AP Physics
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Lab: AP Review Sheets

Chapter 4: Motion in Two Dimensions

Background / Summary

This unit covers motion in two dimensions, focusing on projectile, circular, and relative motion. It emphasizes using equations and vectors to describe how objects move and how their motion is observed from different reference frames.

Major Topics

Projectile motion, uniform circular motion, relative motion, tangential and radial acceleration, parabolic movement

Vocabulary

1. Vector components: the parts of a vector broken into perpendicular directions (usually x and y) that add together to give the original vector
2. Reference frame: a coordinate system or point of view from which position and motion are measured
3. Trajectory: the path that an object follows as it moves through space
4. Uniform circular motion: motion in a circle at a constant speed, where only the direction of velocity changes, not its magnitude
5. Centripetal acceleration: component of acceleration of an object moving in a circle that is directed radially inward toward the center of the circle
6. Projectile motion: motion of an object subject only to the acceleration of gravity
7. Tangential acceleration: the magnitude of which is the time rate of change of speed. Its direction is tangent to the circle.
8. Relative velocity: velocity of an object as observed from a particular reference frame, or the velocity of one reference frame with respect to another reference frame

Formulae

1. $x = vt$
2. $x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$
3. $\Delta x = \int v_x(t)dt$
4. $v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$
5. $v_x = v_{x0} + a_x t$
6. $v_{\frac{A}{B}} = v_A - v_B$
7. $y = vt - \frac{1}{2}at^2$
8. $a_c = \frac{v^2}{r}$
9. $a_t = \frac{dv}{dt}$
10. $F_c = \frac{mv^2}{r}$

Diagrams

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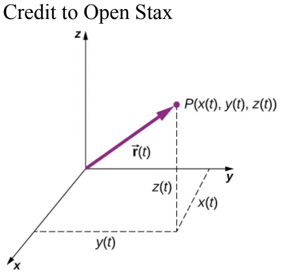

Figure 1: A 3-D coordinate system with a particle at position P(x(t), y(t), z(t))

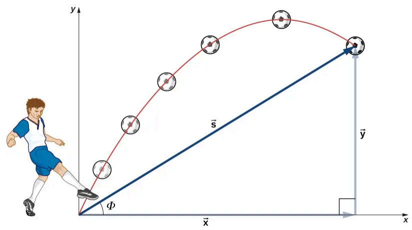

Figure 2: The displacement s of a soccer ball at a point along its path. The vector s has components x and y along the horizontal and vertical axes. Its magnitude is s, and it makes an angle ϕ with the horizontal.

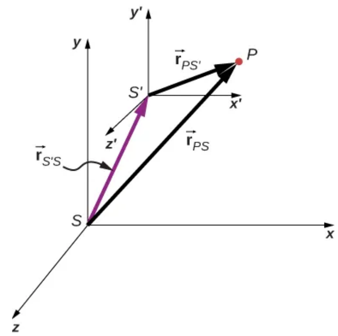

Figure 3: The positions of particle P relative to frames S and S' are r_{PS} and $r_{PS'}$, respectively

Problems

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Example 1: Polar Orbiting Satellite - Chapter 4.1

A satellite is in a circular polar orbit around Earth at an altitude of 400 km—meaning, it passes directly overhead at the North and South Poles. What is the magnitude and direction of the displacement vector from when it is directly over the North Pole to when it is at −45° latitude?

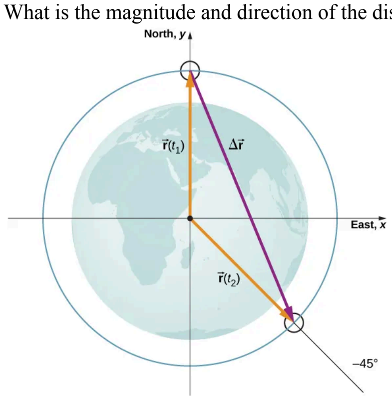

Figure 4: Two position vectors are drawn from the center of Earth, which is the origin of the coordinate system, with the y-axis as north and the x-axis as east. The vector between them is the satellite's displacement.

Solution

1. Although satellites move in 3-D space, they can be graphed in 2D. The position vectors are drawn from the center of the Earth (the origin). The vector between them is the satellite's displacement. We know the Earth's radius is 6370 km, so the length of each position vector is 6770 km.
2. In unit vector notation, the position vectors are
   a. $r(t_1) = 6770.\ km\hat{j}$
      $r(t_2) = 6770.\ km(cos(-45°))\ \hat{i} + 6770.\ km(sin(-45°))\ \hat{j}$
3. Evaluate the sine and cosine
   a. $r(t_1) = 6770.\ \hat{j}$
   b. $r(t_2) = 4787\hat{i} + 4787\hat{j}$
4. Find Δr
   a. $\Delta r = r(t_2) = r(t_1) = 4787\hat{i} - 11557\hat{j}$
5. Magnitude of the displacement is $|\Delta r| = \sqrt{(4787)^2 + (-11557)^2} = 12509\ km$
6. Angle that the displacement makes with the x-axis is $\theta = tan^{-1}(\frac{-11557}{4787}) =- 67.5°$

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Example 2: A Fireworks Projectile Explodes High and Away - Chapter 4.3

During a fireworks display, a shell is shot into the air with an initial speed of 70.0 m/s at an angle of 75.0° above the horizontal. The fuse is timed to ignite the shell just as it reaches its highest point above the ground. (a) Calculate the height at which the shell explodes. (b) How much time passes between the launch of the shell and the explosion? (c) What is the horizontal displacement of the shell when it explodes? (d) What is the total displacement from the point of launch to the highest point?

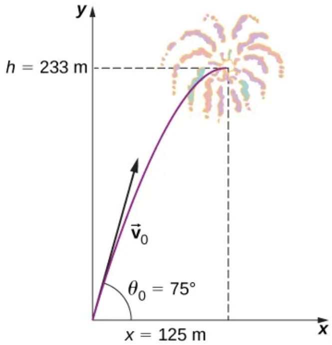

Figure 5: The trajectory of a fireworks shell. The fuse is set to explode the shell at the highest point in its trajectory, which is found to be at a height of 233m and 125m away horizontally.

Solution

(a) By "height" we mean the altitude or vertical position y above the starting point. The highest point in any trajectory, called the apex, is reached when $v_y = 0$. Since we know the initial and final velocities, as well as the initial position, we use the following equation to find y:

(i) $v_y^2 = v_{0y}^2 - 2g(y - y_0)$

Because y_0 and v_y are both zero, the equation simplifies to

(ii) $0 = v_{0y}^2 - 2gy$

Solving for y gives

(iii) $y = \frac{v_{0y}^2}{2g}$

Now we must find v_{0y}, the component of the initial velocity in the y direction. It is given by $v_{0y} = v_0 sin\theta_0$, where v_0 is the initial velocity of 70.0 m/s and $\theta_0 = 75°$ is the initial angle. Thus,

(iv) $v_{0y} = v_0 sin\theta = (70.0 m/s) sin75° = 67.6 m/s$

And y is

(v) $y = \frac{(67.6\, m/s)^2}{2(9.80\, m/s^2)}$

Thus, we have

(vi) y=233m

Note that because up is positive, the initial vertical velocity is positive, as is the max height, but acceleration resulting from gravity is negative. Note also that the max height depends only on the vertical component of the initial velocity, so that any projectile with a 67.6-m/s initial vertical component of velocity reaches a maximum height of 233 m (neglecting air resistance). The numbers in this example are reasonable for large fireworks displays, the shells of which do reach such heights before exploding. In practice, air resistance is not completely negligible, so the initial velocity would have to be somewhat larger than that given to reach the same height.

(b) As in many physics problems, there is more than one way to solve for the time the projectile reaches its highest point. In this case, the easiest method is to use $v_y = v_{0y} - gt$. Because $v_y = 0$ at the apex, this equation reduces to simply

(i) $0 = v_{0y} - gt$ or $t = \frac{v_{0y}}{g} = \frac{67.6 m/s}{9.80 m/s^2} = 6.90s$

This time is also reasonable for large fireworks. If you are able to see the launch of fireworks, notice that several seconds pass before the shell explodes. Another way of finding the time is by using

(ii) $y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$

(c) Because air resistance is negligible, $a_x = 0$ and the horizontal velocity is constant, as discussed earlier. The horizontal displacement is the horizontal velocity multiplied by time as given by $x = x_0 + v_x t$, where x_0 is equal to zero. Thus,

(i) $x = v_x t$

where v_x is the x-component of the velocity, which is given by

(ii) $v_x = v_0 cos\theta = (70.0 m/s) cos75° = 18.1 m/s$

Time t for both motions is the same, so x is

(iii) $x = (18.1 m/s) 6.90 s = 125 m$

Horizontal motion is a constant velocity in the absence of air resistance. The horizontal displacement found here could be useful in keeping the fireworks fragments from falling on spectators. When the shell explodes, air resistance has a major effect, and many fragments land directly below.

(d) The horizontal and vertical components of the displacement were just calculated, so all that is needed here is to find the magnitude and direction of the displacement at the highest point:

(i) $s = 125\hat{i} + 233\hat{j}$

(ii) $|s| = \sqrt{125^2 + 233^2} = 264 m$

(iii) $\phi = tan^{-1}(\frac{233}{125}) = 61.8°$

Example 3: Motion of a Car Relative to a Truck - Chapter 4.5

A truck is traveling south at a speed of 70 km/h toward an intersection. A car is traveling east toward the intersection at a speed of 80 km/h. What is the velocity of the car relative to the truck?

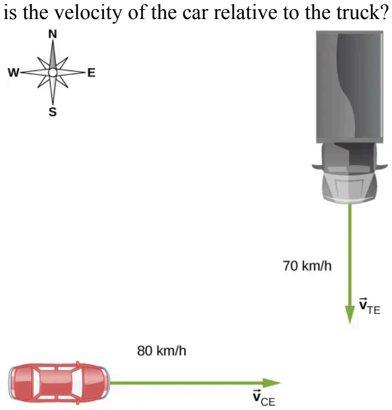

Figure 6: A car travels east toward an intersection while a truck travels south toward the same intersection

Solution

1. The velocity of the car with respect to Earth is $v_{CE} = 80 km/h\hat{i}$. The velocity of the truck with respect to Earth is $v_{TE} = -70 km/h\hat{j}$. Using the velocity addition rule, the relative motion equation we are seeking is
   a. $v_{CT} = v_{CE} + v_{ET}$
2. Here, v_{CT} is the velocity of the car with respect to the truck, and Earth is the connecting reference frame. Since we have the velocity of the truck with respect to Earth, the negative of this vector is the velocity of Earth with respect to the truck: $v_{ET}=-v_{TE}$.
3. We can now solve for the velocity of the car with respect to the truck:
   a. $|v_{CT}| = \sqrt{(80.0 km/h)^2 + (70.0 km/h)^2} = 106. km/h$
   b. $\theta = tan^{-1}(\frac{70.0}{80.0}) = 41.2°$ north of east.