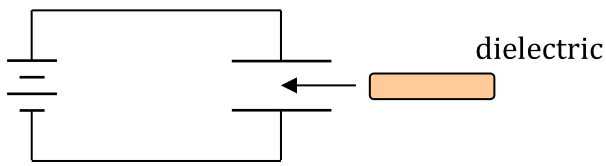
This test covers electric potential energy, electric potential, and capacitance, with some problems requiring a knowledge of basic calculus.

Part I. Multiple Choice

1. Two large, parallel conducting plates are separated by a distance *d*. The two plates are given each given an equal magnitude of charge *Q*, but with opposite polarities, so a constant electric field *E* exists between the plates. A particle of mass *m* and charge +*q* is released from rest at the surface of the positively-charged plate. Its velocity just before it reaches the surface of the negative plate is:
    a. $\sqrt{\frac{2qEd}{m}}$
    b. $\sqrt{\frac{qEd}{m}}$
    c. $\sqrt{\frac{2QEd}{m}}$
    d. $\frac{2qEd}{m}$
    e. $\frac{2QEd}{m}$

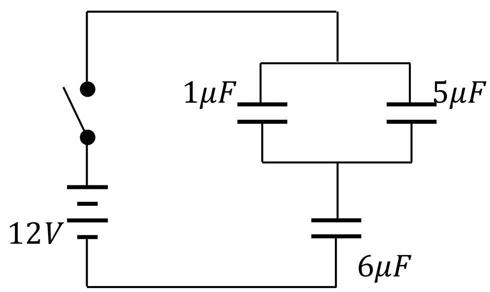
2. Two metal plates of area *A* and separated by a distance *d* are placed in parallel near each other to form a capacitor with capacitance *C*. The plates are connected to a voltage source with potential *V* and allowed to charge completely. The voltage is then removed, and the plates moved so that they are now separated by a distance 2*d*. Which of the following statements is true?
    a. The charge on the plates has changed, and the electric field between them has increased.
    b. The charge on the plates is the same, and the potential between them has decreased.
    c. The potential between the plates has increased, and the electric field between them has decreased.
    d. The capacitance of the plates has decreased, and the energy stored in the capacitor has increased.
    e. The capacitance of the plates has decreased, and the electric field between them has decreased.

3. The electric field in a region of space is given by the function $E = -30x + 2$, where *x* is in meters and *E* is in Volts/meter. What is the electric potential at *x* = 2 meters, relative to the origin?
    a. +56 V
    b. +60 V
    c. -30 V
    d. -60 V
    e. -56 V



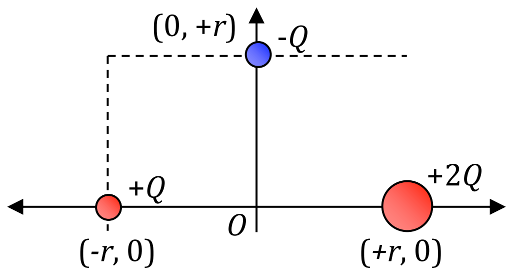
4. A battery is connected to a capacitor in order to set up a potential difference across the plates. While the battery remains connected, a dielectric is inserted between the plates of the capacitor. Which of the following statements concerning the capacitor with the dielectric is false?
   a. The potential difference across the plates of the capacitor is the same as before.
   b. The amount of charge on the plates has increased.
   c. The capacitance of the capacitor has increased.
   d. The net electric field between the plates has increased.
   e. The capacitor stores more energy.

5. An electric field does 4 J of work on a charged particle, moving it from a potential of 1 V to a potential of 3 V. The particle has a charge of:
   a. -8 C
   b. +8 C
   c. -2 C
   d. +0.5 C
   e. -0.5 C



6. Three capacitors, of capacitance $1\mu F$, $5\mu F$, and $6\mu F$, are arranged in a circuit with a switch and a 12-V battery as shown above. The equivalent capacitance of the three capacitors is:
   a. 2 μF
   b. 3 μF
   c. 6 μF
   d. 11/6 μF
   e. 11/12 μF

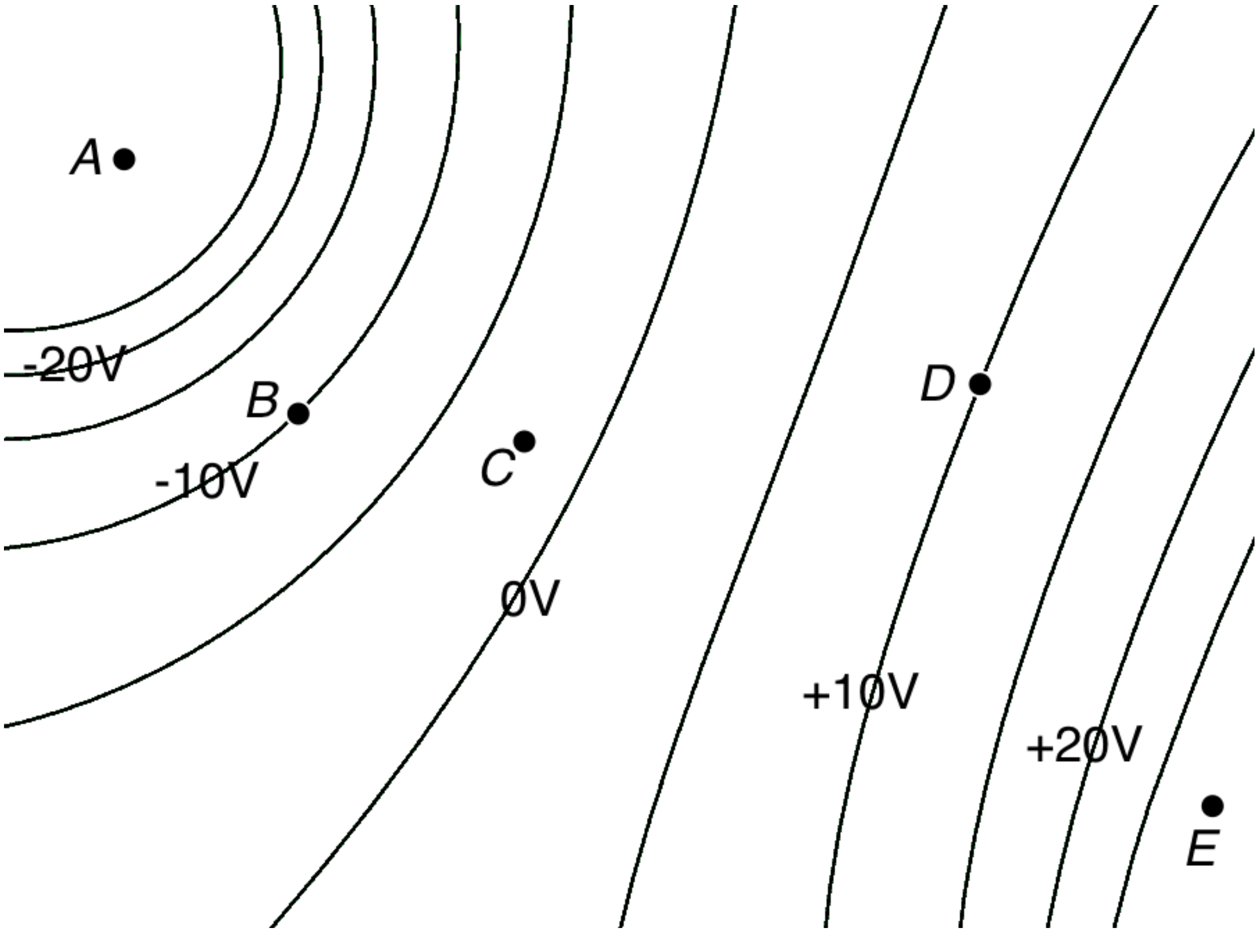
7. A capacitor is fully charged by a 10-Volt battery, and has 20 milliJoules of energy stored in it. The charge on each conducting plate of the capacitor is:
   a. 2×10^{-3} *Coulombs*
   b. 4×10^{-3} *Coulombs*
   c. 2×10^{-4} *Coulombs*
   d. 4×10^{-2} *Coulombs*
   e. 2×10^{-2} *Coulombs*



8. Three point charges of +Q, +2Q, and –Q are each located a distance r away from the origin, as shown above. The magnitude of the electric field at the origin due to these charges is:
   a. $\frac{2kQ}{r^2}$
   b. $\frac{2kQ^2}{r^2}$
   c. $\frac{2kQ}{r}$
   d. $\frac{\sqrt{2}kQ}{r^2}$
   e. $\frac{\sqrt{2}kQ}{2r^2}$

9. Three point charges of +Q, +2Q, and –Q are each located a distance r away from the origin, as shown above. The electric potential at the origin due to these charges is:
   a. $\frac{2kQ}{r}$
   b. $\frac{3kQ}{r}$
   c. $\frac{4kQ}{r}$
   d. $\frac{\sqrt{2}kQ}{r}$
   e. 0

Part II. Free Response



10. The electric potential diagram above shows equipotentials for a 2-dimensional region of space.

    a. At which point—*A*, *B*, *C*, *D*, or *E*—would an electron have the highest electric potential energy? Briefly explain your answer.

    b. In which general direction does the electric field point in this diagram? Briefly explain your answer.

c. Based on the equipotentials, draw a sketch on the diagram above of the electric field, including at least five field lines.

d. At which point—*A*, *B*, *C*, *D*, or *E*—is the magnitude of the electric field the greatest? Briefly justify your answer.

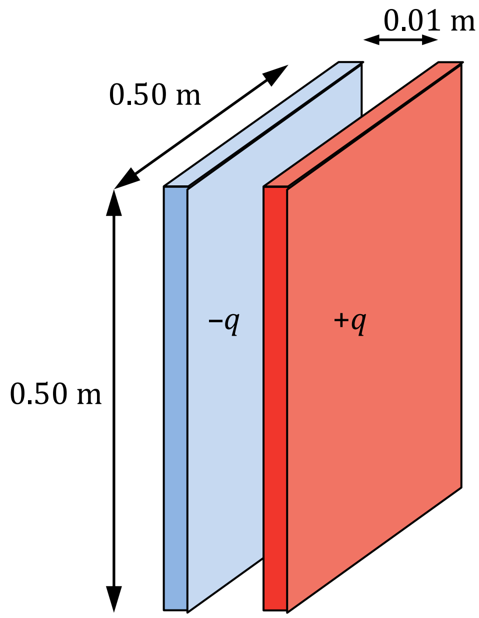
e. A proton is released from rest at point *C*. Qualitatively describe the proton's subsequent:

i. direction of motion

ii. speed

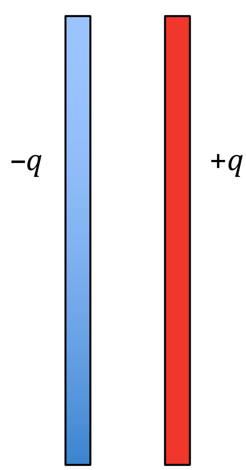
iii. acceleration

f. For the proton in part *e*, calculate its velocity after having moved through a potential difference of 10 V.



11. A parallel plate capacitor is created by placing two large square conducting plates of length and width 0.50 meters facing each other, separated by a 1.00-centimeter gap. A source of potential is connected to the two plates so that a charge of $+q = +\sigma A$ is placed on the left plate and a charge of $-q = -\sigma A$ is on the right plate, where $\sigma = 500 \times 10^{-6} C / m^2$. The potential source is then removed from the plates. The plates are close together so electric field fringe effects at the edges are negligible.

    a. In the space below, draw electric field lines in the vicinity of the plates.



b. Use Gauss's Law to determine the magnitude of the electric field between the plates.

c. Calculate the electric potential V between the two plates.

d. Calculate the capacitance of this capacitor.

A dielectric of $\kappa = 2.00$ is now inserted between the isolated plates while the same amount of charge Q remains on each plate.

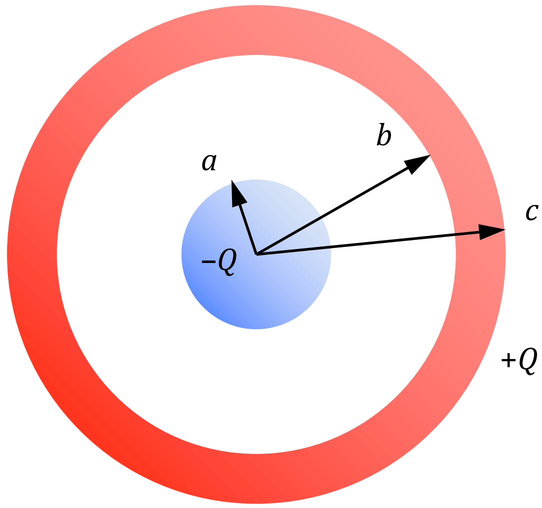
e. Calculate the new capacitance of the system with the dielectric between the plates.

f. The electric field strength between the plates has (check one):
____ increased
____ decreased
____ remained the same

g. The electric potential between the plates has (check one):
____ increased
____ decreased
____ remained the same

h. The energy stored in the capacitor has (check one):
____ increased
____ decreased
____ remained the same

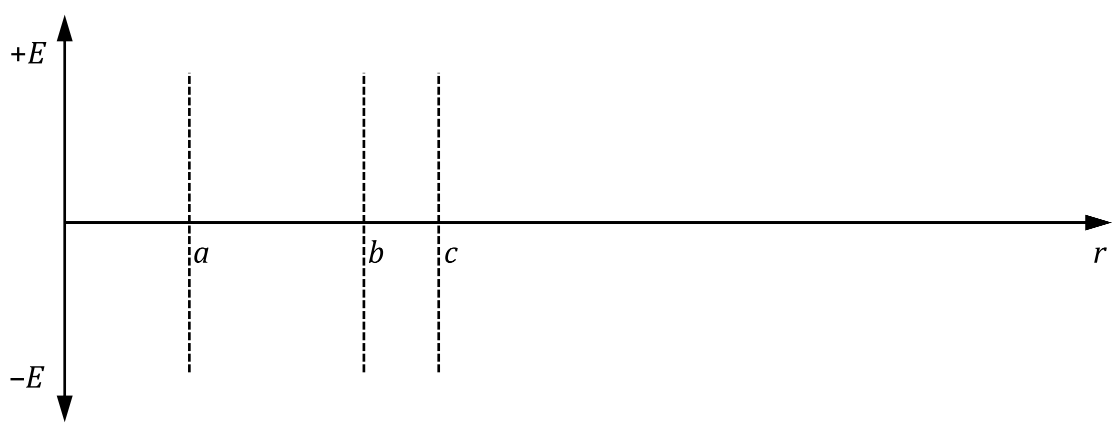
i. How would your answers to *f*, *g*, and *h* change if the dielectric had been inserted *while the voltage supply was still connected to the capacitor*? Explain.



12. A spherical capacitor is constructed of concentric conducting spheres. The interior sphere is solid, with a radius *a* and a charge of –*Q*. The exterior sphere is a hollow shell with inner radius *b* and outer radius *c*, and charge +*Q*. Give symbolic answers as a function of the given variables and fundamental constants.

    a. Use Gauss's Law to calculate the electric field *E*, magnitude and direction, for the space between the spheres, where $a < r < b$.

    b. On the axes below, sketch a graph of electric field *E* as a function of *r* for the range 0 to 2*c*, where $E > 0$ is away from the center.



c. Calculate the magnitude of the difference in electric potential V between the two spheres.

d. Calculate the capacitance of this conducting-sphere system.

e. The spheres are discharged, and then connected to a source of electric potential of magnitude $2V$. Calculate how much Work must be done to fully charge the capacitor under these new conditions.

1. The correct answer is *a*. The particle is accelerated by the electric field, and so its velocity can be determined either by using the Work done on the particle by the field, or by calculating the electric force's acceleration and solving using kinematics. Here, we'll use the energy analysis:

$$U_{initial} = K_{final}$$

$$U = qV,\ V = Ed,\ and\ K = \frac{1}{2}mv^2$$

$$qEd = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2qEd}{m}}$$

The kinematics-based analysis arrives at the same result by using $F_{net} = ma$, $F_e = qE$, and $v_f^{\,2} = v_i^{\,2} + 2ax$.

2. The correct answer is *d*. Once the plates are removed from the voltage source, they have a charge $Q = CV$. Moving the plates farther apart decreases their capacitance by a factor of 2 according to $C = \frac{A}{d}\varepsilon_0$. Although the capacitance of the system has changed, the amount of charge stored on the plates has not: we conclude that the potential between the plates V must have *increased* as a result of the inverse relationship between C and V in $Q = CV$. The electric field between the two plates is described by the relationship $V = -Ed$, and because V and d have both increased by a factor of 2, we conclude that the electric field E between the plates is unchanged. The energy stored in the capacitor can be calculated as a function of any two of Q, C, and V. In this case, $U_C = \frac{1}{2}QV$ reveals that, because V has increased, the energy stored in the capacitor has increased as well. From a Conservation of Energy perspective, this extra energy arises as a result of the Work done in moving the two plates farther apart. The only answer consistent with the above analysis is *d*.

3. The correct answer is *a*. The relationship between electric field and electric potential is given by the integral $V = -\int E \bullet dr$, and determined for this problem as follows:

$$\Delta V = -\int_{x_i}^{x_f} E \bullet dx$$

$$\Delta V = -\int_{0}^{2m} -30x + 2 \bullet dx$$

$$\Delta V = -(-15x^2 + 2x)\Big|_0^2 = (60-4)-(0-0) = +56V$$

4. The correct answer is *d*. Inserting the dielectric into the capacitor increases the capacitance of the parallel plates. The battery maintains the potential across the plates V, so according to $Q=VC$, we can see that the amount of charge stored on the plates will increase. The electric field between the plates remains *constant*, however, based on $\Delta V=Ed$. The capacitor, by virtue of its increased capacitance, stores more energy, according to $U = \frac{1}{2}CV^2$.

5. The correct answer is *c*. The electric field does work on the particle, moving it from a position of high electric potential energy to low electric potential energy according to $\Delta V = \frac{\Delta U}{q}$.

$$q = \frac{\Delta U}{\Delta V} = \frac{U_f - U_i}{V_f - V_i} = \frac{-4J}{2V} = -2C$$

The negative sign on the charge can also be deduced by the fact that the field is doing Work on the particle in moving it from a lower potential to a higher potential. If it were a positive charge, the field would be doing Work in going from higher to lower potential.

6. The correct answer is *b*. The equivalent capacitance of the three capacitors can be found by calculating the $1\mu F$ and $5\mu F$ in parallel, and then putting the $6\mu F$ capacitor in series with that:

$$C_{parallel} = C_1 + C_5$$

$$C_{parallel} = 1\mu F + 5\mu F = 6\mu F$$

Now we put the equivalent capacitance from the parallel capacitors in series with the remaining capacitor:

$$\frac{1}{C_{series}} = \frac{1}{C_{parallel}} + \frac{1}{C_6} =$$

$$\frac{1}{C_{series}} = \frac{1}{6\mu F} + \frac{1}{6\mu F} = \frac{2}{6\mu F}$$

$$C_{series} = 3\mu F$$

7. The correct answer is *b*. The charge stored in the plates of the capacitor can be determined using the formula $U = \frac{1}{2}QV$:

$$U = \frac{1}{2}QV$$

$$Q = \frac{2U}{V} = \frac{2(20 \times 10^{-3} J)}{10V} = 4 \times 10^{-3}\, Coulombs$$

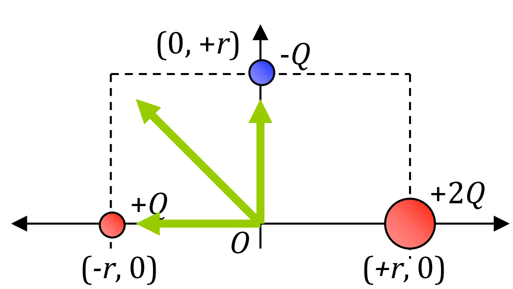
8. The correct answer is *d*. The electric field at the origin can be found by considering the sum of the electric fields of the three particles. This can be quickly determined by considering that there is an electric field in the $-x$ direction due to a net+1Q at the $(+r, 0)$ position, and a an electric field in the $+y$ direction due to the $1Q$ charge at $(0, +r)$. Add these components together to get the answer. Or, mathematically:

$$E_{net} = \sum E_{+Q} + E_{+2Q} + E_{-Q}$$

$$E_x = +\frac{kQ}{r^2} - \frac{k2Q}{r^2} = -\frac{kQ}{r^2} \text{(in the } x\text{-direction)}$$

$$E_y = +\frac{kQ}{r^2} \text{(in the } y\text{-direction)}$$

$$E_{net} = \sqrt{\left(-\frac{kQ}{r^2}\right)^2 + \left(+\frac{kQ}{r^2}\right)^2} = \frac{\sqrt{2}kQ}{r^2}$$



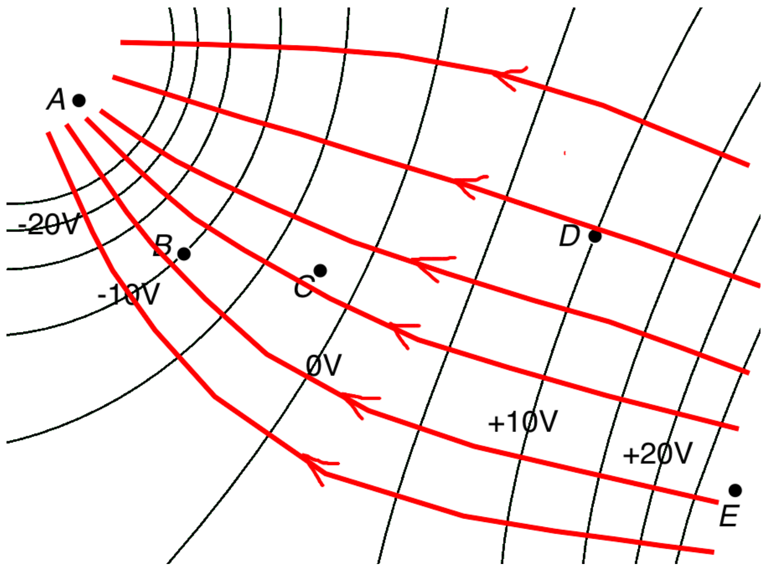
9. The correct answer is *a*. The electric potential, or energy per unit charge, represents the amount of energy per unit charge it takes to move a test particle from infinitely far away to a given location. For a point charge, the absolute potential is calculated using $V = \frac{kQ}{r}$ for each charge, and the electric potential at a given location is simply the sum of the individual potentials for each charge in the vicinity:

$$V = \sum V_{+Q} + V_{+2Q} + V_{-Q}$$

$$V = \frac{k(+Q)}{r} + \frac{k(+2Q)}{r} + \frac{k(-Q)}{r} = \frac{2kQ}{r}$$

10.

a. At **A**. Electron has highest potential energy where electric potential is lowest, the opposite of what a positive test charge experiences.
b. The electric field points from lower-right to upper-left. Electric field lines point in the direction of high-to-low electric potential.
c. Field lines should be perpendicular to equipotentials.



d. At **A**. Field lines become more closely spaced there.
e.
   i. The proton accelerates in the same direction of field, ie, towards upper left of diagram.
   ii. The proton has a speed that *increases* over time; the electric field applies a force to it, causing it to accelerate.
   iii. The proton's acceleration *increases* as it moves into an area with an increasingly strong electric field.

f. This is a conservation of energy problem, with the field doing Work on the charge to increase its kinetic energy. Here, we can look at the change in electric potential energy and see how that converts to kinetic energy.

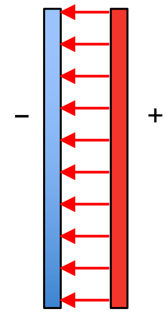
$$\Delta U = \Delta K$$

$$q\Delta V = K_f - K_i = \frac{1}{2}mv^2 - 0$$

$$v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2(1.60e-19C)(10V)}{(1.67e-27kg)}} = 4.38e4m/s$$

11.

a. With the plates in close proximity and negligible fringing:

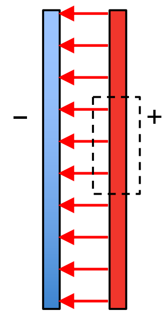


b. Gauss's law can be used to determine the electric field for one plate, and then that effect can be doubled to account for the other plate, or Gauss's law can be used with the two plates as arranged. Here, I'm drawing a Gaussian surface that has field lines from the positively-charged plate going through one surface of area *A*.

$$\oint E \bullet dA = \frac{q_{in}}{\varepsilon_0}$$

$$EA = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{500e-6C/m^2}{8.85e-12C^2/(N \bullet m^2)} = 5.65e7N/C$$



c. For a constant electric field:

$$V = -Ed$$

$$V = (5.65e7N/C)(0.01m) = 5.65e5V$$

d. The capacitance can be calculated either theoretically using the dimensions of the plates, or using the definition of capacitance and the values we've determined here.

Theoretically:

$$C = \frac{A\varepsilon_0}{d} = \frac{(0.5m)^2(8.85e-12C^2/(N \bullet m^2)}{0.01m} = 2.21e-10F$$

Using definition of capacitance:

$$C = \frac{Q}{V} = \frac{\sigma A}{V} = \frac{(500e-6N/m^2)(0.5m)^2}{5.65e5V} = 2.21e-10F$$

e. The dielectric increases the capacitance of the system by a factor $\kappa = 2$.

$$C = \kappa C_0$$

$$C = (2.00)(2.21e-10F) = 4.42e-10F$$

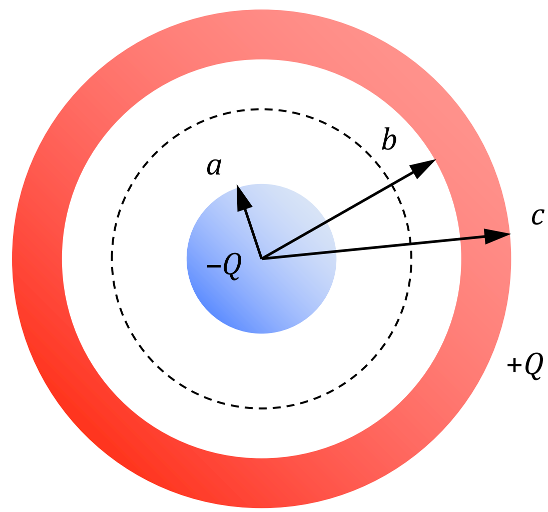
f. The problem states that the plates are isolated when the dielectric is inserted, and retain the same charge. As the insulating dielectric is polarized by the electric field between the plates, it develops an internal electric field in the opposite direction, thus *decreasing* the net electric field between the plates.

g. The electric potential between the plates has *decreased* as well, as a result of $V = -Ed$, or by considering that the capacitance has increased. By $Q = VC$, if capacitance has increased, the potential between the plates has decreased.

h. The energy stored in the capacitor has *decreased*, according to $U = \frac{1}{2}QV$.

i. If a voltage source is still connected during the dielectric insertion, the potential across the plates will be maintained as more charge flows into the surfaces of the plates. Electric field will *remain the same*, electric potential will *remain the same*, and energy stored in the capacitor will *increase*.

12.



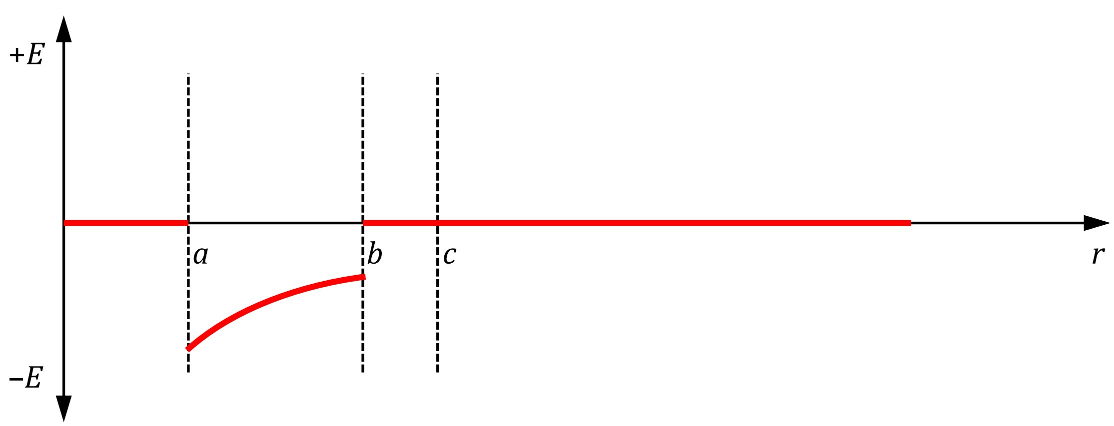
a. Construct a Gaussian sphere in the space between the two conducting spheres:

$$\oint E \bullet dA = \frac{q_{in}}{\varepsilon_0}$$

$$E(4\pi r^2) = -q_{in}(4\pi k)$$

$$E = -k\frac{q}{r^2}\hat{\mathbf{r}}, \text{ or } k\frac{q}{r^2} \text{ toward the center}$$

b. Note that there is no electric field outside the capacitor, and no electric field in the conductors themselves. Here, there is only an electric field between the two charged conductors.



c. Using the position integral of electric field E calculated above:

$$V = -\int E \bullet dr$$

$$V = -\int_b^a \frac{k(-Q)}{r^2} = kQ\left(\frac{1}{b} - \frac{1}{a}\right)$$

Because only the magnitude of the potential was requested, $kQ\left(\frac{1}{a}-\frac{1}{b}\right)$ is also acceptable.

d. Using the definition of capacitance, we would calculate

$$C = \frac{Q}{V} = \frac{Q}{kQ\left(\frac{1}{b}-\frac{1}{a}\right)} = \frac{1}{k}\left(\frac{ab}{a-b}\right)$$

but this would give us a negative value for capacitance, and capacitance is by definition always positive. Adjust the formula to use ($b - a$) in denominator to get the necessary positive result:

$$C = \frac{1}{k}\left(\frac{ab}{b-a}\right)$$

e. Potential has doubled, and because $Q = VC$, the charge on the capacitor has doubled as well.

$$U = \frac{1}{2}QV = \frac{1}{2}(2Q)\left(2\left(kq\left(\frac{1}{b}-\frac{1}{a}\right)\right)\right)$$

Simplifying this leads to $U = 2kQ^2\left(\frac{a-b}{ab}\right)$, which yields a negative answer (because $a < b$).
"Work done to charge the capacitor" implies work being done on the capacitor to place the charge there, so it's better to express this as a positive result:

$$U = 2kQ^2\left(\frac{b-a}{ab}\right)$$