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AP Physics
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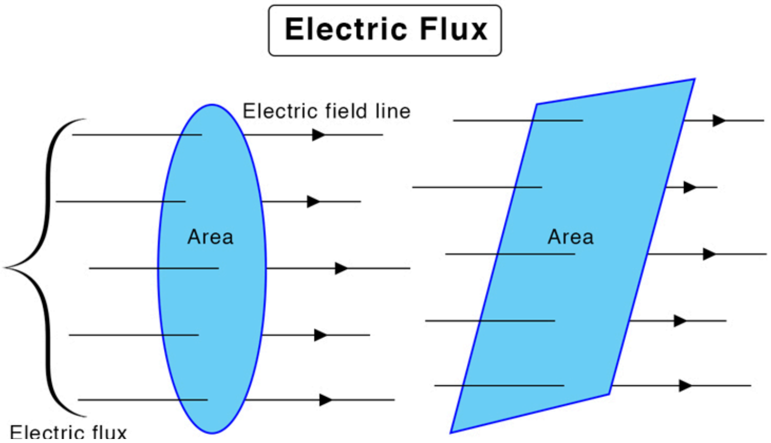
Gauss's Law

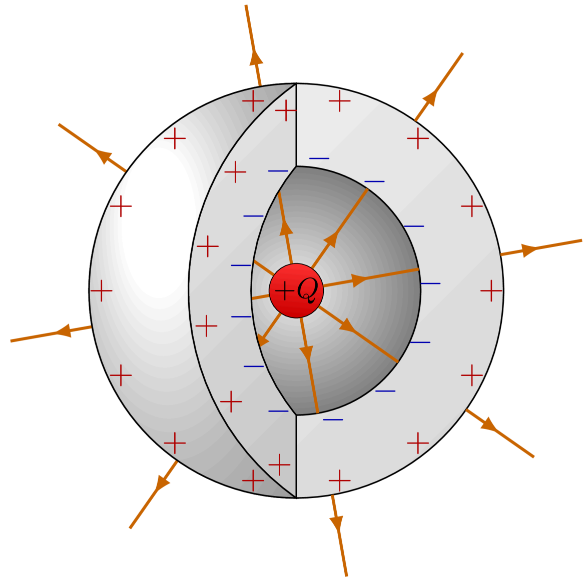
Background:

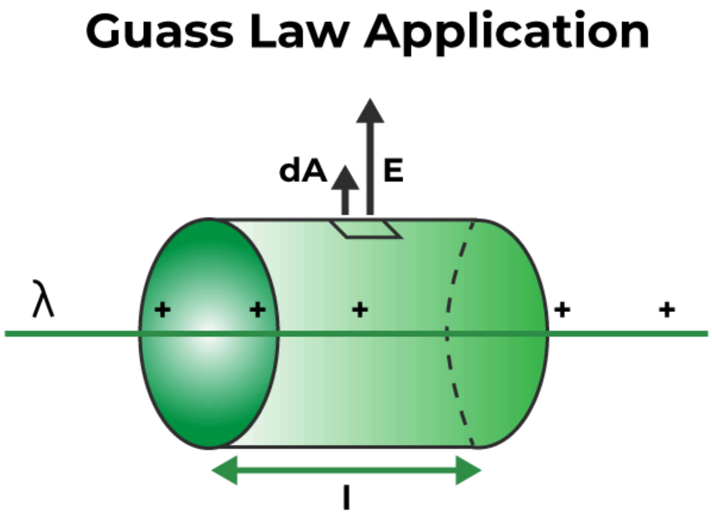
In this unit, we look at Gauss's Law, which helps us understand how electric fields relate to electric charge. You'll learn how to find the electric field by looking at how it passes through different shapes, like spheres and cylinders. Gauss's Law is a useful tool for solving electric field problems more easily, especially when the charge is spread out evenly.

Topics:	**Vocabulary:**	**Formulas:**
Electric Flux	Electric field (E): A field around charged particles that exerts force on other charges.	$\varepsilon_0 \approx 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}$
Gauss's Law Statement	Electric flux (ΦE): A measure of how much electric field passes through a surface.	$\Phi_E = \oint E \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$
Closed Surfaces (Gaussian Surfaces)	Gaussian surface: An imaginary closed surface used to apply Gauss's Law.	$\Phi_E = E \cdot A \cdot cos(\theta)$
Symmetry in Charge Distributions	Closed surface: A surface that completely encloses a volume, like a sphere or cube.	$F_E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$
Applications of Gauss's Law	Uniform charge distribution: When charge is spread out evenly over a region.	$E = \frac{\lambda}{2\pi\varepsilon_0 r}$
Limitations of Gauss's Law	Permittivity of free space (ε_0): A constant that appears in Gauss's Law.	$E = \frac{\sigma}{2\varepsilon_0}$
Conductors in Electrostatic Equilibrium		$\lambda = \frac{q}{L}$
		$\sigma = \frac{q}{A}$
		$\rho = \frac{q}{V}$

Diagrams:







Problems:

(All questions have been borrowed from the Chapter 6 review questions from University Physics Volume 2 by Jeff Sanny, Samuel J. Ling, and William Moebs)

1. A net flux of 1 x 10^4 N m^2 / C passes inward through the surface of a sphere of radius 5 cm. (a) How much charge is inside the sphere? (b) How precisely can we determine the location of the charge from this information?
2. A total charge 5 x 10^-6 C is distributed uniformly throughout a cubical volume whose edges are 8.0 cm long. (a) What is the charge density in the cube? (b) What is the electric flux through a cube with 12.0-cm edges that is concentric with the charge distribution? (c) Do the same calculation for cubes whose edges are 10.0 cm long and 5.0 cm long. (d) What is the electric flux through a spherical surface of radius 3.0 cm that is also concentric with the charge distribution?
3. A charge of -30 μC is distributed uniformly throughout a spherical volume of radius 10.0 cm. Determine the electric field due to this charge at a distance of (a) 2.0 cm, (b) 5.0 cm from the center of the sphere.

Solutions:

1. a. How much charge is inside the sphere?

Given: $\Phi_E = -1.0\times10^4\ N\cdot m^2/C$ (negative b/c the flux is inward)

$\varepsilon_0 = 8.85\times10^{-12}\ C^2/N\cdot m^2$

$$\Phi_E = \frac{q_{in}}{\varepsilon_0}$$

Solve for q_{in}:

$$q_{in} = \Phi_E \cdot \varepsilon_0 = (-1\times10^4)(8.85\times10^{-12}) = \boxed{-8.85\times10^{-8}\ C}$$

b. How precisely can we determine the location of the charge?

We cannot determine the precise location of the charge from this information. Gauss's Law only tells us the total net charge enclosed by the sphere, not its location.

2. Given:

$q = 5.0\times10^{-6}\ C$

$L_{cube} = 8cm = 8\times10^{-2}\ m$

$\varepsilon_0 = 8.85\times10^{-12}\ C^2/N\cdot m^2$

a. what is the charge density in the cube?

$$\rho = \frac{q}{V}$$

we know q but not V:

$$V = L^3 = (8\times10^{-2})^3 = 5.12\times10^{-4}\ m^3$$

$$\rho = \frac{q}{V} = \frac{5\times10^{-6}}{5.12\times10^{-4}} = \boxed{9.77\times10^{-3}\ C/m^3}$$

b. Electric flux through a 12 cm cube

$$\Phi_E = \frac{q}{\varepsilon_0} = \frac{5\times10^{-6}}{8.85\times10^{-12}} = \boxed{5.65\times10^5\ N\cdot m^2/C}$$

c. Flux through:

10 cm cube: Same work as part b. $\boxed{\Phi_E = 5.65\times10^5\ N\cdot m^2/C}$

5 cm cube: $V = L^3 = (0.05)^3 = 1.25\times10^{-4}\ m^3$

$q = \rho\cdot V = (9.77\times10^{-3})(1.25\times10^{-4}) = 1.22\times10^{-6}\ C$

$$\Phi = \frac{q}{\varepsilon_0} = \frac{1.22\times10^{-6}}{8.85\times10^{-12}} = \boxed{1.38\times10^5\ N\cdot m^2/C}$$

d. Flux through sphere with radius 3.0 cm

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.03)^3 = 1.13\times10^{-4}\ m^3$$

$$q = \rho\cdot V = (9.77\times10^{-3})(1.13\times10^{-4}) = 1.11\times10^{-6}\ C$$

$$\Phi = \frac{q}{\varepsilon_0} = \frac{1.11\times10^{-6}}{8.85\times10^{-12}} = \boxed{1.25\times10^5\ N\cdot m^2/C}$$

3. Given:

$q = -30\,\mu C = -30\times10^{-6}\,C$

$r = 10\,cm = 0.1\,m$

$\varepsilon_0 = 8.85\times10^{-12}\,C^2/N.m^2$

a. Electric field due to charge at $r = 2.0\,cm$

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.1)^3 = 4.14\times10^{-3}\,m^3$

$\rho = \frac{q}{V} = \frac{-30\times10^{-6}}{4.14\times10^{-3}} = -7.16\times10^{-3}\,C/m^3$

$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$ (where $q = \rho.\frac{4}{3}\pi r^3$)

$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\rho\frac{4}{3}\pi r^3}{r^2} = \frac{\rho.r}{3\varepsilon_0}$

$E = \frac{\rho.r}{3\varepsilon_0} = \frac{(-7.16\times10^{-3})(0.02)}{3(8.85\times10^{-12})} = \boxed{5.4\times10^{6}\,N/C}$

b. at $r = 5\,cm$

$E = \frac{\rho.r}{3\varepsilon_0} = \frac{(-7.16\times10^{-3})(0.05)}{3(8.85\times10^{-12})} = \boxed{-1.35\times10^{7}\,N/C}$