Lab: AP Review Sheets Kayla Chu
Gauss's Law

Overview:
Gauss's Law can be used to determine the electric field around charges. In this review sheet, we will delve into the concepts of *Electric Flux*, *Gauss's Law*, and *Electric Field and Conductors.*

Key Vocabulary to Keep in Mind:

- *Electric Flux*
  - Electric flux is related to the electric field (magnitude and direction) passing through a given surface area.
- *Flux*
  - The measure of both the amount of field through an area and the total area through which the field is passing.
- *Permeability of free space*
  - Refers to the ability of a material—in this case, free space, a vacuum—to transmit (or "permit") an electric field.
- *Gauss's Law*
  - The net electric flux through any closed surface is equal to the net charge inside the surface divided by ε_0.
- $k = 8.99 \times 10^9 \frac{N \cdot m^2}{C^2}$
- $8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}$

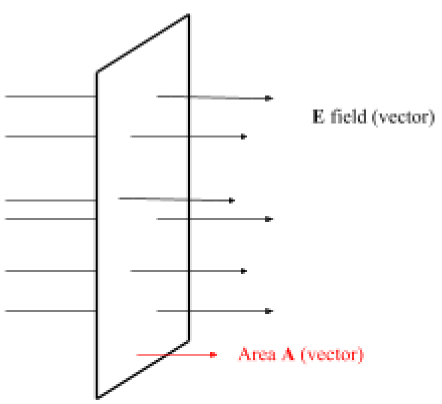
Electric Flux:
Electric flux can be defined with the equation:

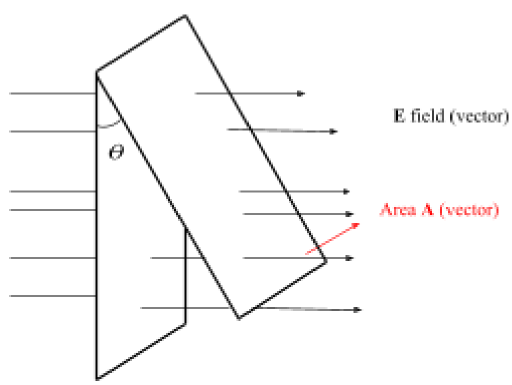
$$\Phi_e = E \bullet A$$

$$\Phi_e = EAcos\theta$$

The net flux through a closed surface is proportional to the net number of lines through the surface:

$$\Phi_e = \oint E \bullet dA$$





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Calculate the net flux for an internal charge:
Find the electric flux passing through the surface of an imaginary sphere of radius R if a charge of +q is located at its center.

Field lines are parallel, so $cos\theta = cos\ 0 = 1$. Evaluate the integral:

$$\Phi_e = \oint E \bullet dA \rightarrow \Phi_e = \oint E\, dA$$

$$\Phi_e = E \oint dA \rightarrow \Phi_e = \frac{kq}{r^2} \oint dA$$

$$\Phi_e = \frac{kq}{r^2}(4\pi r^2) \rightarrow \Phi_e = 4k\pi q$$

$$\Phi_e = \frac{q_{in}}{\epsilon_0}$$

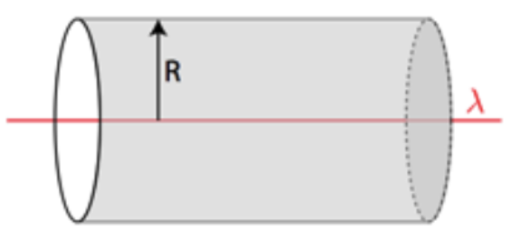
Gauss's Law:
Using what we know thus far, Gauss's Law is defined as:

$$\Phi_e = \oint E \bullet dA = \frac{q_{in}}{\epsilon_0}$$

We can further use Gauss's Law to identify electric fields:

1. Choose an enclosing Gaussian surface based on the arrangement of charges.
2. The shape must have some flux passing through it, it must be perpendicular to the surface, and at a position where the electric field can be calculated.
3. The shape may have other surfaces, through which no field lines are passing (parallel). Because there is no flux through that area, it is not part of the calculation.

Electric Field due to a Line Charge:



$$\Phi_e = \oint E \bullet dA = \frac{q_{in}}{\epsilon_0}$$

$$Q = \lambda L$$

$$E \oint dA = (\lambda L)(4\pi k) \rightarrow E(2\pi r L) = (\lambda L)(4\pi k)$$

$$E = 2k\frac{\lambda}{r}$$

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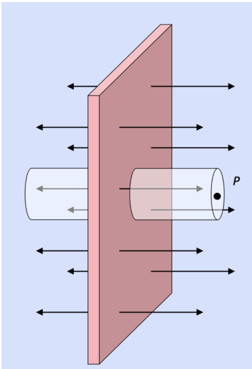
Electric Field of a Non Conducting Plane of Charge:



$$\Phi_e = \oint E \bullet dA = \frac{q_{in}}{\epsilon_0}$$

$$E \oint dA = \frac{\sigma A}{\epsilon_0}$$

$$E2A = \frac{\sigma A}{\epsilon_0} \rightarrow E = \frac{\sigma}{2\epsilon_0}$$

Electric Fields and Conductors:

Consider "electrostatic" situations, in which charges may move in a conductor. They will do so relatively quickly in order to achieve an electrostatic equilibrium.

4 Rules for Conductors in Electrostatic Equilibrium

1. The electric field is zero everywhere inside a conductor.
2. Any charge on an isolated conductor resides on its surface.
3. The electric field just outside a charged conductor is perpendicular to the surface there, and with a magnitude $E = \frac{\sigma}{\epsilon_0}$.
4. On an irregularly-shaped conductor, the charge tends to accumulate at locations where the radius of curvature of the surface is the smallest, ie. at sharp points.

Problems:

1. A point charge of −2μC is located at the center of a cube with sides L = 5 cm. What is the net electric flux through the surface?
2. A 3.5 cm-radius hemisphere contains a total charge of 6.6×10^{-7} C. The flux through the rounded portion of the surface is $9.8\times10^4 \frac{N\cdot m^2}{C}$. What is the flux through the flat base of the hemisphere?
3. A particle with charge Q is placed at the center of a cube with edges of L.
   a. What is the electric flux that passes through the entire cube?
   b. Find the electric flux through one of the faces of the cube.

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Answers:

1. We will use Gauss's Law to solve this problem.

   $$\Phi_e = \oint E \bullet dA = \frac{q_{in}}{\epsilon_0}$$

   $$\Phi_e = \frac{q_{in}}{\epsilon_0}$$

   Plug in the known values into the equation. Make sure to convert all the units.

   $$\frac{-2 \times 10^{-6}\, C}{8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}} = 2.26 \times 10^5 \frac{N \cdot m^2}{C}$$

2. Define Gauss's Law.

   $$\Phi_e = \oint E \bullet dA = \frac{q_{in}}{\epsilon_0}$$

   $$\Phi_e = \frac{q_{in}}{\epsilon_0}$$

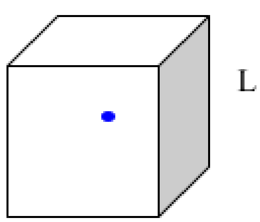
   Because the hemisphere has a rounded and flat base, we need to add the two electric fluxes together to find the total electric flux.

   $$\Phi_{e-rounded} + \Phi_{e-flat} = \frac{q_{in}}{\epsilon_0}$$

   Now, we can plug in the known values and solve for the electric flux through the flat base.

   $$9.8 \times 10^4 \frac{N \cdot m^2}{C} + \Phi_{e-flat} = \frac{6.6 \times 10^{-7}\, C}{8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}}$$

   $$\Phi_{e-flat} = -2.34 \frac{N \cdot m^2}{C}$$

3. 

   a. Using Gauss's Law, we find that the electric flux passing through the entire cube is

   $$\Phi_e = \frac{q_{in}}{\epsilon_0}$$

   b. Electric flux passes equally through all the faces of the cube. Since there are six faces, we can simply solve the problem by dividing the total flux by 6.

   Defining Gauss's Law, $\Phi_e = \frac{q_{in}}{\epsilon_0}$

   $$\Phi_e = \frac{q_{in}}{6\epsilon_0}$$