Lab: AP Review Sheet
Chapter 6: Circular Motion
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Summary:

While much of everyday physics can be described through means of *linear motion*, or moving in a straight line within a frame of reference, motion that involves movement in a circle *without* rotation, such as in the case of a car moving along a curved path or a ball swinging around from a rope. This unit involves critical forces related to such movement like kinetic and static friction, centripetal force, and gravity.

Key Terms & Equations:

- Centripetal Force
- Radial Acceleration
- Tangential
- Friction
  - Static
  - Kinetic
- Mu
  - μ_{static}
  - $\mu_{kinetic}$

$$F_c = \frac{mv^2}{r}$$

$$a_c = \frac{v^2}{r}$$

$$\mu = \frac{friction}{normal}$$

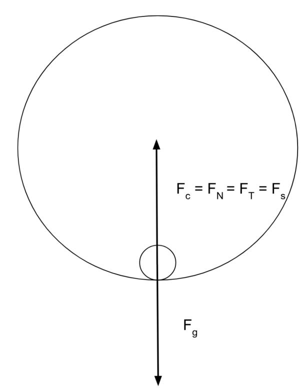
$$f_s = \mu_s \bullet F_N$$

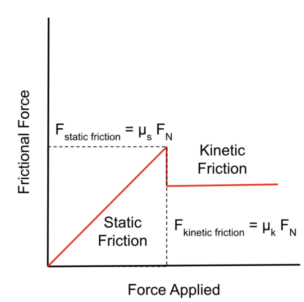
$$f_k = \mu_k \bullet F_N$$

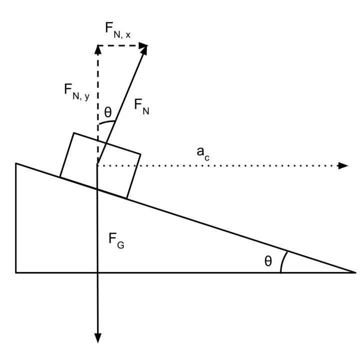
What is Centripetal Force?

Centripetal Force, while not describing a single, specific force, is a standardized term for any force that induces within a circularly-moving object a *radial acceleration,* or an acceleration directed toward the center of the circle. Centripetal Force is most often friction, tension, gravity, or the normal force, though any force that induces radial acceleration is considered to be centripetal.

Diagrams:







Practice Problems:

Easy:

- A 500 kg car on an icy road is attempting to make a turn of radius 11.0 meters. The coefficient of friction $\boldsymbol{\mu}_s$ between the car's tires and the road is 0.24.
    - Calculate the force of *static friction* acting on the car.
    - If the *radial acceleration* of the car is 10 m/s^2, how fast must the car be traveling?

Medium:

- Now, the car is traveling along a banked curve with no friction of radius 35.0 meters. If the angle of the turn above the ground is 22 degrees, what is the maximum speed the car can travel to move about the curve without slipping?

Hard:

- The road has gained friction, and the car has accelerated to a velocity of 20 m/s. Calculate the coefficient of friction required to keep the car on the road.

Solutions:

Easy

$$f_s = \mu_s \cdot F_N$$

$$F_N = mg$$

$$f_s = \mu_s mg$$

$$= (0.24)(500\ kg)(9.8\ m/s^2)$$

$$\boxed{f_s = 1177.2\ N}$$

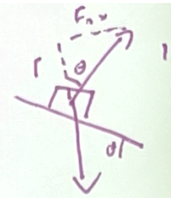
$$a = \frac{v^2}{r}$$

$$v = \sqrt{ar}$$

$$= \sqrt{(10\ m/s^2)(11.0\ m)}$$

$$\boxed{v = 10.49\ m/s}$$

Medium



$$F_c = F_{Nx} = \frac{mv^2}{r}$$

$$F_{Nx} = F_N \sin\theta$$

$$F_N \sin\theta = \frac{mv^2}{r}$$

$$F_{Ny} = F_N \cos\theta = mg$$

$$F_N = \frac{mg}{\cos\theta}$$

$$\frac{\cancel{m}g\sin\theta}{\cos\theta} = \frac{\cancel{m}v^2}{r}$$

$$v^2 = \frac{r\cancel{m}g\tan\theta}{\cancel{m}} = rg\tan\theta$$

$$v = \sqrt{rg\tan\theta}$$

$$= \sqrt{(35.0\ m)(9.8\ m/s^2)(\tan(22))}$$

$$\boxed{v = 11.8\ m/s}$$

Hard:

- First of all, we know that:

$$F_c = F_{Nx} + f_{sx} = \frac{mv^2}{r}$$

- To find f_{static}, we must first find F_{Normal} and f_{static} (not components)

$$\frac{mv^2}{r} = F_N \sin\theta + f_s \cos\theta$$

- Now we can find F_N in the context of f_s. Knowing that:

$$F_g = mg = F_{Ny} + f_{sy}$$

$$mg = F_N \cos\theta + f_s \sin\theta$$

- So F_N equals:

$$F_N = \frac{mg - f_s \sin\theta}{\cos\theta}$$

- Substituting F_N:

$$\frac{mv^2}{r} = \left(\frac{mg - f_s \sin\theta}{\cos\theta}\right)\sin\theta + f_s \sin\theta$$

- Now to solve for static friction:

$$\frac{mv^2}{r} = (mg - f_s \sin\theta)\left(\frac{\sin\theta}{\cos\theta}\right) + f_s \sin\theta$$

$$\sim = (mg - f_s \sin\theta)\tan\theta + f_s \sin\theta$$

$$\sim = mg\tan\theta - f_s \sin\theta\tan\theta + f_s \sin\theta$$

$$\frac{mv^2}{r} - mg\tan\theta = -f_s \sin\theta\tan\theta + f_s \sin\theta$$

$$\sim = f_s(\sin\theta - \sin\theta\tan\theta)$$

- Finally:

$$f_s = \frac{\frac{mv^2}{r} - mg\tan\theta}{\sin\theta - \sin\theta\tan\theta}$$

- However, the question asks for μ_s, not f_s, so we must substitute.

$$\mu_s F_N = \frac{\frac{mv^2}{r} - mg\tan\theta}{\sin\theta - \sin\theta\tan\theta}$$

$$\mu_s = \frac{\frac{mv^2}{r} - mg\tan\theta}{(F_N)(\sin\theta - \sin\theta\tan\theta)}$$

- Now to substitute F_N:

$$\mu_s = \frac{\frac{mv^2}{r} - mg\tan\theta}{\left(\frac{mg - f_s\sin\theta}{\cos\theta}\right)(\sin\theta - \sin\theta\tan\theta)}$$

- Finally, substitute variables for values: $r \times F\sin\theta$

$$\mu_s = \frac{\frac{(500\,kg)(20\,m/s)^2}{(35\,m)} - (500\,kg)(9.8\,m/s^2)\tan(22)}{\left(\frac{(500\,kg)(9.8\,m/s^2) - (3462.6)(\sin(22))}{\cos(22)}\right)(\sin(22) - \sin(22)\tan(22))}$$

$$\int_0^L r F\,dr$$

$$F \cdot \int_0^L \tfrac{1}{2} r^2 \Big|_0$$

$$F \tfrac{1}{2} L^2$$

$$\frac{1}{2} mg L^2$$

$$\boxed{\mu_s = 0.89}$$