Lab: AP Review Sheet
Chapter 11: Angular Momentum
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Summary:

Chapter 11 describes the motion of objects that are moving *rotationally*, or rotating. A major part of angular momentum is the relationship between rotational motion and translational motion, which can be analyzed through forces or energy. Ch. 11 returns to the idea of *moment of inertia*, and expands upon that as part of calculating angular momentum. Additionally, this unit introduces the new but related topic of *conservation of angular momentum*, which functions similarly to other types of conservation previously discussed.

Key Terms & Equations:

- Angular Momentum (L)
- Moment of Inertia (I)
- Torque (τ)

$$s = r\theta$$

$$v = r\omega$$

$$a = r\alpha$$

$$K_{\text{total}} = K_{\text{rot}} + K_{\text{trans}}$$

$$K_{\text{total}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$L_{\text{particle}} = r \times p = rmvsin\theta$$

$$L_{\text{composite}} = I\omega$$

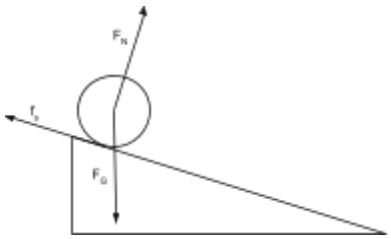
$$\tau = \frac{dL}{dt}$$

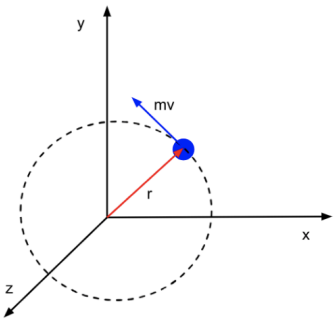
$$\Sigma L_i = \Sigma L_f$$

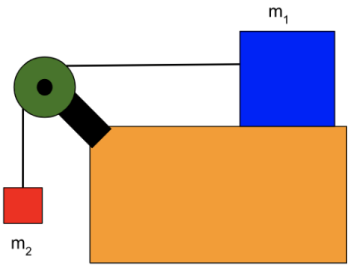
What is Angular Momentum?

Angular momentum is basically a representation of how much rotation an object possesses. This momentum, somewhat like its linear variant, depends on the mass, the angular velocity of the object, and the mass distribution of the object.

Diagrams:







Practice Problems:

Easy:

Using Conservation of Energy, determine the final velocity of a perfect sphere rolling down a ramp of height 5 meters. Assume that air friction is negligible and the ball rolls without slipping.

Medium:

Solve for the acceleration of the system depicted in Diagram 3. Assume the air resistance is negligible and the table and pulley are frictionless.

Hard:

A disk of mass M and radius R, free to rotate about its center, is struck tangentially by a solid sphere of mass m that sticks to it. Solve for the rotational velocity of the system immediately after the collision.

Solutions:

Easy:

$U_i + K_i = U_f + K_f + K_{rot_f}$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$I = \frac{2}{5}mr^2, \quad \omega = \frac{v}{r}$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2$

$mgh = \frac{1}{2}mv^2 + \frac{2}{10}mv^2$

$mgh = \frac{2}{10}mv^2$

$gh = \frac{2}{10}v^2$

$v^2 = \frac{10}{2}gh$

$v = \sqrt{\frac{10}{2}gh}$

$v = \sqrt{\frac{10}{2}(9.8\ m/s^2)(5\ m)} = \boxed{6.37\ m/s}$

Medium:

$E_{total} = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\omega^2 - m_2gd$

$\omega = \frac{v}{R}$

$0 = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 - m_2gd$

$a = \frac{v^2}{2d}$

$$\boxed{a = \frac{m_2 g}{m_1 + m_2 + \frac{1}{R^2}}}$$

Hard:

$I_{disk} = \frac{1}{2}MR^2$, $I_{sphere} = mR^2$

$I_{tot} = \frac{1}{2}MR^2 + mR^2$

$L_{final} = I\omega = \left(\frac{1}{2}MR^2 + mR^2\right)\omega$

$L_{final} = mvR$

$mvR = \left(\frac{1}{2}MR^2 + mR^2\right)\omega$

$$\omega = \frac{mv}{R\left(\frac{1}{2}M + m\right)}$$