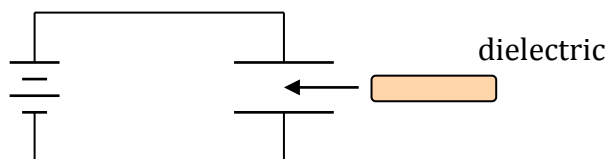


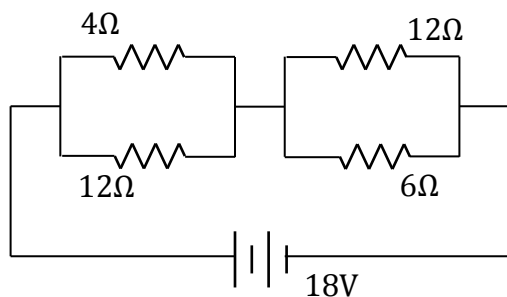
This test covers capacitance, electrical current, resistance, emf, electrical power, Ohm's Law, Kirchhoff's Rules, and RC Circuits, with some problems requiring a knowledge of basic calculus.

### Part I. Multiple Choice

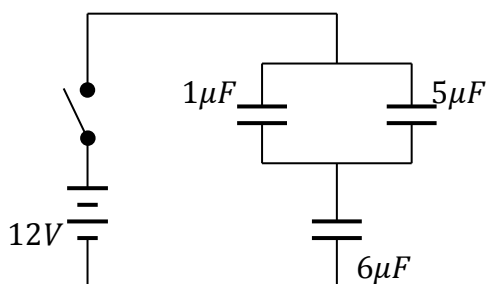
- Two metal plates of area  $A$  and separated by a distance  $d$  are placed in parallel near each other to form a capacitor with capacitance  $C$ . The plates are connected to a voltage source with potential  $V$  and allowed to charge completely. The voltage is then removed, and the plates moved so that they are now separated by a distance  $2d$ . Which of the following statements is true?
  - The charge on the plates has changed, and the electric field between them has increased.
  - The charge on the plates is the same, and the potential between them has decreased.
  - The potential between the plates has increased, and the electric field between them has decreased.
  - The capacitance of the plates has decreased, and the energy stored in the capacitor has increased.
  - The capacitance of the plates has decreased, and the electric field between them has decreased.

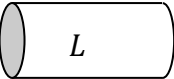

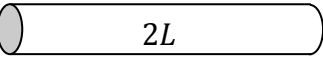
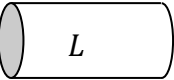
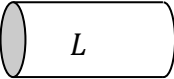
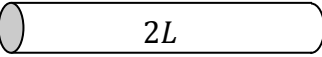
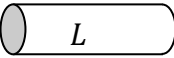

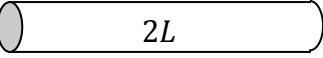



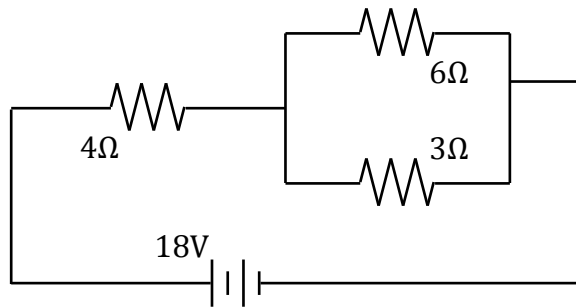
- A battery is connected to a capacitor in order to set up a potential difference across the plates. While the battery remains connected, a dielectric is inserted between the plates of the capacitor. Which of the following statements concerning the capacitor with the dielectric is false?
  - The potential difference across the plates of the capacitor is the same as before.
  - The amount of charge on the plates has increased.
  - The capacitance of the capacitor has increased.
  - The net electric field between the plates has increased.
  - The capacitor stores more energy.



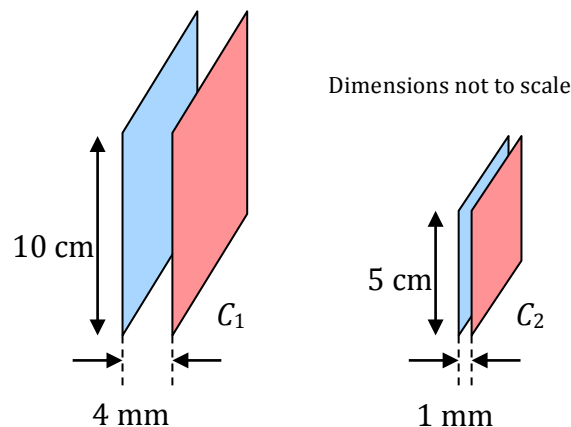
- The equivalent resistance of the four resistors, connected as shown above, is:
  - $7\ \Omega$
  - $4\ \Omega$
  - $2.7\ \Omega$
  - $8.5\ \Omega$
  - $14.4\ \Omega$



4. Three capacitors, of capacitance  $1\mu F$ ,  $5\mu F$ , and  $6\mu F$ , are arranged in a circuit with a switch and a 12-V battery as shown above. The equivalent capacitance of the three capacitors is:
- $2\mu F$
  - $3\mu F$
  - $6\mu F$
  - $11/6\mu F$
  - $11/12\mu F$
5. Two conducting wires,  $W_1$  and  $W_2$ , are made of two different materials, the first with a resistivity of  $\rho_1$ , the second with resistivity  $\rho_2 = \frac{\rho_1}{2}$ . Which of the following pairs of cylindrical wires, with indicated cross sectional area and length, will have equal resistances?
- | Wire $W_1$  | Wire $W_2$  |
|---|---|
| a. $2A$  | $A$    |
| b. $A$   | $2A$   |
| c. $2A$  | $A$   |
| d. $A$   | $2A$  |
| e. $A$   | $A$    |
6. A capacitor is fully charged by a 10-Volt battery, and has 20 millijoules of energy stored in it. The charge on each conducting plate of the capacitor is:
- $2 \times 10^{-3}$  Coulombs
  - $4 \times 10^{-3}$  Coulombs
  - $2 \times 10^{-4}$  Coulombs
  - $4 \times 10^{-2}$  Coulombs
  - $2 \times 10^{-2}$  Coulombs

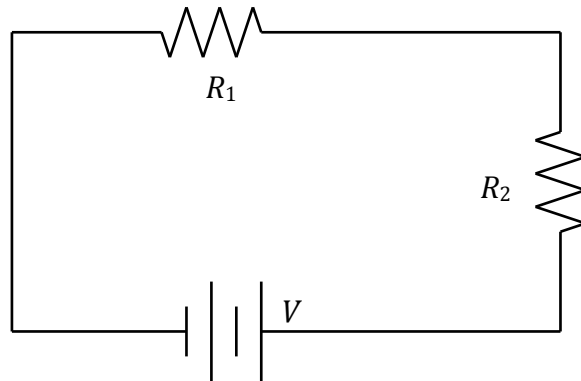


7. Three resistances of  $4\Omega$ ,  $3\Omega$ , and  $6\Omega$  are connected to an  $18\text{V}$  battery as shown in the circuit above. The power dissipated by the  $3\Omega$  resistor is:
- $3\text{W}$
  - $12\text{W}$
  - $8\text{W}$
  - $6\text{W}$
  - $4\text{W}$
8. The cost of the electricity for running a  $120\text{-V}$  computer for  $10$  hours is a little less than  $25$  cents. If electricity costs  $10$  cents per kilowatt-hour, what is the current running through the computer?
- About  $2\text{ A}$
  - About  $4\text{ A}$
  - About  $1\text{ A}$
  - About  $3\text{ A}$
  - About  $0.3\text{ A}$



9. Capacitor  $C_1$  consists of square parallel plates  $10\text{-centimeters}$  on a side, and separated by a distance of  $4$  millimeters, as shown. Capacitor  $C_2$  has square parallel plates  $5\text{-centimeters}$  on a side, and separated by a distance of  $1$  millimeter. What is the ratio between the capacitances of the two parallel-plate systems?
- $C_1:C_2 = 8:1$
  - $C_1:C_2 = 4:1$
  - $C_1:C_2 = 2:1$
  - $C_1:C_2 = 1:1$
  - $C_1:C_2 = 1:2$

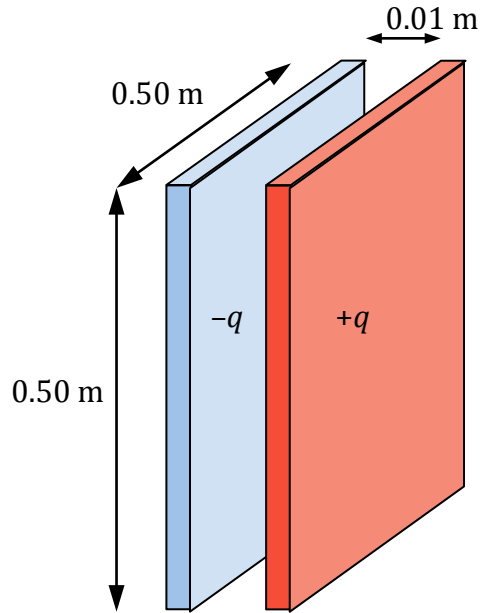
## Part II. Free Response



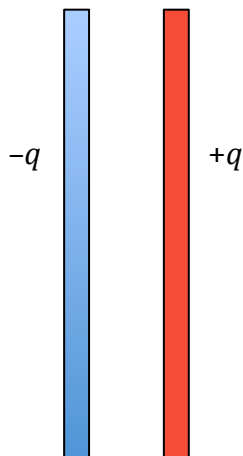
10. Two resistors are placed in series with a battery of potential  $V$  as shown above.
- On the diagram, draw wires and a  $\text{V}$  to show where you would connect a voltmeter to measure the potential across  $R_1$ . If any existing wires need to be disconnected in the circuit, draw an X over those wires in the diagram.
  - On the diagram, draw wires and an  $\text{A}$  to show where you would connect an ammeter to measure the current traveling through  $R_2$ . If any existing wires need to be disconnected in the circuit, draw an X over those wires in the diagram.

Resistance  $R_2$  has a value of  $1.00 \times 10^3 \Omega$ . The voltmeter shows the potential across  $R_1$  to be 3.00V and the ammeter shows the current to be 5.00 mA through  $R_2$ .

- Calculate the value of  $R_1$ .
- Calculate the power dissipated by  $R_2$ .
- Calculate the terminal voltage  $V$  of the battery.
- The battery is disconnected from the circuit, and its potential measured to be 8.20 V. Determine the internal resistance of the battery when it was connected in the circuit.



11. A parallel plate capacitor is created by placing two large square conducting plates of length and width  $0.50$  meters facing each other, separated by a  $1.00$ -centimeter gap. A source of potential is connected to the two plates so that a charge of  $+q = +\sigma A$  is placed on the left plate and a charge of  $-q = -\sigma A$  is on the right plate, where  $\sigma = 500 \times 10^{-6}\text{ C/m}^2$ . The potential source is then removed from the plates. The plates are close together so electric field fringe effects at the edges are negligible.
- a. In the space below, draw electric field lines in the vicinity of the plates.



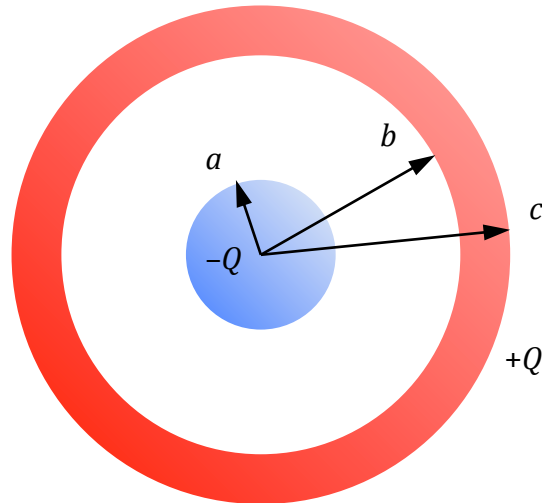
b. Use Gauss's Law to determine the magnitude of the electric field between the plates.

c. Calculate the electric potential  $V$  between the two plates.

d. Calculate the capacitance of this capacitor.

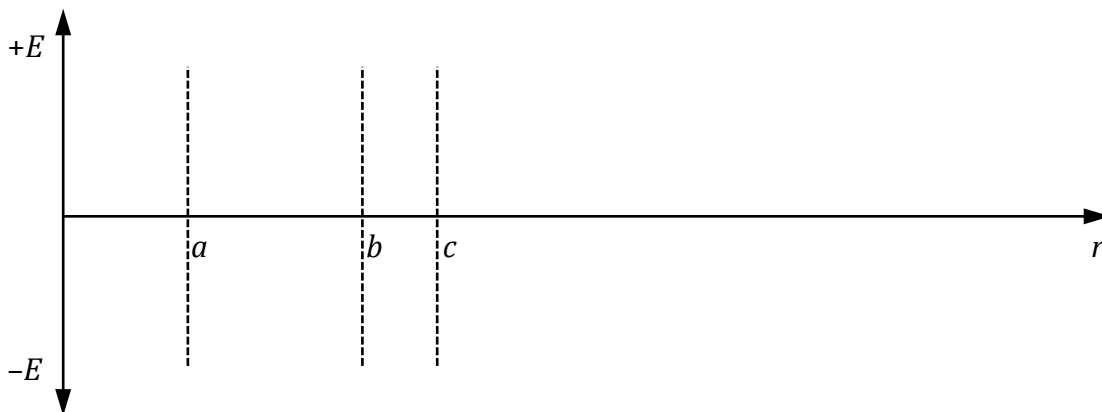
A dielectric of  $\kappa = 2.00$  is now inserted between the isolated plates while the same amount of charge  $Q$  remains on each plate.

- e. Calculate the new capacitance of the system with the dielectric between the plates.
- f. The electric field strength between the plates has (check one):  
 increased  
 decreased  
 remained the same
- g. The electric potential between the plates has (check one):  
 increased  
 decreased  
 remained the same
- h. The energy stored in the capacitor has (check one):  
 increased  
 decreased  
 remained the same
- i. How would your answers to *f*, *g*, and *h* change if the dielectric had been inserted *while the voltage supply was still connected to the capacitor*? Explain.



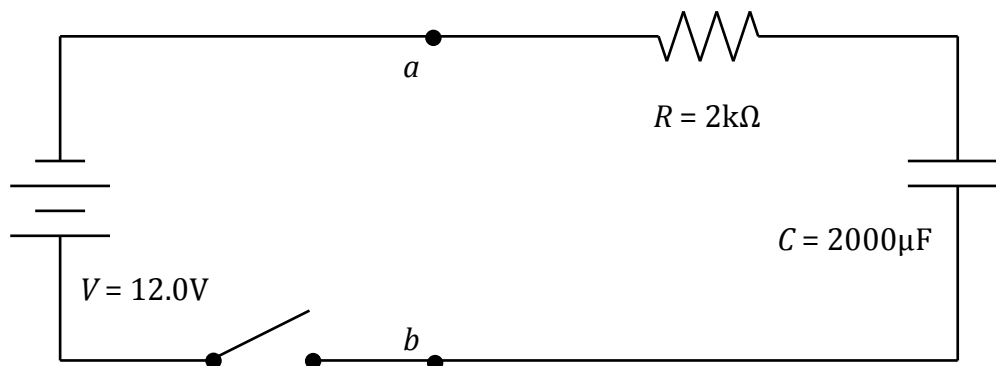
12. A spherical capacitor is constructed of concentric conducting spheres. The interior sphere is solid, with a radius  $a$  and a charge of  $-Q$ . The exterior sphere is a hollow shell with inner radius  $b$  and outer radius  $c$ , and charge  $+Q$ . Give symbolic answers as a function of the given variables and fundamental constants.
- Use Gauss's Law to calculate the electric field  $E$ , magnitude and direction, for the space between the spheres, where  $a < r < b$ .

- On the axes below, sketch a graph of electric field  $E$  as a function of  $r$  for the range  $0$  to  $2c$ , where  $E > 0$  is away from the center.

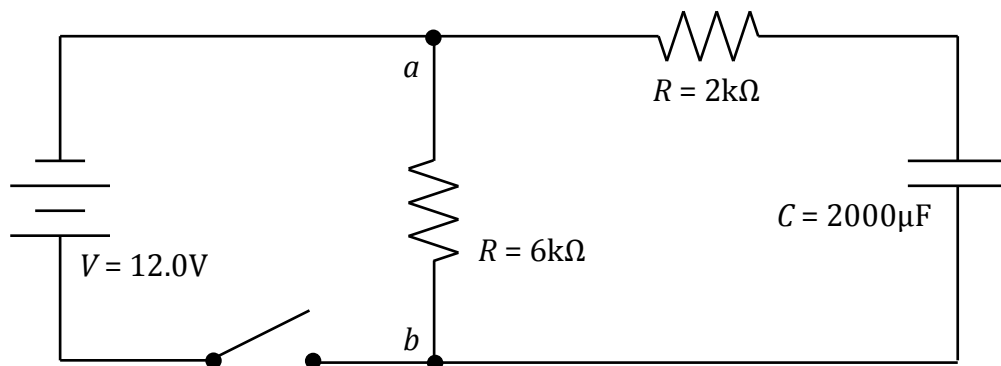




- c. Calculate the magnitude of the difference in electric potential  $V$  between the two spheres.
- d. Calculate the capacitance of this conducting-sphere system.
- e. The spheres are discharged, and then connected to a source of electric potential of magnitude  $2V$ . Calculate how much Work must be done to fully charge the capacitor under these new conditions.



13. The circuit shown above is constructed with a 12.0-Volt power supply, a  $2.00 \times 10^3 \Omega$  resistor, and a  $2.00 \times 10^{-3} F$  capacitor, initially uncharged. The switch is open, but then closed at time  $t = 0$ .
- Calculate the time constant for the circuit.
  - Calculate the initial flow of current, both magnitude and direction, when the switch is first closed.
  - Write and solve a differential equation to find the charge  $Q$  on the plates of the capacitor as a function of time  $t$ .



The switch is opened again, and a  $6000\Omega$  resistor placed across the circuit from point  $a$  to point  $b$  at a new time  $t = 0$ .

- d. What is the time constant of this new circuit?
  
  
  
  
  
  
  
  
  
  
- e. At what time is the potential across the capacitor  $6.00\text{ V}$ ?
  
  
  
  
  
  
  
  
  
  
- f. Calculate the total energy dissipated by the  $6000\Omega$  resistor. (Assume that any heating effects of the wire are negligible.)

1. The correct answer is *d*. Once the plates are removed from the voltage source, they have a charge  $Q = CV$ . Moving the plates farther apart decreases their capacitance by a factor of 2 according to  $C = \frac{A}{d}\epsilon_0$ . Although the capacitance of the system has changed, the amount of charge stored on the plates has not: we conclude that the potential between the plates  $V$  must have *increased* as a result of the inverse relationship between  $C$  and  $V$  in  $Q = CV$ . The electric field between the two plates is described by the relationship  $V = -Ed$ , and because  $V$  and  $d$  have both increased by a factor of 2, we conclude that the electric field  $E$  between the plates is unchanged. The energy stored in the capacitor can be calculated as a function of any two of  $Q$ ,  $C$ , and  $V$ . In this case,  $U_C = \frac{1}{2}QV$  reveals that, because  $V$  has increased, the energy stored in the capacitor has increased as well. From a Conservation of Energy perspective, this extra energy arises as a result of the Work done in moving the two plates farther apart. The only answer consistent with the above analysis is *d*.
2. The correct answer is *d*. Inserting the dielectric into the capacitor increases the capacitance of the parallel plates. The battery maintains the potential across the plates  $V$ , so according to  $Q=VC$ , we can see that the amount of charge stored on the plates will increase. The electric field between the plates remains *constant*, however, based on  $\Delta V=Ed$ . The capacitor, by virtue of its increased capacitance, stores more energy, according to  $U = \frac{1}{2}CV^2$ .
3. The correct answer is *a*. Solve first for the pairs of resistors in parallel, then use those equivalent resistances to get the effective, or equivalent, resistance for the circuit as a whole.

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_{\text{equivalent}}}$$

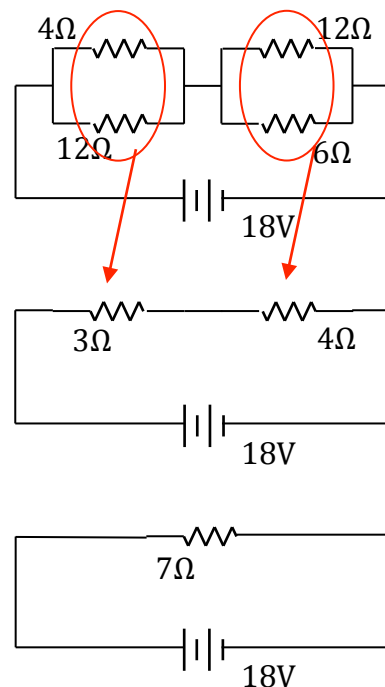
$$\frac{1}{4} + \frac{1}{12} = \frac{1}{R_{\text{equivalent}}} \rightarrow R_{\text{equivalent}} = 3\Omega$$

$$\frac{1}{12} + \frac{1}{6} = \frac{1}{R_{\text{equivalent}}} \rightarrow R_{\text{equivalent}} = 4\Omega$$

Now use these two equivalent resistances in series to determine the overall resistance.

$$R_1 + R_2 = R_{\text{equivalent}}$$

$$R_{\text{equivalent}} = 3\Omega + 4\Omega = 7\Omega$$



4. The correct answer is *b*. The equivalent capacitance of the three capacitors can be found by calculating the  $1\mu F$  and  $5\mu F$  in parallel, and then putting the  $6\mu F$  capacitor in series with that:

$$C_{parallel} = C_1 + C_5$$

$$C_{parallel} = 1\mu F + 5\mu F = 6\mu F$$

Now we put the equivalent capacitance from the parallel capacitors in series with the remaining capacitor:

$$\frac{1}{C_{series}} = \frac{1}{C_{parallel}} + \frac{1}{C_6} =$$

$$\frac{1}{C_{series}} = \frac{1}{6\mu F} + \frac{1}{6\mu F} = \frac{2}{6\mu F}$$

$$C_{series} = 3\mu F$$

5. The correct answer is *a*. The resistance of a wire can be calculated using the formula  $R = \frac{\rho L}{A}$ . With half as big a resistivity in wire  $W_2$ , wire  $W_1$  needs to either be half as long as  $W_2$ , or have twice as large a cross-sectional area. Choice *a* satisfies this requirement.

6. The correct answer is *b*. The charge stored in the plates of the capacitor can be determined using the formula  $U = \frac{1}{2}QV$ :

$$U = \frac{1}{2}QV$$

$$Q = \frac{2U}{V} = \frac{2(20 \times 10^{-3} J)}{10V} = 4 \times 10^{-3} \text{ Coulombs}$$

7. The correct answer is *b*. Analysis of the effective resistance of the resistor shows that the current coming from the battery is 3 Amps, resulting in a potential decrease of 12V over the  $4\Omega$  resistor. Thus, there is a 6V decrease over the  $3\Omega$  and  $6\Omega$  resistors.

The Power dissipated by that resistor can then be calculated using  $P = \frac{V^2}{R} = \frac{6^2}{3} = 12W$ .

A similar Power analysis may be performed by using  $P = IV$  or  $P = I^2R$ , with appropriate values substituted in.

8. The correct answer is *a*. Based on the total cost of the energy, the time, and the price per kWh, we can determine about how much Power the computer is using:

$$\frac{\sim 25 \text{cents}}{1} \times \frac{kW \cdot hr}{10 \text{cents}} \times \frac{1}{10 \text{hrs}} = 0.25 kW = \sim 250W$$

Now, using  $P=IV$ , we can determine the current being drawn by the computer:

$$P = IV, \text{ so } I = \frac{P}{V}$$

$$I = \frac{\sim 250W}{120V} = \sim 2A$$

9. The correct answer is  $d$ . The capacitance of a parallel-plate system can be determined using  $C = \frac{\epsilon_0 A}{d}$ , where  $A$  is the surface area of the plates, and  $d$  is the separation between the plates. Here,  $C_1$  has 4 times the surface area of  $C_2$ , but also 4 times the distance. This has the effect of giving the two sets of plates equal capacitance.

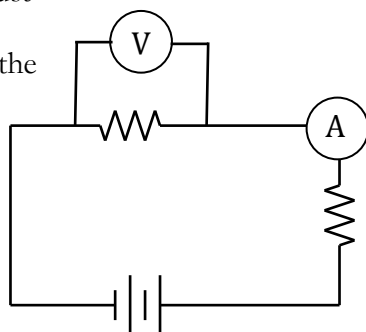
$$C = \frac{\epsilon_0 A}{d}$$

$$\frac{C_1}{C_2} = \frac{\epsilon_0 A_1 / d_1}{\epsilon_0 A_2 / d_2}$$

$$\frac{C_1}{C_2} = \frac{A_1 d_2}{A_2 d_1} = \frac{(2x)^2 (d)}{(x^2)(4d)} = \frac{1}{1}$$

10.

a. The voltmeter must be placed *over* the resistor to measure the potential difference before and after.



b. The ammeter must be inserted *into* the circuit so that there is only one path for current to follow. The ammeter may be placed anywhere in the circuit to measure current because it's a series circuit and the current is the same everywhere. But it makes most sense intuitively to place the meter near the resistor.

1. The two resistors are in series, so the current  $I$  is the same throughout the circuit. Using Ohm's Law and the information we have for  $R_1$ :

$$V_1 = IR_1$$

$$3V = (0.005A)R_1$$

$$R_1 = \frac{3}{0.005} = 600\Omega$$

2. Using the stated current and resistance:

$$P = I^2 R$$

$$P = (0.005A)^2 (1000\Omega) = 2.5e-2W = 25mW$$

3. The terminal voltage of the battery can be calculated using the total  $I$  and  $R$  for the circuit, along with Ohm's Law:

$$V = IR$$

$$V = (0.005A)(600\Omega + 1000\Omega) = 8.00V$$

4. Terminal voltage is emf less the potential drop due to the internal resistance:

$$V = \epsilon - Ir$$

$$8 = 8.2 - (0.005)r$$

$$r = 40\Omega$$

11.

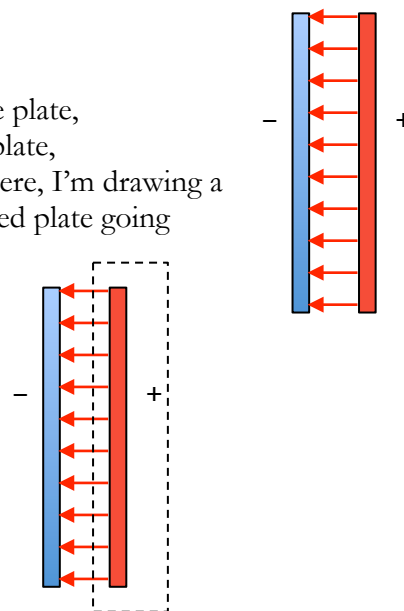
a. With the plates in close proximity and negligible fringing:

b. Gauss's law can be used to determine the electric field for one plate, and then that effect can be doubled to account for the other plate, or Gauss's law can be used with the two plates as arranged. Here, I'm drawing a Gaussian surface that has field lines from the positively-charged plate going through one surface of area  $A$ .

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{\epsilon_0} = \frac{500e-6C/m^2}{8.85e-12C^2/(N \cdot m^2)} = 5.65e7N/C$$



c. For a constant electric field:

$$V = -Ed$$

$$V = (5.65e7N/C)(0.01m) = 5.65e5V$$

d. The capacitance can be calculated either theoretically using the dimensions of the plates, or using the definition of capacitance and the values we've determined here.

Theoretically:

$$C = \frac{A\epsilon_0}{d} = \frac{(0.5m)^2(8.85e-12C^2/(N \cdot m^2))}{0.01m} = 2.21e-10F$$

Using definition of capacitance:

$$C = \frac{Q}{V} = \frac{\sigma A}{V} = \frac{(500e-6N/m^2)(0.5m)^2}{5.65e5V} = 2.21e-10F$$

e. The dielectric increases the capacitance of the system by a factor  $\kappa = 2$ .

$$C = \kappa C_0$$

$$C = (2.00)(2.21e-10F) = 4.42e-10F$$

f. The problem states that the plates are isolated when the dielectric is inserted, and retain the same charge. As the insulating dielectric is polarized by the electric field between the plates, it develops an internal electric field in the opposite direction, thus *decreasing* the net electric field between the plates.g. The electric potential between the plates has *decreased* as well, as a result of  $V = -Ed$ , or by considering that the capacitance has increased. By  $Q = VC$ , if capacitance has increased, the potential between the plates has decreased.h. The energy stored in the capacitor has *decreased*, according to  $U = \frac{1}{2}QV$ .i. If a voltage source is still connected during the dielectric insertion, the potential across the plates will be maintained as more charge flows into the surfaces of the plates. Electric field will *remain the same*, electric potential will *remain the same*, and energy stored in the capacitor will *increase*.

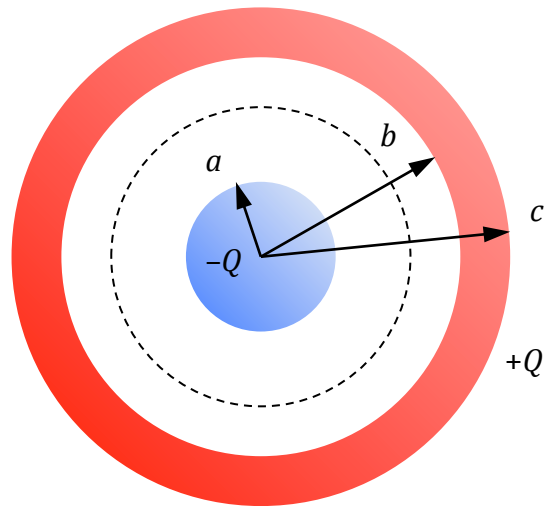
12.

- a. Construct a Gaussian sphere in the space between the two conducting spheres:

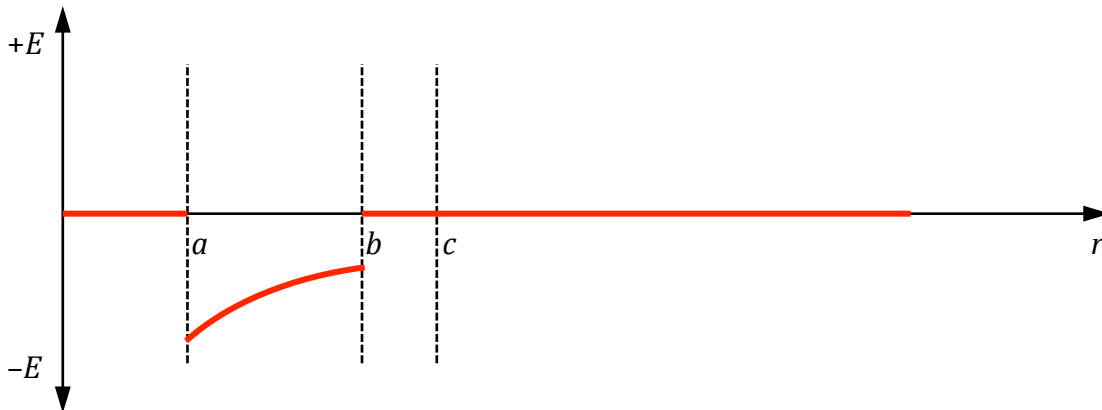
$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{in}}{\epsilon_0}$$

$$E(4\pi r^2) = -q_{in}(4\pi k)$$

$$E = -k \frac{q}{r^2} \hat{\mathbf{r}}, \text{ or } k \frac{q}{r^2} \text{ toward the center}$$



- b. Note that there is no electric field outside the capacitor, and no electric field in the conductors themselves. Here, there is only an electric field between the two charged conductors.



- c. Using the position integral of electric field  $E$  calculated above:

$$V = -\int \mathbf{E} \cdot d\mathbf{r}$$

$$V = -\int_b^a \frac{k(-Q)}{r^2} = kQ \left( \frac{1}{b} - \frac{1}{a} \right)$$

Because only the magnitude of the potential was requested,  $kQ \left( \frac{1}{a} - \frac{1}{b} \right)$  is also acceptable.

- d. Using the definition of capacitance:

$$C = \frac{Q}{V} = \frac{Q}{kQ \left( \frac{1}{a} - \frac{1}{b} \right)} = \frac{1}{k} \left( \frac{ab}{b-a} \right)$$

Note that capacitance formula must use  $(b-a)$  in denominator to produce positive result, because capacitance is always positive.

- e. Potential has doubled, and because  $Q = VC$ , the charge on the capacitor has doubled as well.

$$U = \frac{1}{2} QV = \frac{1}{2} (2Q) \left( 2kQ \frac{ab}{b-a} \right) = 2kQ^2 \left( \frac{ab}{b-a} \right)$$



13.

a.  $\tau = RC = (2000e - 6F)(2000\Omega) = 4.00s = 4.00\Omega F$

b. Initial current flow based on capacitor initially being uncharged:

$$I_0 = \frac{V}{R} = \frac{12.0V}{2000\Omega} = 0006A = 6.00mA$$

c. Using the changes in potential over the loop:

$$V_{\text{battery}} - V_{\text{resistor}} - V_{\text{capacitor}} = 0$$

$$V - IR - \frac{q}{C} = 0$$

$$V - \frac{dq}{dt}R - \frac{q}{C} = 0$$

This is the initial set-up. Now rearrange to get  $dq$  and  $dt$  on opposite sides:

$$V - \frac{dq}{dt}R - \frac{q}{C} = 0$$

$$\frac{dq}{dt}R = V - \frac{q}{C}$$

$$\frac{dq}{dt} = \frac{CV}{RC} - \frac{q}{RC}$$

$$\frac{dq}{dt} = \frac{q - CV}{-RC}$$

$$\frac{dq}{q - CV} = \frac{1}{-RC} dt$$

It helps to know where you're going in that derivation! Now integrate both sides, the left for the charge on the plates, from 0 to  $Q$ , and the right for time  $t = 0$  to  $t$ .

$$\int_0^Q \frac{dq}{q - CV} = \int_0^t \frac{1}{-RC} dt$$

$$\ln(q - CV)_0^Q = \frac{-t}{RC}$$

$$\ln(Q - CV) - \ln(0 - CV) = \frac{-t}{RC}$$

$$\ln\left(\frac{Q - CV}{-CV}\right) = \frac{-t}{RC}$$

$$\ln\left(1 - \frac{Q}{CV}\right) = \frac{-t}{RC}$$

$$1 - \frac{Q}{CV} = e^{-t/RC}$$

$$\frac{Q}{CV} = 1 - e^{-t/RC}$$

$$Q = CV(1 - e^{-t/RC})$$

- d. Now the capacitor is in series with two resistors, and is discharging through both of them. The time constant for this circuit is calculated just as before:

$$\tau = RC = (2000\Omega + 6000\Omega)(2000e - 6F) = 16s$$

- e. This question is a bit complex. We don't have a way of determining  $V$  across the capacitor as a function of time, although we can get current flow  $I$  as a function of time if we calculate or know that  $I = I_0 e^{-t/RC}$ . Using this fact with Ohm's Law allows us to determine the time at which  $V = 6.00V$ :

$$I_{inst} = \frac{V_0}{R} e^{-t/RC}$$

$$\frac{V_{inst}}{R} = \frac{V_0}{R} e^{-t/RC}$$

$$\frac{V_{inst}}{V_0} = e^{-t/RC}$$

$$\ln\left(\frac{V_{inst}}{V_0}\right) = -\frac{t}{RC}$$

$$t = -RC \ln\left(\frac{V_{inst}}{V_0}\right) = -16s \ln\left(\frac{6.00V}{12.00V}\right) = 11.1s$$

- f. The total amount of energy stored in the capacitor is calculated based on its capacitance and the total potential across its plates, which is 12 V when fully charged:

$$U = \frac{1}{2} CV^2$$

$$U = \frac{1}{2} (2000e - 6F)(12V)^2 = 0.144J$$

That energy is dissipated through the two resistors in the circuit according to  $P = I^2 R$  for each resistor. Because the current flow through the resistors in series is identical, the Power dissipated by each resistor is proportional to its resistance. The  $6000\Omega$  resistor is 75% of the total  $8000\Omega$  resistance, so it dissipates 75% of the 0.144J, or  $0.75 \times 0.144J = 0.108J$ .