momentum.

$$L = I_{total}\omega = (3.727\,\mathrm{kg\,m^2})(4.0\,\mathrm{rad/s})$$

$$L = 14.91\,\mathrm{kg\cdot m^2/s}$$

The direction is perpendicular to the plane of rotation (out of the page).