Chapter 9: Momentum

AP Physics
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Background/Summary: In this unit, we will learn what momentum and impulse is and how they relate to forces. We will also look at the conservation of momentum in different types of collisions as well as center of mass.

| Key Points |
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| What is linear momentum? <br> - Quantity of motion <br> - Caused by a force applied over time |
| What is impulse? <br> - Amount of force applied over a given time <br> - A change in momentum |
| Conservation of Momentum <br> - Similarly to conservation of energy, this states that when two or more particles interact without outside forces, their total linear momentum stays the same <br> - $\mathrm{p}_{1}+\mathrm{p}_{2}=\mathrm{p}_{1}{ }^{\prime}+\mathrm{p}_{2}{ }^{\prime}$ |
| Types of Collisions <br> - Inelastic collisions <br> - Collision where some energy is "lost"(converted) to heat <br> - Perfectly inelastic collision <br> - An inelastic collision, but the objects stick together after their collision <br> - Elastic collision <br> - A collision where the amount of energy converted to heat is negligible |


| Important Formulae | Uses |
| :---: | :---: |
| $p=m v$ | Equation to calculate momentum |
| $\mathrm{F}=\mathrm{dp} / \mathrm{dt}$ | Equation relating force to change in momentum |
| $J=\int_{t i}^{t f} F d t=\Delta p$ | Ways to calculate impulse/change in momentum |
| $p_{1}+p_{2}=p_{1}{ }^{\prime}+p_{2}{ }^{\prime}$ | Law of conservation of momentum |
| $\begin{aligned} & p_{1}+p_{2}=p_{1}{ }^{\prime}+p_{2}^{\prime} \\ & K_{1}+K_{2}=K_{1}^{\prime}{ }^{\prime}+K_{2}^{\prime} \end{aligned}$ | Used in conjunction to solve elastic collisions |
| $\begin{aligned} & p_{1}+p_{2}=p^{\prime} \\ & m_{1} v_{1}+m_{2} v_{2}=\left(m_{1}+m_{2}\right) v^{\prime} \end{aligned}$ | Used to solve a perfectly inelastic collision |
| $p_{1}+p_{2}=p_{1}{ }^{\prime}+p_{2}{ }^{\prime}$ | Used to solve inelastic collision |
| $c m=\frac{\Sigma m_{i} x_{i}}{\Sigma m_{i}}$ | Finding center of mass |
| $c m=\frac{1}{M} \int x d m$ | Finding center of mass for continuous distribution |

## Free Response Question \# 1



1. A 10.0 kg ball hits a wall going $20 \mathrm{~m} / \mathrm{s}$ and rolls away at $5 \mathrm{~m} / \mathrm{s}$. First, calculate the impulse that the wall exerts on the ball. Then, given that the collision takes .156 seconds, calculate the average force exerted on ball during the collision

To calculate impulse, we can use the equations given above to see that $\mathrm{J}=\Delta \mathrm{p}$.

$$
\begin{aligned}
& J=m v_{f}-m v_{i} \\
& =10.0(-5)-(10.0)(20) \\
& =-250 \mathrm{~kg} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

To calculate average force, we can use equation $\mathrm{J}=\mathrm{F} \Delta \mathrm{t}$

$$
F=J / \Delta t=-250 / .156
$$

$$
F=-1600 N
$$

## Free Response Question \# 2


2. A bullet with mass $m$ hits a ballistic pendulum with length $L$ and mass $M$ and lodges in it. When the bullet hits the pendulum it swings up from the equilibrium position and reaches an angle $\alpha$ at its maximum. Determine the bullet's velocity when it hits the pendulum

Using the key word lodges in it, we know that this is a perfectly inelastic collision problem.
$m v_{1}+M v_{2}=(m+M) v^{\prime}$
$\mathrm{V}_{2}=0$ because pendulum starts at rest
$v_{1}=\frac{(m+M) v^{\prime}}{m}$
When the pendulum swings upwards, there is a conservation of energy where all the initial kinetic energy is converted to potential energy at its maximum
$K_{i}=U_{f}$
$\frac{1}{2}(m+M) v^{\prime 2}=(m+M) g h$
$\mathrm{h}=\mathrm{L}-\mathrm{L} \cos (\boldsymbol{\alpha})=\mathrm{L}(1-\cos (\boldsymbol{\alpha}))$
$v^{\prime}=\sqrt{2 g L(1-\cos (\boldsymbol{\alpha}))}$
Therefore,
$v_{1}=\frac{(m+M) \sqrt{2 g L(1-\cos (\boldsymbol{\alpha}))}}{m}$

## Free Response Question \# 3


3. Calculate $X_{\mathrm{cm}}$ and $\mathrm{Y}_{\mathrm{cm}}$ for the system above

Using $c m=\frac{\Sigma m_{i} x_{i}}{\Sigma m_{i}}$ equation from our equation list above, we can calculate the component center of masses for the system
$X_{\mathrm{cm}}=\mathrm{m}_{1} \mathrm{X}_{1}+\mathrm{m}_{2} \mathrm{X}_{2}+\mathrm{m}_{3} \mathrm{X}_{3} /\left(\mathrm{m}_{1}+\mathrm{m}_{2}+\mathrm{m}_{3}\right)$
$=\frac{12(3)+24(0)+36(7)}{12+24+36}$
$=\frac{288}{72}$
$X_{c m}=4$
$Y_{\mathrm{cm}}=\mathrm{m}_{1} \mathrm{y}_{1}+\mathrm{m}_{2} \mathrm{y}_{2}+\mathrm{m}_{3} \mathrm{y}_{3} /\left(\mathrm{m}_{1}+\mathrm{m}_{2}+\mathrm{m}_{3}\right)$
$=\frac{12(0)+24(2)+36(5)}{12+24+36}$
$=\frac{228}{72}$
$\mathrm{Y}_{\mathrm{cm}}=3.17$

