Capacitance

Cormac Murran

Key Equations

$C = Q / V$

* This is really the entire unit. Often used in conjunction with Gauss's law to find capacitance

$C_{(equivalent)} V_{(Battery)} = C_1V_1 + C_2V_2$

$E = -D$

$C \text{ total (series)} = 1/C_1 + 1/C_2 + \ldots$

$C \text{ total (parallel)} = C_1 + C_2 + \ldots$

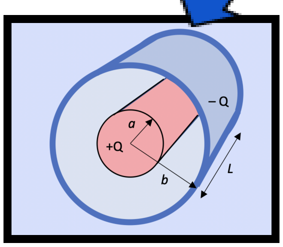
*These equations for capacitance in series and parallel can be remembered as the opposite as for resistance.

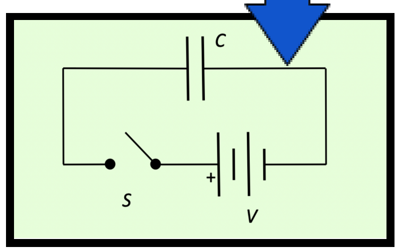
Basic Background

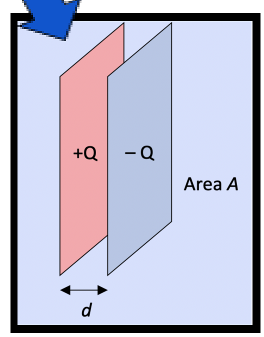
Our unit on capacitance combines previous knowledge of Gauss's Law, Electric Force, Field, Potential, and Potential Energy with new knowledge on capacitors to analyze circuits with capacitors and isolated capacitors. Capacitance is a measurement of the ability for a capacitor to store charge. This unit we mostly look at parallel plate and cylindrical capacitors in and out of circuits.

Major Topics

- Theorectical Capacitance of a system
- Capacitors in Series and Parallel
- Charging and Energy Storage in Capacitor
- Electric Fields and Potentials in Capacitor
- Dielectrics in Capacitors

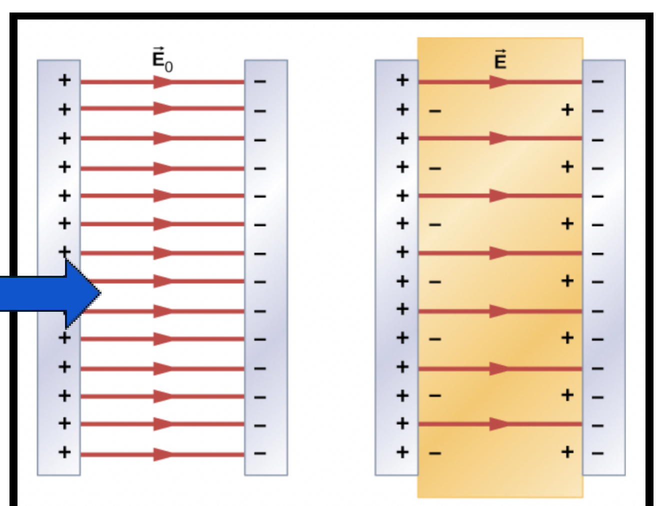






Key Vocabulary

- Capacitance = Amount of charge per unit volt (Farads)
- Capacitors store electrical charge and energy (or capacitance)
- Dielectric = insulating material that decreases electric potential when placed between plates.
- Equivalent or Effective capacitance is the amount of capactiance created by capacitors in series or parallel



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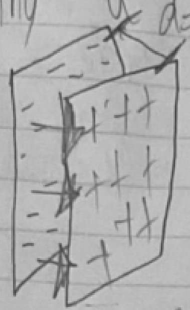
What is the strength of the Electric field between 2 parallel conducting plates seperated by .01m and having a ΔV of 1.5e4V



E

parallel conducting plates with a Electric field between them have a ΔV caused by opposite charges

$F = qE \quad \Delta V = \frac{\Delta U}{q} \quad$ AND $\quad \Delta U = -\int_{x}^{x'} F\,ds$

$\Delta U = -\int_{0}^{d} qE\,ds$ — ★ we treat x_i as 0 to simplify equation and x_f

$\Delta U = -qEd$

$\Delta V = \frac{-qEd}{q} \qquad \Delta V = Ed \qquad E = \frac{\Delta V}{d} = \frac{1.5e4V}{.01m}$

$$E = 1.5e6\ N/C$$

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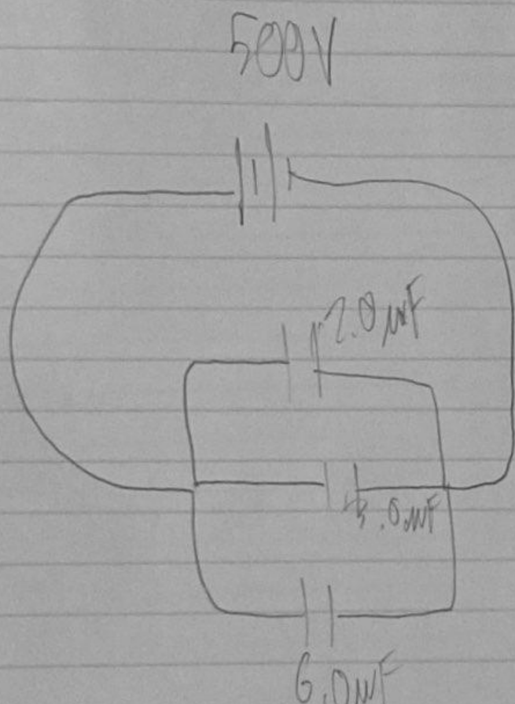
Three capacitors, with capacitances of $C_1 = 2.0\,\mu F$, $C_2 = 3.0\,\mu F$, $C_3 = 6.0\,\mu F$ are connected in parallel. 500V is applied across the combination. Determine voltage across each capacitor and it's charge. Also find the effective capacitance



The potential difference across all capacitors in a circuit is ~~they~~ the same, so the $\Delta V = 500$ across each

Charge on each capacitor plate:

$Q_1 = C_1V_1 = 2e\text{-}6(500) = \boxed{1.0e\text{-}3\,C}$

$Q_2 = C_2V = (3e\text{-}6)(500) = \boxed{1.5e\text{-}3\,C}$

$Q_3 = C_3V = (6e\text{-}6)(500) = \boxed{3.0e\text{-}3\,C}$

$C_{Effective} = C_1 + C_2 + C_3$ for capacitors in Parallel ($C_{effective} = \frac{1}{C_1} + \frac{1}{C_2} \ldots$ for series)

$C_{effective} = 2e\text{-}6 + 3e\text{-}6 + 6e\text{-}6 = \boxed{11e\text{-}6\,F}$

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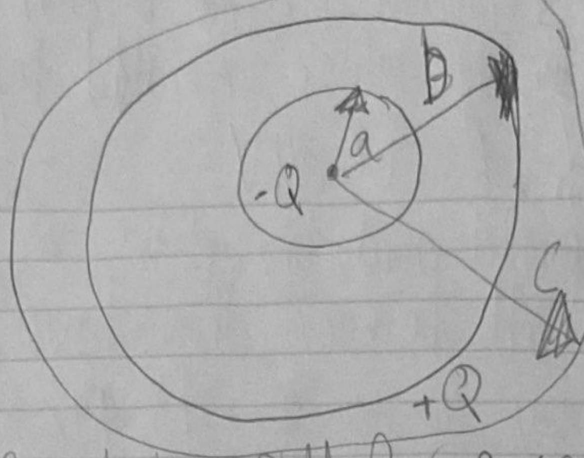

A Spherical capacitor is constructed of concentric conducting spheres. The interior is solid with a radius a and a charge of -Q. The exterior is a hollow sphere with inner radius b, outer radius c, and charge +Q

a. Determine electric field for $a < r < b$:

* here we use gauss's law and given constraints

$$\oint E \cdot dA = \frac{q_{in}}{\varepsilon_0} \rightarrow E(4\pi r^2) = -q_{in}(4\pi K)$$

$$\boxed{E = -K\frac{q}{r^2}\hat{r}}$$ towards center

→ $q_{in} = -Q$ because the gausian sphere surronds the inner cylinder

d. Calculate capacitance of the conducting sphere system

* Some of this uses parts of Equipotentials lots of capacitance is other units too!!

$C = \frac{Q}{V}$, but we are missing V, therefore we must find the ΔV or potential difference between the two spheres.

Since we know $V = -\int E\,dr$ we can use our E value above:

$$V = -\int_b^a \frac{K(-Q)}{r^2} = KQ\left(\frac{1}{b} - \frac{1}{a}\right)$$

* We could also have gone from a to b. Must make capacitance positive at end though

Now we plug in:

$$C = \frac{Q}{V} = \frac{Q}{KQ\left(\frac{1}{a} - \frac{1}{b}\right)} = \boxed{\frac{1}{K}\left(\frac{ab}{b-a}\right)}$$