AP Physics C: Mechanics Lab: Review Sheet

Chapter 13: Universal Gravitation

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Background:	Major Topics:
This unit goes over how everything exerts gravitational forces on every other thing, which is more specifically known as Newton's Law of Universal Gravitation, how this law relates to potential energy, and how orbits and distance really affect gravitational forces.	• Universal Law of Gravitation • Potential Energy of Universal Gravitation • Kepler's Laws

Unit Vocabulary

- *Aphelion:* the point in the orbit of a body of mass at which it is furthest away from the larger body of mass it orbits.
- *Perihelion:* the point in the orbit of a body of mass at which it is closest to the larger body of mass it orbits.
- *Gravity field:* an area where an object's gravity applies a non-negligible gravitational force on another object.
- *Satellite:* a general term used for any body that is in orbit around another

Important Formulae / Constants:

$$F_g = -G\frac{Mm}{r^2}\hat{r} \text{ or } F_g = G\frac{Mm}{r^2}$$

$$F_c = \frac{mv^2}{r}$$

$$U = M_{earth}\frac{-Gm}{r}$$

$$v = \sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$$

$$v_{escape} = \sqrt{\frac{2GM}{R}}$$

$$a_g = \frac{GM}{r^2}$$

$$W_g = -\Delta U = \int_{xi}^{xf} F_g\, dx$$

$$U_{total} = \Sigma\ U_i$$

$$G = 6.67 \times 10^{-11}\ N\frac{m^2}{kg^2}$$

$$M_{Earth} = 5.97 \times 10^{24}\ kg$$

$$R_{Earth} = 6.37 \times 10^{6}\ \text{m}$$

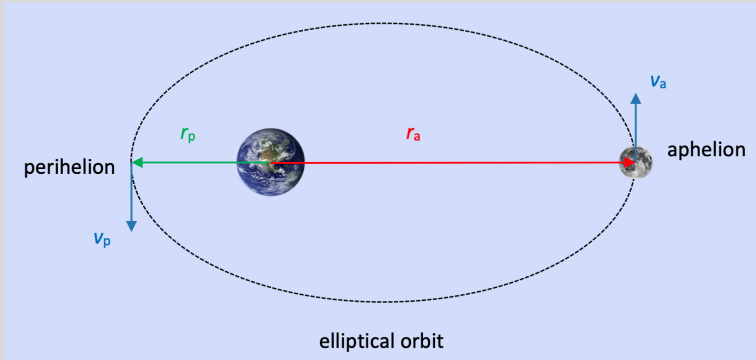
Universal Law of Gravitation:

- Every particle attracts every other particle in the universe with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between their centers of mass
- Basically, everything in the universe attracts each other, where large distances between objects lower the gravitational force, and small masses lower the gravitational force. That's why an apple doesn't exert a noticeable gravitational field, and that's why a planet at one end of the Milky Way doesn't affect one at the other.

Important Points:

Work from gravity is always going to be negative because a satellite infinitely far away from another body, where it has zero gravitational potential energy, loses potential energy as it gets pulled closer to the larger body. This means that the gravitational potential energy of a system has a negative value because gravitational potential energy decreases as more gravity pulls the satellite closer.	As you get further from Earth, the acceleration due to gravity doesn't stay 9.8 m/s, hence, we use equations regarding Universal Gravitation to find the acceleration due to gravity at different points.

Orbits
Satellites tend to move in elliptical orbits, where conservation of angular momentum can be applied to find the aphelion and perihelion. To find the energy of a satellite moving through a gravitational field, we can use this equation, $U = M_{earth} \frac{-Gm}{r}$ or $-\Delta U = \int_{xi}^{xf} F_g \, dx$, where x_i is infinity.



Still, sometimes we will analyze circular orbits because this is an intro physics class.

Kepler's Laws

1. All planets move in elliptical orbits, with the Sun at one of the focal points.
2. The radius vector drawn from the Sun to a planet sweeps out equal areas in equal time intervals.
3. For two different planets orbiting the sun at different distances, use the equation:

$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$$

Questions:

1. Two planets have masses of $4.3. \times 10^{24}$ kg and 7.6×10^{23} kg respectively, and are 5.2×10^{7} m apart. What is the gravitational force between them?

2. A satellite of mass 1350 kg is in a circular orbit around the Earth 450 km above the Earth's surface.
   a. Determine the speed of the satellite.
   b. Find the period of its orbit.

3. Calculate the escape velocity from the surface of a planet with a mass of 6.2×10^{24} kg and a radius of 7.34×10^{6} m.

Answers:

1. Use $F = G\frac{m_1 m_2}{r^2}$

$G = 6.674e{-11} \frac{Nm^2}{kg^2}$

$m_1 = 4.3e24$ kg

$m_2 = 7.6e23$ kg

$r = 5.2e7$ m

$$F = G\frac{(4.3e24)(7.6e23)}{(5.2e7)^2}$$

$$\boxed{F = 8.06e22 \text{ N}}$$

2. a) Use $v = \sqrt{\frac{GM}{r}}$

$G = 6.674e{-11} \frac{Nm^2}{kg^2}$

$M_e = 5.97e24$ kg

$R_e = 6.37e6$ m

$$v = \sqrt{\frac{G(5.97e24)}{(6.37e6)}}$$

$$\boxed{v = 7.91e3 \text{ m/s}}$$

b) $$T = \frac{2\pi r}{v} = \frac{2\pi(6.37e6)}{(7.91e3)} = \boxed{5.06e3 \text{ seconds}}$$

3. Use $V_{esc} = \sqrt{\frac{2GM}{r}}$

$$V_{esc} = \sqrt{\frac{2(6.67e{-11})(6.2e24)}{(7.34e6)}}$$

$$\boxed{V_{esc} = 1.06e4 \text{ m/s}}$$