AP Physics C Review
Chapter 6: Gauss's Law

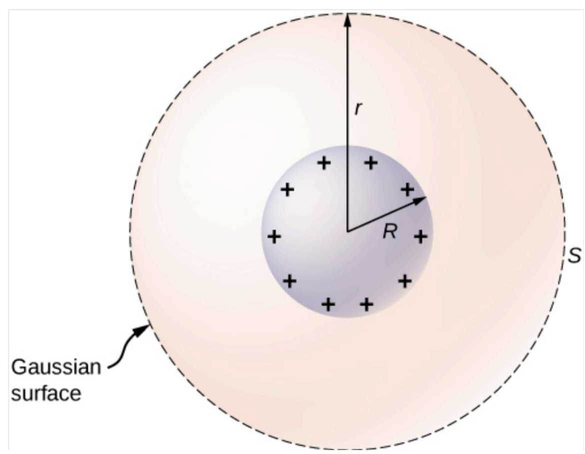
Jack Naliboff

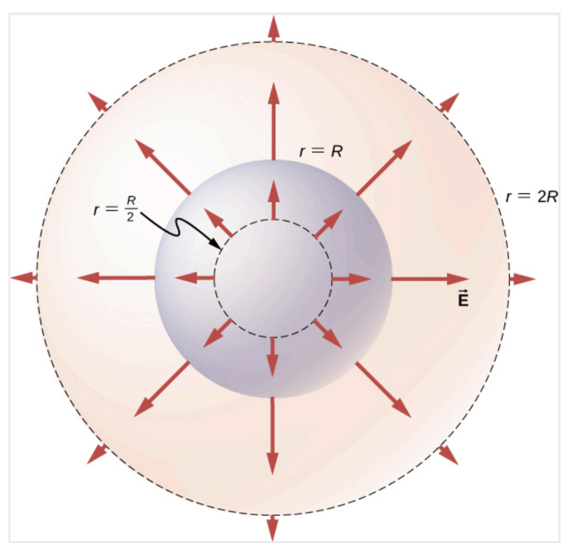
Background / Summary: In this chapter, physics students explore how to find and measure electric fields using electric flux and Gauss's Law. These tools enable students to relate charge, electric field, surface area of a charged object, and electric flux, all handy when solving E&M problems.

Key Ideas:

- Electric flux refers to the magnitude and direction of an electric field over a given area. With the E and A both being vector quantities flux is equal to $\oint E \bullet dA$.
- Gauss's law states that the electric flux of a closed surface is equal to it's enclosed charge divided by the permittivity of free space, an expression written as $\frac{q_{enc}}{\varepsilon_0}$.
- Conductors in equilibrium have all of their charge at the surface. Inside of the shell, there is no electric field. Outside, the electric field behaves like any other object and can be found with Gauss's Law.
- When using Gauss's law and the electric flux integral, it is important to choose an appropriately symmetrical Gaussian surface that matches the charged object it is enclosing or being compared to.
- There are three commonly used Gaussian surfaces: spheres, planes, and cylinders, that can each model various charge distributions.
- For each Gaussian surface, the area calculation is different in accordance to the area of each shape.
- Some calculations for enclosed charge require integration and use of charge density λ.

$$\boxed{\begin{aligned}\Phi_E &= \int \vec{E} \cdot d\vec{A} \\ \oint \vec{E} \cdot d\vec{A} &= \frac{q_{\text{enc}}}{\varepsilon_0}\end{aligned}}$$





Some Key Vocabulary:

Area vector - vector with magnitude equal to the area of a surface and direction perpendicular to the surface

Gaussian surface - any enclosed (usually imaginary) surface

Problem Set:

Easy

37. The electric flux through a spherical surface is $4.0 \times 10^4 \text{ N} \cdot \text{m}^2/\text{C}$. What is the net charge enclosed by the surface?

Medium

56. Charge is distributed throughout a spherical shell of inner radius r_1 and outer radius r_2 with a volume density given by $\rho = \rho_0 r_1 / r$, where ρ_0 is a constant. Determine the electric field due to this charge as a function of *r*, the distance from the center of the shell.

Hard

94. A spherical rubber balloon carries a total charge *Q* distributed uniformly over its surface. At $t = 0$, the radius of the balloon is *R*. The balloon is then slowly inflated until its radius reaches 2*R* at the time t_0. Determine the electric field due to this charge as a function of time (a) at the surface of the balloon, (b) at the surface of radius *R*, and (c) at the surface of radius 2*R*. Ignore any effect on the electric field due to the material of the balloon and assume that the radius increases uniformly with time.

Solutions:

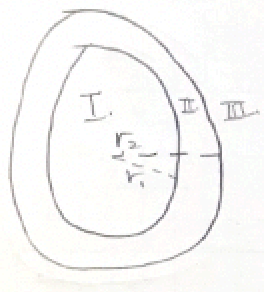
37.

$$\Phi_E = \frac{q_{enc}}{\varepsilon_0}$$ Gauss's Law

$$4.0e4\ \text{Nm}^2/\text{C} = \frac{q_{enc}}{8.85e{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)}$$

$$3.54e{-7}\ \text{C} = q_{enc}$$

56.



$\rho = \rho_0 \frac{r_1}{r}$

Region I., where $r < r_1$:

No electric field because no net charge.

Region II. where $r_1 < r < r_2$:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \qquad q = \int_{r_1}^{R} \rho \, dv$$

$$dv = 4\pi r^2 dr$$

$$\rho = \rho_0 \frac{r_1}{r}$$

$$q = \int_{r_1}^{R} \left(\rho_0 \frac{r_1}{r}\right) 4\pi r^2 \, dr$$

$$q_{enc} = \rho_0 r_1 4\pi \int_{r_1}^{R} r \, dr$$

$$q = (\rho_0 r_1 4\pi)\left(\tfrac{1}{2}R^2 - \tfrac{1}{2}r_1^2\right)$$

$$E \cdot 4\pi r^2 = \frac{\rho_0 r_1 2\pi (r^2 - r_1^2)}{\varepsilon_0}$$

$$E = \frac{\rho_0 r_1 (r^2 - r_1^2)}{2\varepsilon_0 r^2}$$

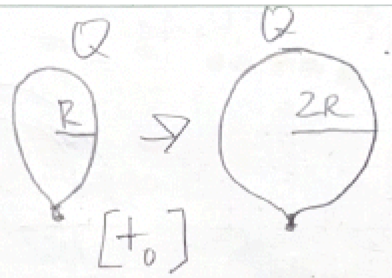
Region III. where $r > r_2$.

Same as region 2, but the integral for q is from r_1 to r_2 instead of r_1 to r.

So,

$$E = \frac{\rho_0 r_1 (r_2^2 - r_1^2)}{2\varepsilon_0 r^2}$$

94.



(a).

$$\frac{dr}{dt} = \frac{\Delta r}{\Delta t}$$

$$\frac{dr}{dt} = \frac{R}{t_0}$$

Rate of change of radius r.

$$r = \frac{R}{t_0} t + R$$

r as a function of t

$$\int E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 4\pi\left(\frac{R}{t_0} t + R\right)^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{kQ}{\left(\frac{R}{t_0} t + R\right)^2}$$

(b) For E at R, plug in $t = 0$,

$$E = \frac{kQ}{R^2}$$

(c) For E at 2R, plug in $t = t_0$

$$E = \frac{kq}{(2r)^2}$$