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Electric Potential

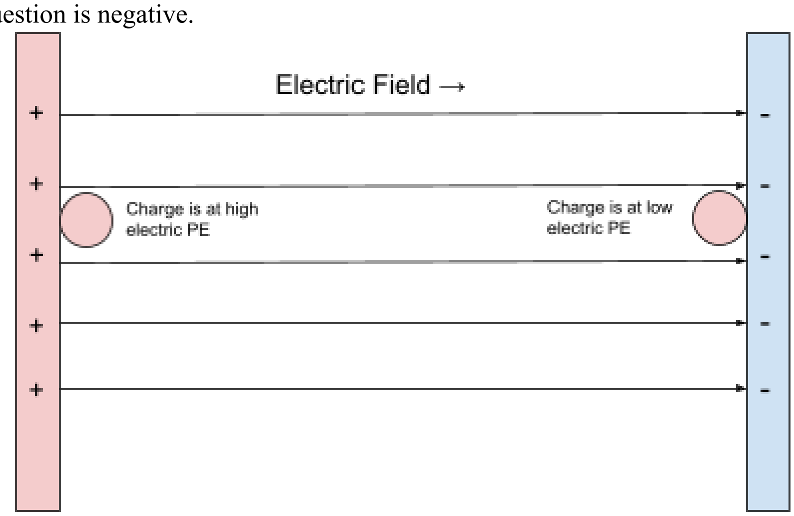
Background: This unit covers the ability of electric fields to do work on charges, and the potential energy that charges have before they move under the influence of these electric fields.

Topics Covered in Unit:

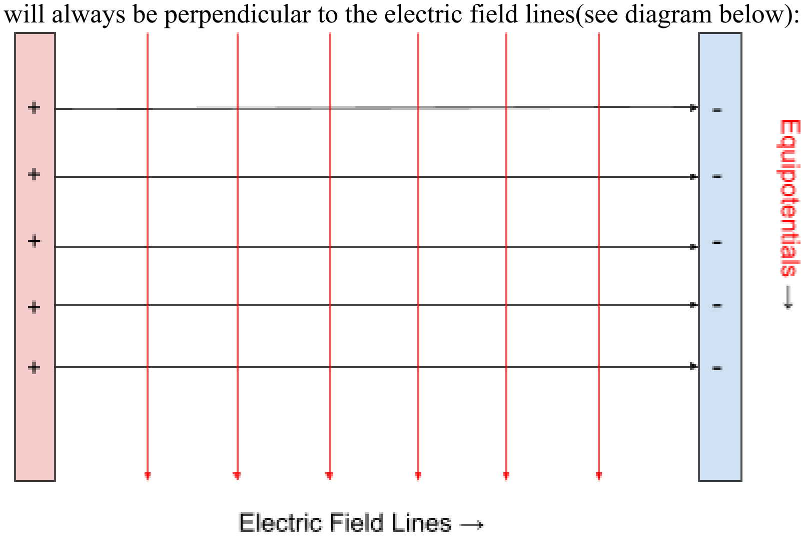
1. Electric Potential Energy
2. Absolute Electric Potential
3. Equipotentials and Their Relationship to Field Lines
4. Calculating Potential/Fields
    a. Includes calculating Potential/Fields within different kinds of shapes as well as conducting/non-conducting materials

Key Takeaways:

1. In the presence of an electric field, positive charges will move with the direction of the field: Away from positive and towards negative
    a. When a positive charge is at the positive end of a field, it has **high electric potential energy** because the field has the potential to do work on the charge and move it towards negative, where it will have **low electric potential energy**. Note that these definitions are flipped if the charge in question is negative.



2. **Change in electric potential** is the change in electric potential energy per unit of charge
3. **Equipotentials** are the lines that show the direction of electric potential that is the same at every point in the line. It will always be perpendicular to the electric field lines(see diagram below):



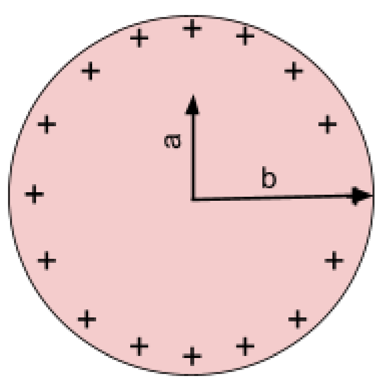
4. Because we know that change in electric potential relates directly with the electric field, we can use electric field values combined with Gauss' Law to calculate electric potential within and on the surfaces of different solids, including conductors and non-conductors

Important Formulas:

Units of Electric Potential	Joules/Coulomb OR Volts(V)
Change in electric potential energy equals the work done by the electric field on a single charge	$F_{electric} = qE$ $W_{done\ by\ electric\ field} = qEd$ $W = -\Delta U =- \int F_{electric} \cdot ds$ $U_f - U_i =- \int_{initial}^{final} qE \cdot ds$
Change in electric potential is the change in electric potential energy per unit of charge(from takeaway #2	$\Delta V = \frac{\Delta U}{q_0}$
Combining equations from above, electric potential varies directly with the electric field.	$\Delta V = \frac{-\int_{initial}^{final} qE \cdot ds}{q_0} =- \int_{initial}^{final} E \cdot ds$ OR $\frac{-dV}{dx} = E_x$
Electric potential of a small, positive, test charge distance *r* from an electric field	$V = \frac{kq}{r}$ where $k = 8.99e^9 \frac{Nm^2}{C^2}$
Electric Potential inside a conducting sphere	Because $E_x = 0$, there is no change in V, so it will stay the same

Practice Problems

1. A conducting sphere with a total charge of +Q is shown below:



   a. Given that the voltage from a distance from the center of the sphere *r,* where $0<r<a$, is a value V_0, what would be the voltage from a distance *r* where $a<r<b$?
   b. What would be the voltage from a distance *r* where $r>b$? How does this value compare to the one found in part a?

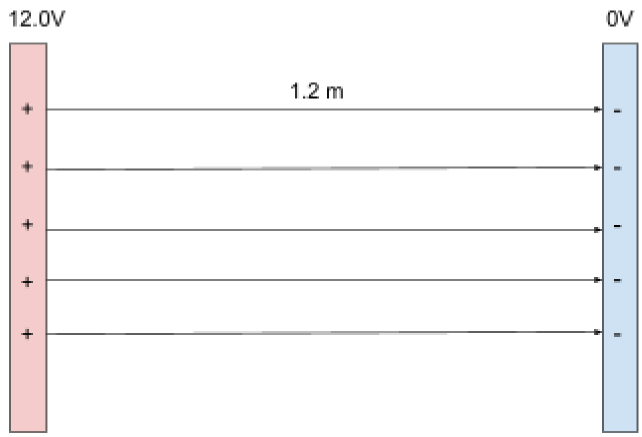
Answers:

1a.

Because the change in potential is proportional to the electric field inside the conductor($\frac{-dV}{dx} = E_x$) and the electric field is zero inside a conductor because all the charges are on the surface, the potential from these radii values would be the same as from $0<r<a$, which would be V_0

1b.
Because this radii range is greater than the radius of the sphere itself, the whole sphere can now be treated like a point charge.
Using the given charge +Q, $V = \frac{kQ}{r}$
The voltage measured past r=b will begin at a value very close to V_0 and then will begin to decline as the distance from the sphere increases.

2. There is a potential difference of 12.0V across two plates, which are 1.20 meters apart
   a. What is the magnitude of the electric field?
   b. How much work would be done on the proton if it were to be moved across the plates?
   c. If the proton starts at rest at the starting plate, how fast is the proton moving at the moment it arrives at the other plate? Would the plate that the proton ends at be positive or negative?

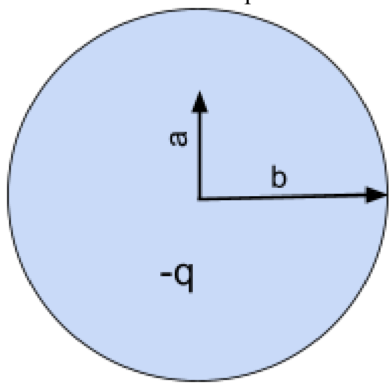


Answers:
2a.
Since $W_{done\ by\ electric\ field} = qEd$, AND $\Delta V = -\frac{\Delta U}{q_0} = -\frac{\Delta W}{q_0}$, $\Delta V = Ed$
Because the question is just asking for magnitude, signs can be ignored
$V_f - V_i = Ed$
$12.0V - 0V = E(1.20m)$

$E = 10.0N/C$
2b.
$\Delta V = -\frac{\Delta W}{q_0}$, where $q_0 = 1.602e^{-24}C$
$12.0V = -\frac{\Delta W}{1.602e^{-24}}$
Because work is a scalar quantity, signs can be ignored
$W = 1.92e^{-23}J$
2c.
Here, we need to use the conservation of energy:
$U_f - U_i = K_f - K_i$
Because the proton starts at rest, $K_i = 0$, and because $U_f - U_i = -W$, we can use the work value we found in part b:
$-W = \frac{1}{2}mv_f^2$
Ignoring signs because work is scalar, and plugging in the mass of a proton:
$1.92e^{-23}J = \frac{1}{2}(9.11e^{-31}kg)v^2$
$v = 6490m/s$

Positive charges move away from positive and towards negative, so the plate the proton would end at the negative plate

3. A non-conducting sphere with uniformly distributed total charge of -q is shown below:
    a. Determine the voltage at a distance *a* in the sphere where *a*<*b*



Answer:

3a.

In order to find the electric potential, we first need to find the electric field. To find the electric field, we need to find the charge of the section of sphere, which will be a ratio of volume of the sphere with radius *a* to the volume of the full sphere with radius *b*.

$\frac{Vol._a}{Vol._{total}} = \frac{Q_a}{Q_{total}}$ and using the formula of a volume of a sphere$(\frac{4}{3}\pi r^2)$

$\frac{\frac{4}{3}\pi b^3}{\frac{4}{3}\pi a^3} = \frac{Q_a}{Q_{total}}$

$Q_a = Q_{total}\frac{a^3}{b^3} =- q\frac{a^3}{b^3}$

Now using $\frac{kq}{r^2}$, derived from Gauss' Law for a sphere

$E = \frac{kq}{r^2} = \frac{-kq\frac{a^3}{b^3}}{a^2} = \frac{-kqa}{b^3}$,

Now that we have our E, we can calculate our V value:

$\Delta V =- \int_{initial}^{final} E \cdot ds$

$V_f - V_i =- \int_{initial}^{final} - \frac{kqa}{b^3} \cdot da$

Signs cancel out, since we are trying to find the Voltage at distance *a*, *a* will be our final value and *b* will be the initial, or total value:

$V_a - V_{total} = \frac{kq}{b^3}\int_b^a a \cdot da$

Knowing that antiderivative of $\int_b^a a \cdot da = \frac{1}{2}a^2$ evaluated from b to a AND $V_{total} = \frac{kq}{b}$:

$V_a - \frac{kq}{b} = \frac{kq}{b^3} \cdot \frac{1}{2}(a^2 - b^2)$

$V_a =\frac{kq}{2b}(\frac{a^2}{b^2} - \frac{b^2}{b^2}) + \frac{kq}{b}$

Factoring and simplification leads to final answer:

$V_a =\frac{kq}{2b}(\frac{a^2}{b^2} - 3)$

Sources used in making this review sheet:crashwhite.com, openstax textbook volume 2