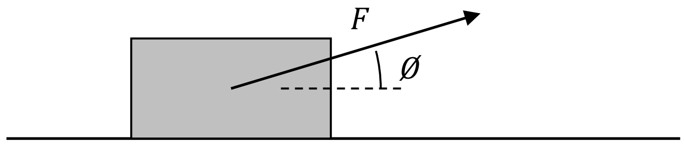
This test covers Work, mechanical energy, kinetic energy, potential energy (gravitational and elastic), Hooke's Law, Conservation of Energy, heat energy, conservative and non-conservative forces, with some problems requiring a knowledge of basic calculus.

Part I. Multiple Choice



1. A force F is exerted at an angle Ø on a box of mass m as it is dragged across the floor at constant velocity. If the box travels a distance x, then the work done by the force F on the box is

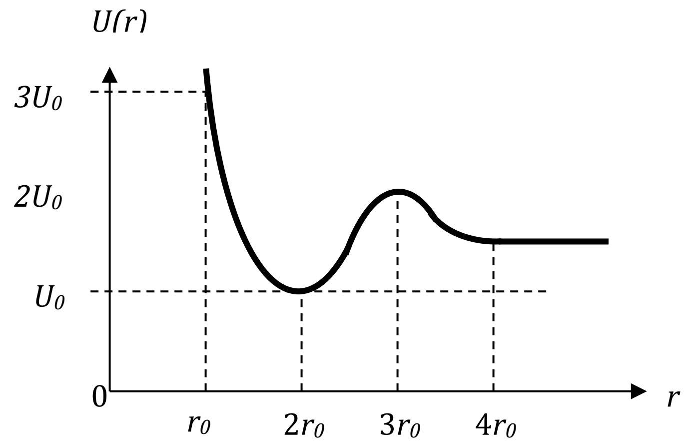
   a. Fx
   b. $Fx \cos Ø$
   c. $mgx \cos Ø$
   d. $Fx \sin Ø$
   e. $Fx \tan Ø$

2. A block of wood, initially moving along a rough surface, is pushed with an applied horizontal force $F_{applied}$ that is less than the friction force $F_{friction}$. Which of the following statements is *false*?

   a. The Work being done by the applied force is negative.
   b. The net Work being done on the block is negative.
   c. The block is slowing down.
   d. The net Work being done on the box decreases its kinetic energy K.
   e. There is an increase in internal energy due to friction.

3. A 300-Watt electric wheelchair has a mass of 50kg, and carries its 50kg occupant at constant velocity up a long ramp. About how much time does it take the wheelchair to reach the top of the 10-meter high ramp?

   a. 3 s
   b. 17 s
   c. 10 s
   d. 333 s
   e. 33 s



4. The graph above represents the potential energy U as a function of position r for a particle of mass m. If the particle is released from rest at position r_0, what will its speed be at position $3r_0$?

a. $\sqrt{\frac{8U_0}{m}}$

b. $\sqrt{\frac{4U_0}{m}}$

c. $\sqrt{\frac{2U_0}{m}}$

d. $\sqrt{\frac{6U_0}{m}}$

e. The particle will never reach position $3r_0$.

5. The behavior of a non-linear spring is described by the relationship $F = -2kx^3$, where x is the displacement from the equlibrium position and F is the force exerted by the spring. How much potential energy is stored in the spring when it is displaced a distance x from equilibrium?

a. $\frac{1}{2}kx^4$

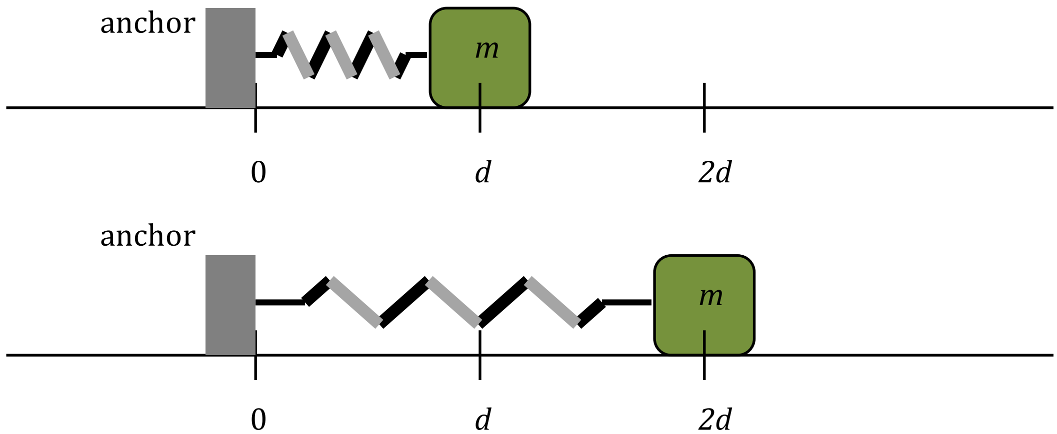
b. $6kx^2$

c. $\frac{1}{3}kx^4$

d. $\frac{1}{3}kx^3$

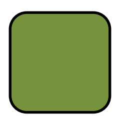
e. $\frac{2}{3}kx^2$

Part II. Free Response

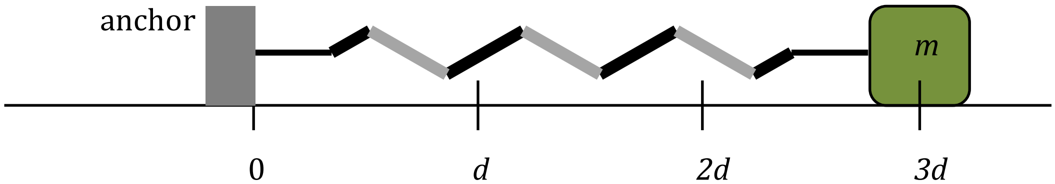


6. A block of mass *m* rests on a rough surface, and has a light spring of spring constant *k* and unstretched length *d* attached to one side as shown, with the other end of the spring attached to an anchor. There is a static coefficient of friction μ_s between the surface and the block, and when the block is placed to the right at position 2*d*, it remains stationary on the surface. Express answers in terms of *m*, *k*, *d*, and fundamental constants.

a. Draw a free-body diagram of the block at the time when it is located at position 2*d*.



b. Determine the friction force acting on the block when it is located at position 2*d*.



c. The block is now moved to position 3*d* and released, where it remains at rest. When the block is moved slightly past this position, the block begins to slide along the surface with a kinetic coefficient of friction μ_k.

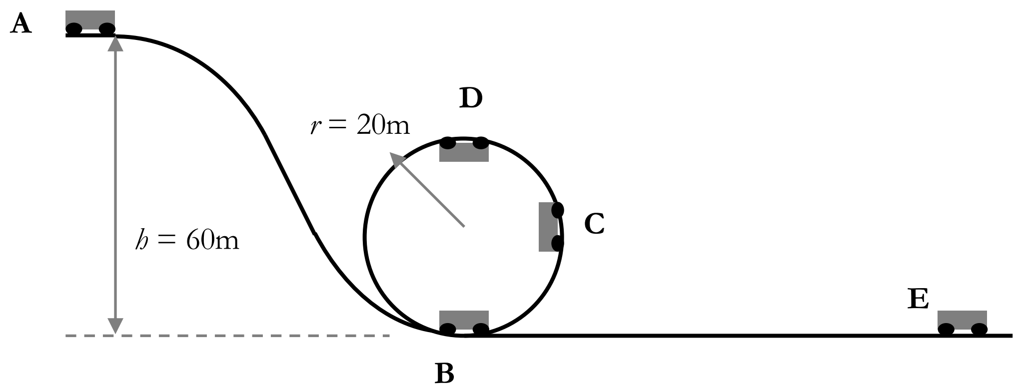
  i. In terms of the variables given, what is the value of μ_s?

  ii. How much potential energy is stored in the mass-spring system just before the block begins to move?

  iii. The block slides a total distance of *d* before coming to a halt again. Determine the coefficient of kinetic friction μ_k.

iv. At what position does the block have its maximum velocity as it slides?

v. What is the maximum velocity of the block as it slides?



7. A roller coaster car of mass m = 200 kg is released from rest at the top of a 60 m high hill (position **A**), and rolls with negligible friction down the hill, through a circular loop of radius 20 m (positions **B**, **C**, and **D**), and along a horizontal track (to position **E**).

a. What is the velocity of the car at position **B**?

b. Determine the velocity of the car at position **C**.

c. Draw a free-body diagram of the car at position **C**.

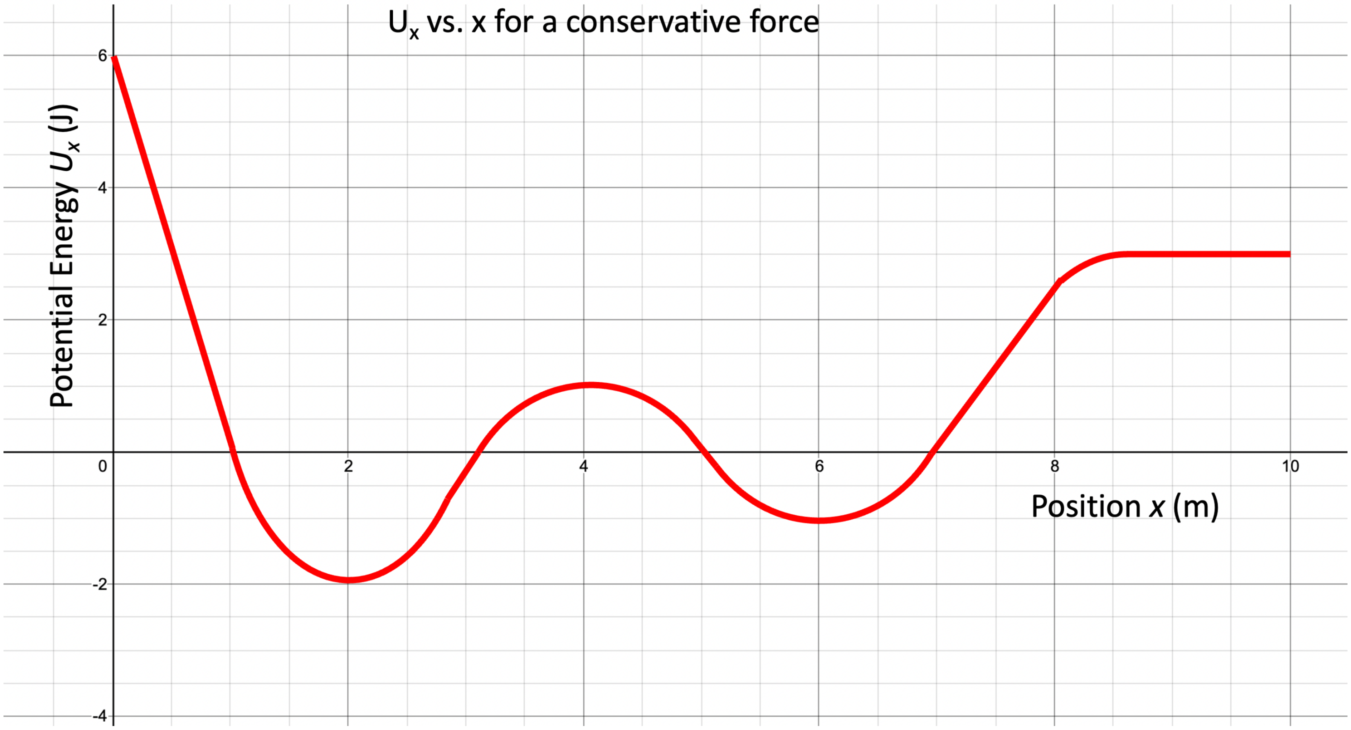
d. Determine the velocity of the car at position **D**.

e. Determine the force (magnitude and direction) of the track on the car at position **D**.

f. After completing the loop, the rollercoaster car is travelling horizontally at velocity v_0 and subjected to a braking force $F_{braking} = -kv$, where k is a constant, v is the instantaneous velocity of the car, and time t is the amount of time that the braking force has been applied.

   i. Write, but do not solve, a differential equation that could be used to evaluate the velocity v of the car as a function of k, m, time t, and fundamental constants as needed, as the car moves along the horizontal track.

   ii. Solve the differential equation from part (f)(i) above, using definite integrals to determine an equation that gives the velocity v of the car as a function of initial velocity v_0, k, m, and time t.



8. A conservative force acts in the x-direction on a particle of mass m = 2.0 kg to produce a potential energy curve as shown above.

a. A particle is released from rest at the 0.25 meter position.

i. What is the potential energy of the particle at this position?

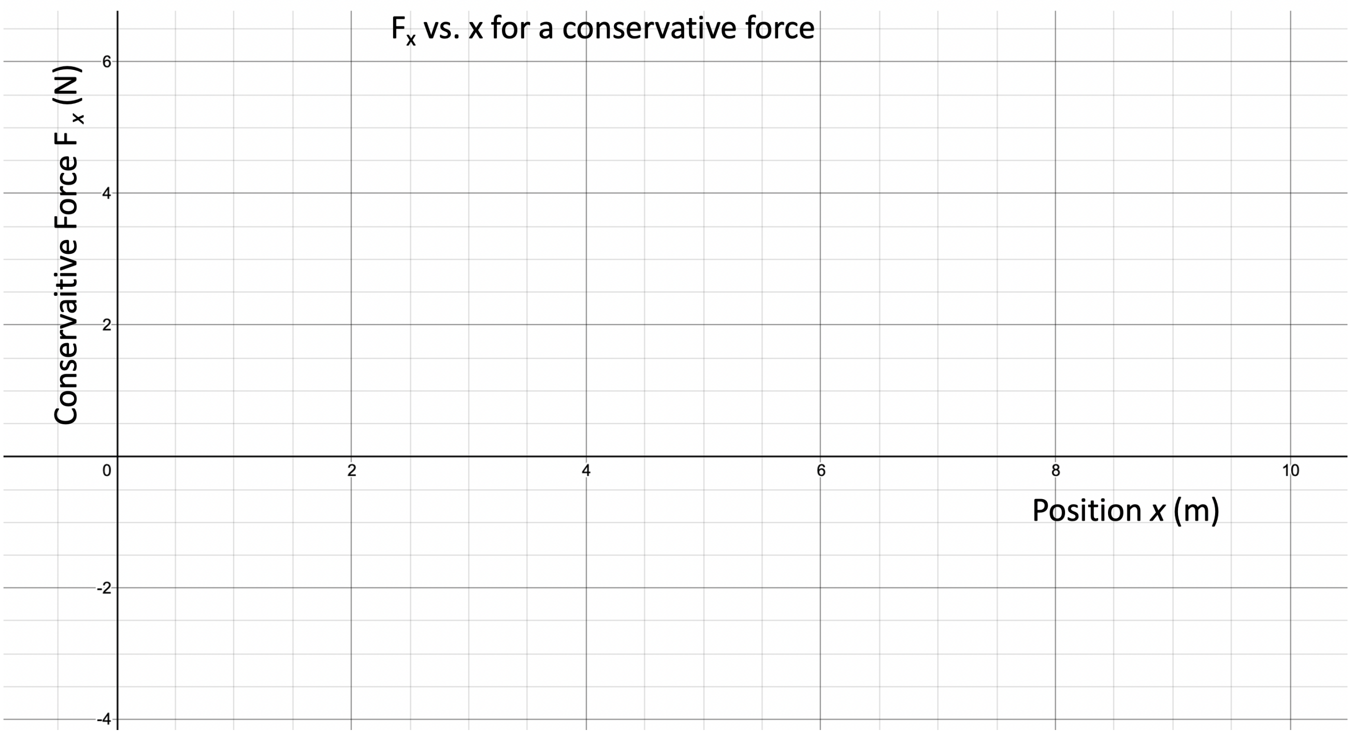
ii. What is the velocity of this particle at x = 2 meters?

iii. Describe the point on the U curve at x = 4m briefly, and what happens when the released particle reaches this position.

iv. Does the released particle reach $x = 9$ m? If not, explain why not. If so, describe the particle's behavior.

b. A second particle, also of mass $m = 2.0$ kg, is released from rest at $x = 9$ m. Briefly describe the behavior of this particle.

c. Sketch a graph of the conservative force that produces this potential energy curve.



1. The correct answer is ***b***. Work done by an object is calculated according to the Work formula W=*F*•*x*, or W=*Fx* cos Ø. There are a couple of distractors in this problem: the mass *m* of the box is not needed in the solution, and the box's constant velocity isn't required.

The fact that the box is traveling at constant velocity implies that there is friction impeding its motion, but evidently the energy "lost" to friction is equal to the work being done by the force *F*, so that the net Work done on the box by *F* and $F_{friction} = 0$. None of this information is necessary, though, to solve the problem.

2. The correct answer is *a*. Statement *a* is false because the work done by any single agent is $W = Fx\cos\theta$. Here, with the applied Force and the displacement of the box being in the same direction (or $\cos\theta = \cos(0) = 1$), the Work being done by the applied force is positive, and goes toward increasing the box's kinetic energy.

Of course, the *net*, or overall, Work being done on the box has to include the force of Friction, which is acting in a direction opposite that of the box's displacement. Thus, Work done by friction is negative, which has the effect of converting some of the box's kinetic energy to internal energy via heat.

3. The correct answer is *e*. The wheelchair carries a total mass of 100kg up to a height of 10m, with 300J of Work being done by the wheelchair each second. The time for the total Work does is calculated as follows:

$$P = \frac{Work}{time}$$

$$time = \frac{Work}{Power} = \frac{mgh}{P}$$

$$time = \frac{(100kg)(10m/s^2)(10m)}{300J/s} = \frac{10000}{300} = 33s$$

4. The correct answer is ***c***. This is a conservation of energy problem, in which we examine the relationship between the particle's potential and kinetic energies.

$$U_i + K_i = U_f + K_f$$

$$3U_0 + 0 = 2U_0 + \tfrac{1}{2}mv^2$$

$$U_0 = \tfrac{1}{2}mv^2$$

$$v = \sqrt{\frac{2U_0}{m}}$$

5. The correct answer is *a*. The potential energy stored in the spring is calculated using the Work integral:

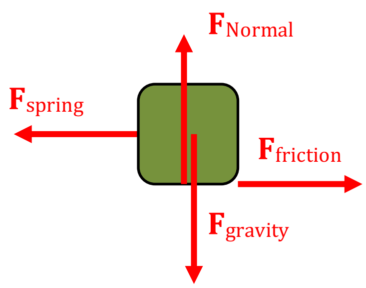
$$U = -\int_{x_i}^{x_f} F \bullet dx$$

$$U = -\int_{0}^{x} -2kx^3 \bullet dx$$

$$U = 2k \left.\frac{x^4}{4}\right|_0^x = \frac{1}{2}kx^4$$

6.

a. The block isn't moving yet, so the force exerted by the spring to the left equals the force exerted by friction to the right. Vectors should be labeled and drawn with lengths proportional to their magnitude. Some instructors wish for vectors to be drawn with their point of origin beginning at the location where that force is applied, and some instructors prefer that force vectors be drawn originating from the center of the (point) mass.



b. The friction force acting on the block at this point is equal in magnitude to the force applied by the spring:

$$F_{net} = ma$$
$$-F_{spring} + F_{friction} = 0$$
$$F_{friction} = kx = kd$$

c.

i. The coefficient of static friction is based on the maximum force of static friction that the block-surface can support. In this case:

$$\mu_{static} = \frac{F_{friction}}{F_{Normal}} = \frac{kx}{mg} = \frac{k(2d)}{mg}$$

ii. The potential energy stored in the spring is simply based on $U_{spring} = \frac{1}{2}kx^2$:

$$U_{spring} = \frac{1}{2}kx^2$$

$$U_{spring} = \frac{1}{2}k(2d)^2 = 2kd^2$$

iii. The coefficient of kinetic friction can be determined by using a Conservation of Energy analysis, taking into account the potential energy of the mass-spring system at the beginning and end, as well as the energy lost to heat in the block's slide:

$$U_{spring-initial} - \Delta E_{internal} = U_{spring-final}$$

$$\frac{1}{2}kx_i^2 - F_{friction}d = \frac{1}{2}kx_f^2$$

$$\frac{1}{2}kx_i^2 - \mu_k mgd = \frac{1}{2}kx_f^2$$

$$\frac{1}{2}k(2d)^2 - \mu_k mgd = \frac{1}{2}kd^2$$

$$\mu_k = \frac{3kd}{2mg}$$

Note that this result reveals that $\mu_k = \frac{3}{4}\left(\frac{2kd}{mg}\right) = \frac{3}{4}\mu_x$, which is consistent with the principal that coefficients of kinetic friction are less than coefficients of static friction.

iv. The block has its maximum velocity where the slope of the velocity-time curve is 0, ie. where acceleration is 0. At that position, the force from the spring and the force of friction are equal to each other:

$$F_{net} = ma$$

$$F_{spring} - F_{friction} = 0$$

$$kx - \mu_k F_{Normal} = kx - \mu_k mg = 0$$

where x is the displacement of the spring from its unstretched length at *d*. Substituting in μ_k from the problem before:

$$kx - \mu_k mg = 0$$

$$kx - \left(\frac{3kd}{2mg}\right)mg = 0$$

$$kx - \frac{3}{2}kd = 0$$

$$x = \frac{3d}{2}, \text{ relative to unstretched length at } d$$

Relative to the origin:

$$\frac{3d}{2} + d = \frac{5}{2}d$$

v. Now that we know the position of the maximum velocity, we can use Conservation of Energy again to calculate that velocity:

$$U_{s-initial} - \Delta E_{\text{int}} = K + U_{s-final}$$

$$\frac{1}{2}kx_i^2 - F_{friction}x = \frac{1}{2}mv^2 + \frac{1}{2}kx_f^2$$

$$\frac{1}{2}k(2d)^2 - \mu_k mg\left(\frac{d}{2}\right) = \frac{1}{2}mv^2 + \frac{1}{2}k\left(\frac{3}{2}d\right)^2$$

$$\frac{1}{2}k(2d)^2 - \left(\frac{3kd}{2mg}\right)mg\left(\frac{d}{2}\right) = \frac{1}{2}mv^2 + \frac{1}{2}k\left(\frac{3}{2}d\right)^2$$

Solve to get $v = \frac{d}{2}\sqrt{\frac{k}{m}}$

7.

a. This is a conservation of energy problem, with the gravitational potential energy of the car *U* being converted to kinetic energy *K* as the car rolls down the hill.

$$U = K$$

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh} = \sqrt{2 \bullet 9.8 \bullet 60} = 34.3 m/s$$

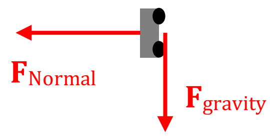
b. At position **C**, some of the car's *K* has converted back to *U*.

$$U_i = K + U_f$$

$$mgh_A = \frac{1}{2}mv_C^{\ 2} + mgh_C$$

$$v_C = \sqrt{2g(h_A - h_C)} = \sqrt{2 \bullet 9.8(60-20)} = 28m/s$$

c. The free-body diagram for the car includes the only two forces acting on the car: the force of gravity, and the force of the track which is providing the centripetal force that keeps the car moving in a circle (usually considered as a Normal force).



d. The velocity of the car at position **D** is determined using the same technique as in (b):

$$U_i = K + U_f$$

$$mgh_A = \frac{1}{2}mv_D^{\ 2} + mgh_D$$

$$v_D = \sqrt{2g(h_A - h_D)} = \sqrt{2 \bullet 9.8(60-40)} = 19.8m/s$$

e. The force of the track on the car at **D** can be determined by using Newton's 2nd Law of Motion to the circular motion:

$$F_c = \frac{mv^2}{r}$$

$$F_{track} + F_g = \frac{mv^2}{r}$$

$$F_{track} = \frac{mv^2}{r} - F_g$$

Here we're considering down (toward the middle of the circle) to be in the positive direction, and we're assuming that the force of the track and the force of gravity are both pointing down: the force of gravity is providing some of the force to keep the car moving in a circle, and the force of the track will provide the remaining.

In some cases, however—if the car is traveling very slowly, for example—a much smaller centripetal force will required, to the point that the track has to provide an *upwards* force. We would realize that this was the case if we calculated a Force for the track that was negative.

Continuing with our calculation:

$$F_{track} = \frac{mv^2}{r} - F_g$$

$$F_{track} = \frac{(200)(19.8)^2}{20} - (200)(9.8) = 1960N$$

f.

i. The horizontal braking force causes the velocity of the car to decrease as a function of time *t*. The integral describing the car's motion is based on applying Newton's 2nd Law of Motion:

$$F_{net} = ma$$
$$-kv = m\frac{dv}{dt}$$
$$\frac{dv}{dt} = \frac{-k}{m}v$$
$$\frac{dv}{v} = \frac{-k}{m}dt$$

ii. Now solve the integral for *v*:

$$\int_{v_0}^{v}\frac{dv}{v} = \int_0^t \frac{-k}{m}dt$$
$$\ln v\Big|_{v_0}^{v} = \frac{-k}{m}t$$
$$\ln v - \ln v_0 = \frac{-k}{m}t$$
$$\ln\left(\frac{v}{v_0}\right) = \frac{-k}{m}t$$
$$\frac{v}{v_0} = e^{\frac{-k}{m}t}$$
$$v = v_0 e^{\frac{-k}{m}t}$$

8. Potential energy can be defined only for conservative forces, and diagrams are one way of describing those forces. The relationship between the conservative force acting on a particle, its displacement, and the potential energy associated with the force is described by the equation $\Delta U = -\int F_x dx$, or $F_x = -\frac{dU}{dx}$.

a.

i. By examining the graph, the potential energy *U* is about 4 Joules.

ii. Using Conservation of Energy:

$$U_i + K_i = U_f + K_f$$
$$4J + 0 = -2J + \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{12}{m}} = \sqrt{\frac{12}{2}} = 2.4 m/s$$

iii. At *x* = 4m there is a position of *unstable equilibrium*. At this point, based on $F_x = -\frac{dU}{dx}$, there is no acceleration applied by the conservative force (the slope of the *U* curve is 0), but here, the particle will continue moving past that position, based on the fact that it has 3 Joules of *K* energy.

iv. The particle *does* reach the position *x* = 9 m, and because it still has 1 Joule of *K* energy, it will continue to move to the right, even though the conservative force is no longer causing any acceleration.

b. A particle released from rest at position *x* = 9 m will not experience any conservative force, and so will not accelerate. It will remain at that location.

c. The graph of the conservative force is effectively a graph of the negative slopes of the *U*-*x* graph. Key points to make sure are included in the Force graph are positions where *U* graph inflects. The Force curve should cross the *x*-axis at equilibrium points (at $x = 2$, 4, and 6 meters, as well as for $x > 8.5$ m). Sketch in the rest of the curve based on your interpretation of the *U* graph.