

Chapter 6: Circular Motion

Background:

Unit 6, Circular motion, describes the movement of an object in the path of a circle. Even when speed is constant, the velocity of the object is changing direction; therefore, there must be an acceleration, which points towards the middle of the circle, known as the centripetal acceleration. This acceleration requires a net inward force, which is known as, surprisingly, centripetal force, which can come from gravity, tension, and more. These are the foundational ideas that one must understand to solve these problems

Key Topics

- **Sliding friction (static v.s. kinetic)**
- **Centripetal force**
- **Horizontal circular motion**
- **Vertical circular motion**
- **Non-uniform circular motion**
- **“Centrifugal force” misconception**
- **Resistive Forces**

Vocabulary:

- **Centripetal Acceleration:** The acceleration of an object moving in a circle that always points inward, changing the direction of the velocity, not necessarily the speed.
- **Centripetal Force:** The net force directed towards the center of the circular path that causes centripetal acceleration. It is **NOT** a new force; rather, caused by the likes of gravity or tension.
- **Tangential Velocity:** The instantaneous velocity of an object moving in a circle, directed along the edge of the circular path.
- **Static Friction:** The frictional force that prevents motion between two surfaces. It adjusts in size to match the applied force up to a maximum value.
- **Kinetic Friction:** The frictional force acting when two surfaces are sliding past each other. It has a constant value.
- **Normal Force:** The force exerted by a surface perpendicular to an object in contact with it.
- **Period T :** The time required to complete one revolution (one full circle). Frequency is inversely related to period $\rightarrow f = 1/T$
- **Angular Velocity ω :** The rate at which an object rotates, measured in radians per second.
 $\omega = 2\pi/T$
- **Centrifugal Force:** Doesn't exist! The movement outwards of the center of the circle is a product of inertia (objects wanting to move in a straight line), and not a force.

Equations:

CENTRIPETAL FORCE + (ACCELERATION)

$$F_c = \frac{mv^2}{r} \Rightarrow \sum F_{\text{inward}} = \frac{mv^2}{r} \quad a_c = \frac{v^2}{r}$$

CIRCULAR MOTION RELATIONSHIPS

$$v = \frac{2\pi r}{T} \quad \omega = \frac{2\pi}{T} \quad v = r\omega \quad a_c = r\omega^2$$

FRICTION

$$F_f = \mu F_n$$

HORIZONTAL CIRCULAR MOTION

$$F_c = F_f = \mu_s mg \quad \frac{mv^2}{r} = \mu_s mg \quad v_{\text{max}} = \sqrt{\mu_s gr}$$

BANKED CURVE

• NO FRICTION $\rightarrow \tan \theta = \frac{v^2}{rg}$; w/ Friction \rightarrow vertical = $F_n \cos \theta$
horizontal = $F_n \sin \theta$
 $+ \sum F_{\text{inward}} = \frac{mv^2}{r}$

VERTICAL CIRCULAR MOTION

$\sum F_{\text{towards center}} = \frac{mv^2}{r}$ \rightarrow top of circle: $\frac{mv^2}{r} = mg + T$
 \rightarrow side of circle: $\frac{mv^2}{r} = T$ \rightarrow bottom of circle: $\frac{mv^2}{r} = T - mg$

DRAG + RESISTIVE FORCES

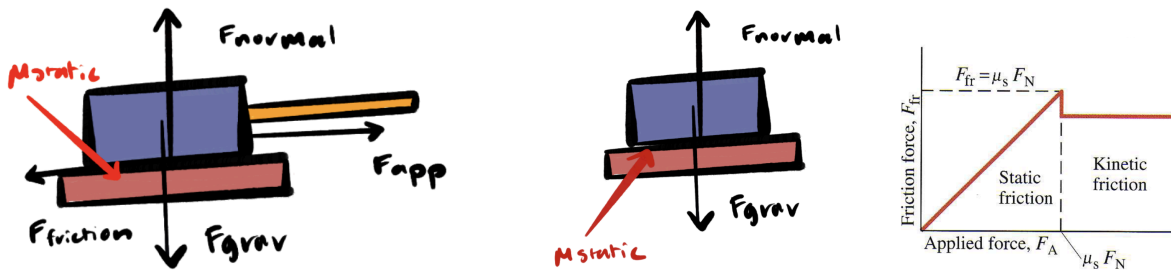
$$F_{\text{drag}} = bv, cv^2$$

Terminal velocity: $\sum F = 0$

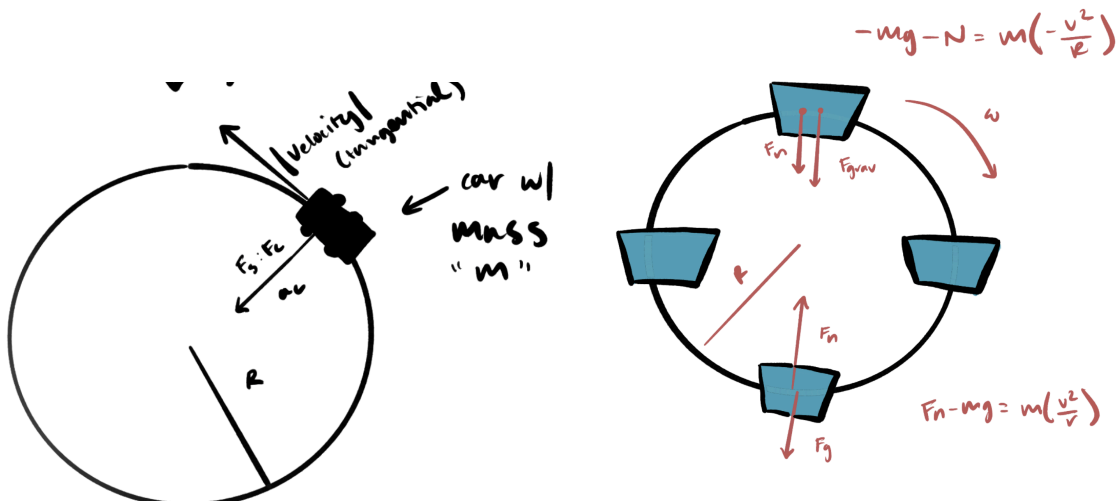
$$\rightarrow mg = F_{\text{drag}}$$

Diagrams:

Friction:



Horizontal and Vertical Circular Motion



3 Problems:

EASY: A 2.0 kg mass is attached to a string and swung in a horizontal circle of radius 1.5 m at a constant speed of 3.0 m/s.

- Find the centripetal acceleration
- Find the centripetal force

MEDIUM: A 1000 kg car travels around a flat circular curve of radius 50 m. The coefficient of static friction is 0.40.

- What is the maximum speed the car can travel without slipping?
- What provides the centripetal force?

HARD: A 0.5 kg ball is attached to a string that is 1.2 m long, swung in a vertical circle. At the bottom, the speed is 6.0 m/s.

- Find the tension at the bottom of the path
- Find the speed at the top of the path
- Determine if the string goes slack at the top

SOLUTIONS:

Easy: a) acceleration: $a_c = \frac{v^2}{r}$

$$\rightarrow \frac{3.0^2}{1.5} = \frac{9}{1.5}$$

$$\boxed{= 6.0 \text{ m/s}^2}$$

b) force: $F_c = ma$

$$\rightarrow (2.0)(6.0) = 12$$

$$\boxed{12 \text{ N}}$$

Medium: a) set centripetal = Friction

$$\frac{mv^2}{r} = Ms mg$$

$$\rightarrow \frac{v^2}{r} = Ms g$$

$$\rightarrow v = \sqrt{Ms gr} = \sqrt{(4)(9.8)(2.0)}$$

$$\boxed{v = 14 \text{ m/s}}$$

b) the centripetal force is provided by static friction

HARD:

a) tension @ bottom:

$$\frac{mv^2}{r} = T - mg$$

$$\rightarrow T = \frac{mv^2}{r} + mg$$

$$\rightarrow T = \frac{(5)(36)}{1.2} + (5)(9.8)$$

$$\rightarrow T = 15 + 4.9 = 19.9$$

$$\boxed{T = 19.9 \text{ N}}$$

c) based on part b, since there is not enough energy to reach the top, the ball cannot complete the circle. Therefore, the string goes slack before reaching the top.

b) speed @ top...
height change = $2r$
 $\rightarrow 2.4 \text{ m}$

$$\frac{1}{2} mv^2 = \frac{1}{2} mv^2 + mg(2r)$$

$$\rightarrow \frac{1}{2}(36) = \frac{1}{2}v^2 + 9.8(2.4)$$

$$18 = \frac{1}{2}v^2 + 23.52$$

$$\frac{1}{2}v^2 = -5.52 \dots$$

impossible \rightarrow
speed is too low