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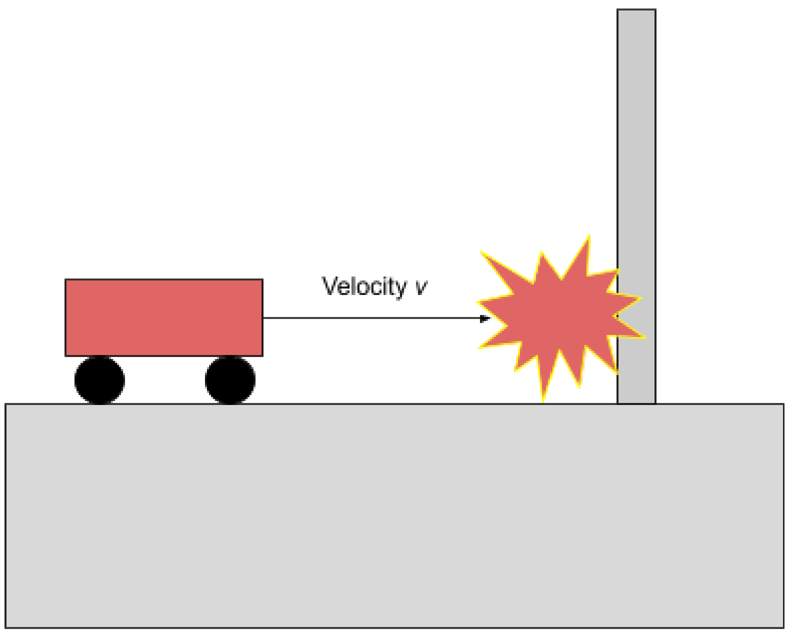
Momentum

Background: This unit covers how to measure an object's quantity of motion, or momentum, when it is affected by a force. It also covers an important law:the law of conservation of momentum, which is very important analyzing objects that interact through forces.

1. Linear Momentum
2. Impulse
3. Conservation of Momentum
4. Collisions: Elastic and Inelastic
5. Center of Mass

Key Takeaways:

1. **Linear Momentum** is measured as an object's quantity of motion, and is a product of an object's mass and velocity
2. **Impulse** is the force applied on an object by another object for a period of time, such as a moving cart crashing into a wall:

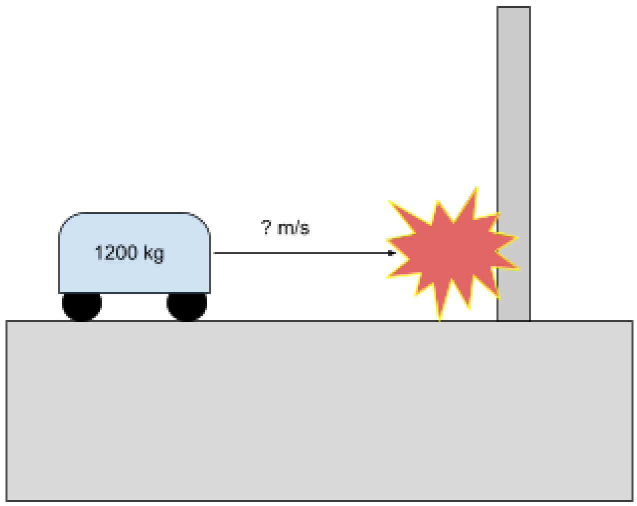


3. Because impulse is just force applied to time, Newton's third law of motion still applies to impulse, in that **an object that applies an impulse will receive an impulse of equal magnitude and opposite direction**.
4. This leads to our definition of **The Law of Conservation of Linear Momentum:** If two or more objects interact with one another in an isolated system, momentum will be conserved
5. Many examples of the transfer of linear momentum are through collisions, and there are different types of collisions in which are encountered when doing these kinds of collisions:
    a. **Elastic Collision:**A collision where little to no kinetic energy is lost to heat in the act of the collision, so kinetic energy K is completely conserved.
    b. **Inelastic Collision**:A collision where some energy is lost to heat in the act of the collision, so kinetic energy is not conserved
        i. Collision problems are usually inelastic unless specified.
    c. **Perfectly Inelastic Collision:**A collision where after the two objects collide, they stick together, and therefore have the same velocity as a combined mass
6. **Center of Mass** is defined as the position in an object where the position vectors of an object add up to zero, or where the object is completely balanced when held at that point.

Important Formulae:

Momentum is a vector that is the product of an object's mass and its velocity	$\vec{p} = m\vec{v}$ Units of impulse: Newton-Seconds
Deriving Momentum from Newton's Second Law	$F_{net} = ma$ $F_{net} = (m)\frac{\Delta v}{\Delta t}$ $F_{net} = \frac{mv_f - mv_i}{\Delta t}$ Because $\vec{p} = m\vec{v}$ $F_{net} = \frac{p_f - p_i}{\Delta t}$ $\vec{F} = \frac{d\vec{p}}{dt}$
Impulse is the force applied over a time interval, and also represents change in momentum	$\vec{J} = \int_{t_i}^{t_f} \vec{\ } dt = \Delta\vec{p} = F\Delta t = m\Delta v$
Law of Conservation of Linear Momentum: If two or more objects interact with one another in an isolated system, momentum will be conserved	$\vec{p}_1 + \vec{p}_2 + \ldots \vec{p}_n = \vec{p}_1' + \vec{p}_2' + \ldots \vec{p}_n'$ OR $m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_n\vec{v}_n = m_1\vec{v}_1' + m_2\vec{v}_2' + \ldots + m_n\vec{v}_n'$
Elastic Collision: A collision where little to know kinetic energy is lost to heat in the act of the collision, so kinetic energy *K* is completely conserved.	$\vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2'$ AND $K_1 + K_2 = K_1' + K_2'$
Perfectly Inelastic Collision:A collision where after the two objects collide, they stick together, and therefore have the same velocity with a shared mass	$\vec{p}_1 + \vec{p}_2 = \vec{p}\ '$ $m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\ \vec{v}'$
Calculating Center of Mass in 2-3 dimensions for point masses	$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \ldots + m_nx_n}{m_1 + m_2 + m_3 + \ldots + m_n} = \frac{\Sigma m_ix_i}{\Sigma m_i}$ $y_{cm} = \frac{\Sigma m_iy_i}{\Sigma m_i}$, $z_{cm} = \frac{\Sigma m_iz_i}{\Sigma m_i}$ $r_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k} = \frac{\Sigma m_ix_i\hat{i} + m_iy_i\hat{j} + m_iz_i\hat{k}}{M}$
Calculating Center of Mass for continuous distribution of mass	$x_{cm} = \frac{1}{M}\int x\, dm$, for a given axis(x, y, z)
Density Functions and Constants	Length: $\lambda = \frac{dm}{dL}$, $dm = \lambda\, dL$ Area: $\sigma = \frac{dm}{dA}$, $dm = \sigma\, dA$ Volume: $\rho = \frac{dm}{dV}$, $dm = \rho\, dV$
System in Motion and Center of Mass	$\frac{d}{dt}\vec{r}_{cm} = \frac{d}{dt}\frac{\Sigma m_i\vec{r}_i}{M}$ $\vec{v}_{cm} = \frac{\Sigma m_i\vec{v}_i}{M}$ $M\vec{v}_{cm} = \vec{p}_{total}$

Practice Problems



1. A car weighing 1200kg is moving at an unknown velocity v when it crashes into a wall and comes to rest. During the crash, the car applies a force of 4.5×10^4N to the wall over a time period of .400s.
    a. What is the impulse, magnitude and direction of the car on the wall? The wall on the car?
    b. How fast was the car traveling before it hit the wall?

Answers:

1a.

Impulse is defined as the force applied over a period of time, or $J = F\Delta t$

$J = F\Delta t$

$J = 45000N(.400s) = 18000Ns\ to\ the\ right$

Since there is an equal and opposite impulse from the wall, the impulse from the wall will be 18000Ns to the left

1b.

Knowing that $J = F\Delta t = m\Delta v$

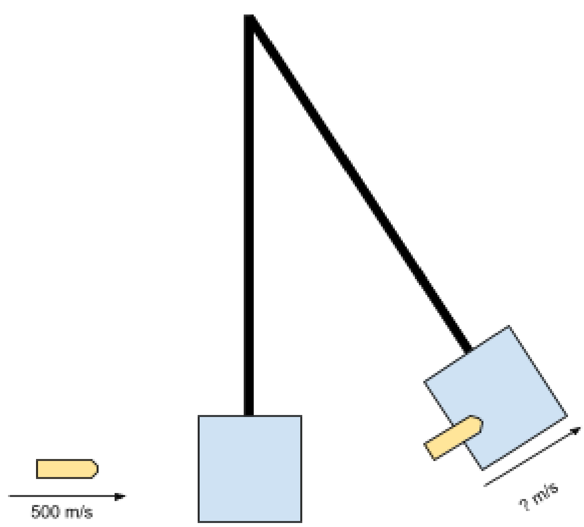
$J = m\Delta v$

Plugging in values(J is negative because the impulse from the wall is what is slowing the car down):

$-18000Ns = 1200kg(0m/s - v_i)$

$-v_i = -\frac{-18000Ns}{1200kg}$

$v_i = 15m/s$



2. A ballistic pendulum system with a wood block is set up(see above). A bullet weighing 4.0 grams moving to the right at a velocity 500 m/s collides with and sticks into the 2.0 kg wood block, which is initially at rest.
    a. How fast is the woodblock-bullet unit moving immediately after the collision?
    b. What is the maximum height, in centimeters, of the woodblock unit as it swings upward?

Answers:

2a.

This is an example of a perfectly inelastic collision, so we know that

$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\,\vec{v}'$

$m_{bullet}\vec{v}_{bullet\ initial} + m_{block}\vec{v}_{block\ initial} = (m_{bullet} + m_{block})\,\vec{v}'$

$\vec{v}' = \frac{m_{bullet}\vec{v}_{bullet\ initial} + m_{block}\vec{v}_{block\ final}}{(m_{bullet}+m_{block})}$

$\vec{v}' = \frac{(.004kg)(500m/s)+(2.0kg)(0m/s)}{(2.0kg+.004kg)}$

$\vec{v}' = .998\,m/s$

2b.

Now that we know the initial velocity of the unit, we can use the conservation of energy to find this maximum height:

$U_i + K_i = U_f + K_f$

$mgh_i + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv_f^2$

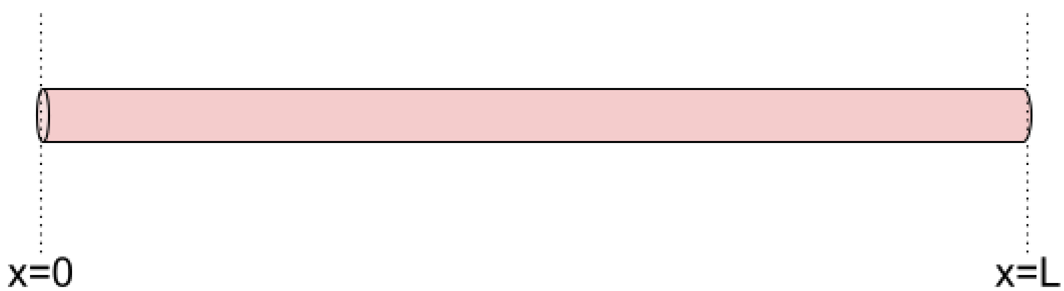
v_f is zero because the block is at maximum height, and considering height initial to be zero:

$\frac{1}{2}mv_i^2 = mgh_f$

$h_f = \frac{\frac{1}{2}mv_i^2}{mg} = \frac{v_i^2}{2g}$

Plugging in values:

$h_f = \frac{\frac{1}{2}(.998m/s)^2}{9.80m/s^2} = .0508m = 5.08cm$



3. Determine the center of mass of a long rod(see above) with a changing density $\lambda = \frac{1}{3}x^2$

Answers:

3.

First, we must define how to calculate the center of mass with continuous distribution of density:

$x_{cm} = \frac{1}{M}\int x\,dm$

Using the density constant equation to substitute for dm:

$\lambda = \frac{dm}{dx},\ dm = \lambda\,dx = \frac{1}{3}x^2dx$

$x_{cm} = \frac{1}{M}\int_0^L \frac{1}{3}x^3dx$

$x_{cm} = \frac{1}{12M}\,(L^4 - 0^4)$

$x_{cm} = \frac{L^4}{12M}$

Now using density constant equation to find expression for M in terms of x

$M = \int_0^L dm = \int_0^L \frac{1}{3}x^2dx = \frac{1}{9}L^3$

Plugging in our expression for M:

$x_{cm} = \frac{L^4}{12M} = \frac{L^4}{12(\frac{1}{9}L^3)}$

$x_{cm} = \frac{3L}{4}$

Sources used in making review sheet: crashwhite.com, openstax textbook volume 1