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Lab: AP Physics Review Sheets
Ch. 10 Rotation of a Rigid Object About a Fixed Axis

Brief summary:
This chapter explores the physics of circular motion and friction, discusses how different types of forces can cause both linear and rotational motion. It covers the key concepts of static and kinetic friction, centripetal force, resistive (drag) forces, and more.

Major topics:
1. Angular kinematics
2. Moment of Inertia
3. Torque
4. Torque and Angular Acceleration
5. Rotational Work and Power

Cross Product (Vector Multiplication)

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

Formulae:

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$s = r\theta$

$v = r\omega$

$a = r\alpha$

$\vec{\tau} = \vec{r} \times \vec{F} = rF\sin\theta$

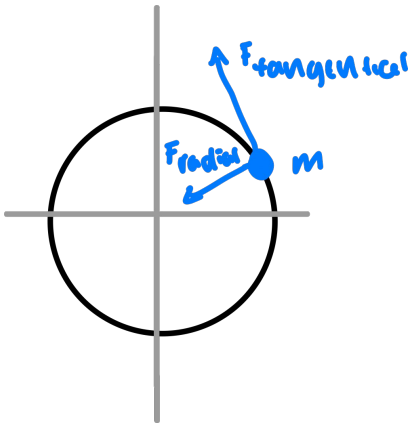
$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum mr^2$

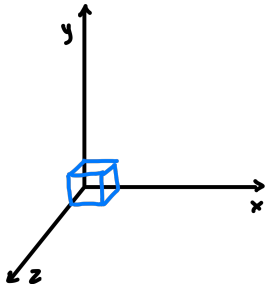
$K = \frac{1}{2} I\omega^2$

Key terms/vocabulary:
Angular position, velocity, and acceleration - their angular forms
Tangential velocity - linear speed along circular path
Moment of inertia (I) - rotational mass resistance
Torque - rotational force
Rotational kinetic energy - energy of a spinning object
Rotational work - Torque x angular displacement (rotational form)
Rotational power - Torque x angular velocity (rotational form)
Unit vectors (i, j, k) - directional basis vectors (x, y, z); used for cross product calculations

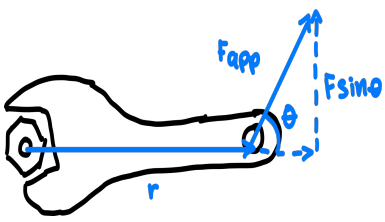
Circular Motion Force:



RHR Cross Product Vectors:



Torque Components:



Practice problems:

Source: *All questions have been borrowed from the Chapter 10 review questions from University Physics Volume 1 by Jeff Sanny, Samuel J. Ling, and William Moebs*

1. [easy]

Question 29
A track star runs a 400-m race on a 400-m circular track in 45 s. What is his angular velocity assuming a constant speed?

2. [medium]

Question 37

A wheel has a constant angular acceleration of 5.0 rad/s^2. Starting from rest, it turns through 300 rad. (a) What is its final angular velocity? (b) How much time elapses while it turns through the 300 radians?

3. [hard]

Question 85

Suppose you exert a force of 180 N tangential to a 0.280-m-radius, 75.0-kg grindstone (a solid disk). (a)What torque is exerted? (b) What is the angular acceleration assuming negligible opposing friction? (c) What is the angular acceleration if there is an opposing frictional force of 20.0 N exerted 1.50 cm from the axis?

Solutions:

1)

distance = 400 m

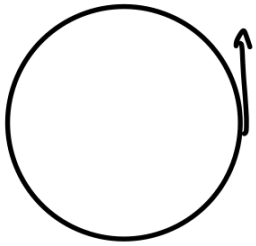
$t = 45\text{ s}$

$\theta = 2\pi$ rad (angular displacement)

$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$\omega = \frac{2\pi}{45} = \boxed{0.140\ \text{rad/s}}$$

2)



$\alpha = 5.0\ \text{rad/s}^2$

$\omega_0 = 0\ \text{rad/s}$

$\theta = 300$ rad

a) kinematic equation (angular version)

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\omega^2 = 0 + 2(5.0)(300) = 3{,}000$$

$$\omega = \sqrt{3{,}000} \approx \boxed{54.77\ \text{rad/s}}$$

b) kinematic equation

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$\omega_0 = 0$ (starts from rest)

$$\theta = \frac{1}{2}\alpha t^2$$

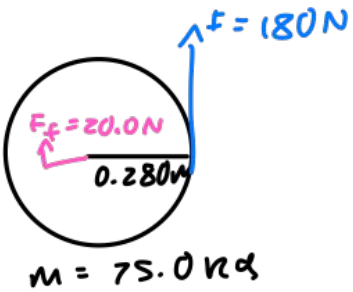
$$t^2 = \frac{2\theta}{\alpha}$$

$$\frac{2(300)}{5.0} = 120$$

$$t = \sqrt{120}$$

$$t = \boxed{10.95\ \text{s}}$$

3.



a) $\tau = F \cdot r$ force to rotational relationship

$\tau = 180(0.280)$

$\tau = \boxed{50.4\ Nm}$

b) another conversion

$\Sigma \tau = I\alpha$

$\alpha = \frac{\Sigma \tau}{I}$

$I = \frac{mr^2}{2}$ moment of inertia

$\alpha = \frac{F_f}{\frac{mr^2}{2}}$

$\alpha = \frac{2F}{mr} = \frac{2(180)}{75(0.280)}$

$\alpha = \boxed{17.1\ rad/s^2}$

c) back to the components

$\alpha = \frac{\Sigma \tau}{\frac{mr^2}{2}} = \frac{Fr - F_f r}{\frac{mr^2}{2}}$

plugging in

$\alpha = \frac{180(0.280) - 20.0(0.0150)}{\frac{75.0(0.280)^2}{2}}$

$\alpha = \boxed{17.0\ rad/s^2}$

↑ angular acceleration