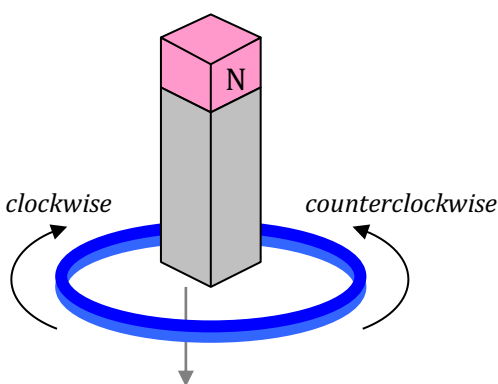This test covers Faraday's Law of induction, motional emf, Lenz's law, induced emf and electric fields, eddy currents, self-inductance, inductance, RL circuits, and energy in a magnetic field, with some problems requiring knowledge of basic calculus.

Part I. Multiple Choice

1. A permanent magnet is dropped, south pole-down, through a conducting loop as shown. As the magnet falls toward—and then falls past—the area of the loop, what will be the direction of the current flow?
   a. Clockwise when falling toward, but counterclockwise as falling away
   b. Clockwise when falling toward, and clockwise as falling away
   c. Counterclockwise when falling toward, but clockwise as falling away
   d. Counterclockwise when falling toward, and counterclockwise as falling away
   e. There will be no current induced, because there is no changing magnetic flux.
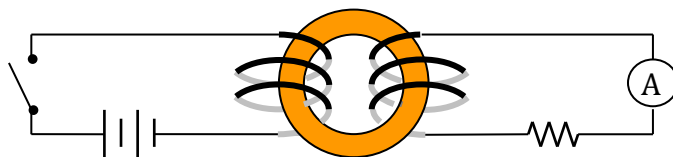
2. The switch on the left side of the diagram above is closed, allowing current to flow through the coil wrapped around a ferromagnetic torus. Which of the statements below correctly describes what happens just after the switch is closed?
   a. There is a clockwise increase in magnetic field in the toroid, and no current flowing through the resistor.
   b. There is a clockwise increase in magnetic field in the toroid, and a current flowing through the resistor to the left.
   c. There is a clockwise increase in magnetic field in the toroid, and a current flowing through the resistor to the right.
   d. There is a counterclockwise increase in magnetic field in the toroid, and a current flowing through the resistor to the left.
   e. There is a counterclockwise increase in magnetic field in the toroid, and a current flowing through the resistor to the right.

glider on frictionless air track
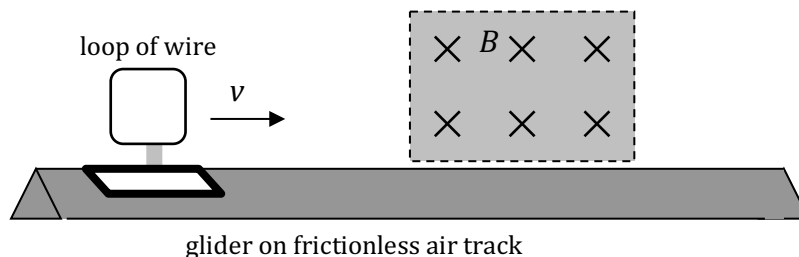
3. A single, continuous loop of conducting wire is mounted on a glider, which travels on a frictionless air track with an initial velocity v. When the front edge of the loop enters the magnetic field B pointing into the page as shown...
   a. there is a clockwise current in the loop, and the glider slows down.
   b. there is a counterclockwise current in the loop, and the glider slows down.
   c. there is a clockwise current in the loop, and the glider speeds up.
   d. there is a counterclockwise current in the loop, and the glider speeds up.
   e. there is no current in the loop, and the glider travels at constant v.
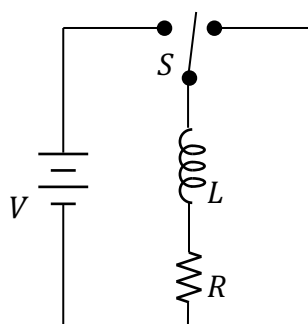

$r = 0.50$ m

B
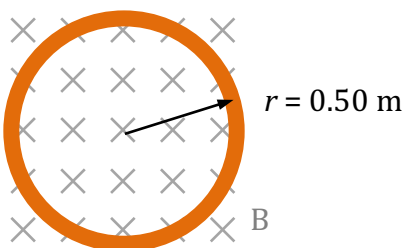
4. A loop of conducting wire in the plane of the page, with a radius of 0.50 m and a net resistance of $R = 0.5 \, \Omega$, is placed in a constant magnetic field B = 4.0 T directed into the page as shown. Which statement is true?
   a. The flux through the loop is 0, and current flows in the counterclockwise direction.
   b. The flux through the loop is 0, and current flows in the clockwise direction.
   c. The flux through the loop is π Webers, and current flows in the clockwise direction.
   d. The flux through the loop is π Webers, and current flows in the counterclockwise direction.
   e. The flux through the loop is π Webers, and the current is 0.
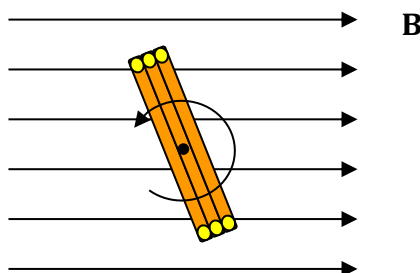
5. A conducting loop consisting of N coils is rotated about an axis parallel to the plane of the coils and through the middle of the loop as shown. The loop has an area $\mathbf{A}$, and rotates with a constant angular velocity ω in the vicinity of a constant magnetic field $\mathbf{B}$, which induces a time-varying emf $\mathcal{E}$ in the coil. What is the magnitude of this emf as a function of time?

    a. $\mathcal{E} = \omega NBA \sin(\omega t)$

    b. $\mathcal{E} = NBA \sin(\omega t)$

    c. $\mathcal{E} = \omega NBA \sin(t)$

    d. $\mathcal{E} = \omega^2 NBA \sin(t)$

    e. $\mathcal{E} = \omega NBA \sin(\omega)$

6. A circuit consists of a battery V, an inductor L, and a resistor R, connected with a switch S as shown. The switch, which has been in the rightmost position for a long time, is quickly moved to the left position so that the battery is connected in series with the inductor and the resistor. Which of the following statements describes the current going through the resistor at two different times?

|	Immediately after the switch is thrown	A long time after the switch is thrown
a.	$I_R = V/R$	$I_R = 0$
b.	$I_R = V/R$	$I_R = V/R$
c.	$I_R = 0$	$I_R = V/R$
d.	$I_R = 0$	$I_R = V/R - V/L$
e.	$I_R = 0$	$I_R = V/R - \sqrt{2L}$
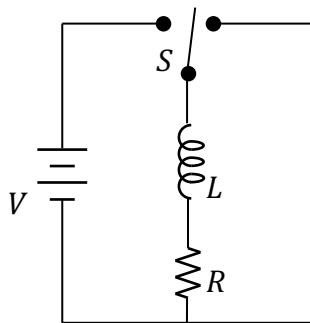
7. A circuit consists of a battery V, an inductor L, and a resistor R, connected with a switch S as shown. The switch, which has been in the leftmost position for a long time, is quickly moved to the right so that the inductor and the resistor are connected only to each other. Which of the following statements is true of the right loop of the circuit, for the time immediately after the switch is moved to the right?
   a. There is no longer a magnetic field in the inductor, so there is no current flowing through the right loop.
   b. There is still a magnetic field in the inductor, but there is no current flowing through the right loop.
   c. There is a decreasing current flowing clockwise through the right loop.
   d. There is a decreasing current flowing counterclockwise through the right loop.
   e. There is an oscillating current flowing through the right loop.
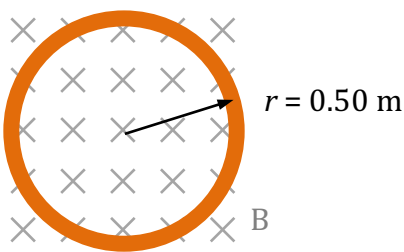
8. A loop of conducting wire in the plane of the page, with a radius of 0.50 m and a net resistance of $R = 0.5\ \Omega$, is placed in a magnetic field that is increasing according to the function $\dfrac{dB}{dt} = 0.5\,T/s$. Which statement is true?
   a. The net flux through the loop is 0, and there is no current flow.
   b. There is a current flow of $\pi/8$ Amps in the clockwise direction.
   c. There is a current flow of $\pi/8$ Amps in the counterclockwise direction.
   d. There is a current flow of $\pi/4$ Amps in the clockwise direction.
   e. There is a current flow of $\pi/4$ Amps in the counterclockwise direction.

Part II. Free Response
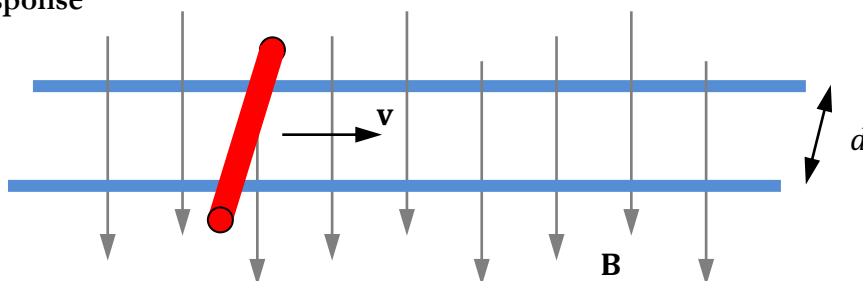
1. A pair of parallel, conducting rails are arranged horizontally as shown in the diagram above, separated by a distance d, and with a constant, vertically-oriented, magnetic field **B** pointing down in the vicinity of the rails. A thin, light, conducting bar of mass m is placed across the rails and pulled across the frictionless rails with constant velocity **v**. Give answers in terms of given variables and fundamental constants.

    a. Determine the magnitude and direction of the induced electric field in the conducting bar as it moves through the magnetic field.

   b. Calculate the potential difference between the two ends of the bar.
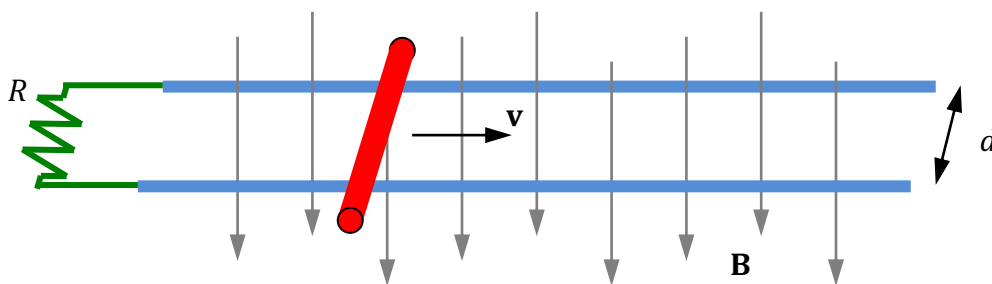
A load resistance R is now connected to the conducting rails as shown, forming a complete circuit with the sliding bar.

    c. Use Faraday's Law to calculate the emf being induced in the circuit as the bar is moved across the rails.

  d. Calculate the power dissipated by the circuit as the bar continues to be pulled with velocity **v**.

The bar being pulled with velocity **v** is now released at time $t = 0$ so that it has an initial velocity of **v**.
  e. Calculate the magnitude and direction of the initial magnetic force acting on the bar as it slides to the right.

  f. Write, but do not solve, a differential equation that could be used to determine the velocity of the sliding bar as a function of time.

  g. Solve the equation in part (f) above to produce an equation that can be used to calculate velocity v_{final} as a function of time t.
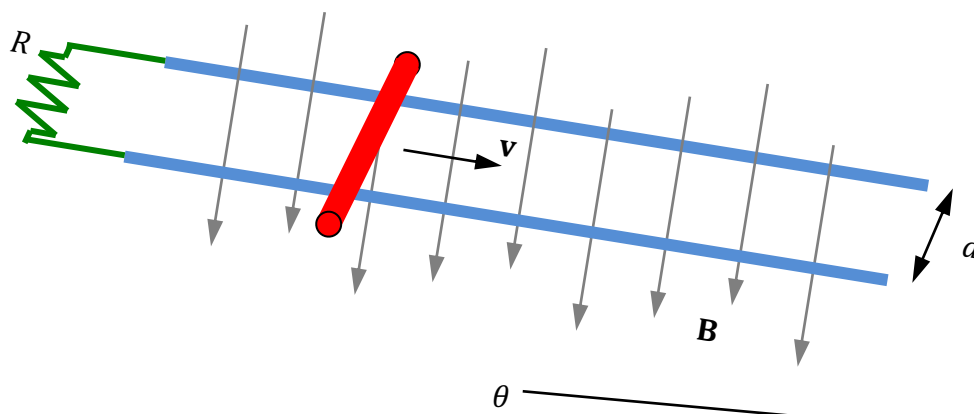
The entire apparatus—rails and magnetic field—is placed on a platform that allows the device to be tilted so that the rails are oriented at an angle θ above the horizontal. The magnetic field **B** remains perpendicular to the plane of the rails.

    h. Draw a free-body diagram of the forces acting on the bar as it slides.

    i. At a certain angle θ, the bar slides down the frictionless rails with a constant speed. Determine this speed.
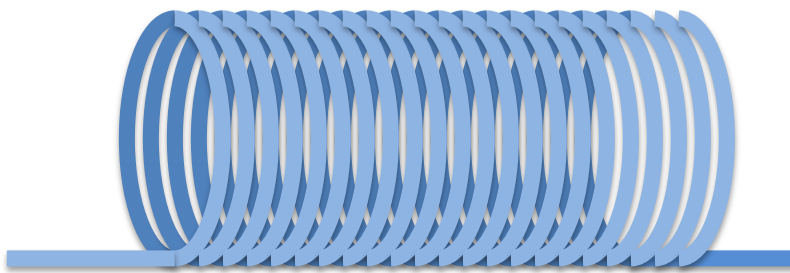
2. A long thin solenoid consists of 20 closely-spaced loops of copper, 10.0-centimeters long with a diameter of 2.00-centimeters. Initially, the solenoid is placed into a circuit with a 12.0-Volt battery and a 10.0-Ω resistor.

    a. Determine the current running through the solenoid while it is in this circuit.

    b. Determine the magnitude of the magnetic field produced by this solenoid while it is in this circuit.

    c. Determine the direction of the magnetic field inside the solenoid if the current through the coils is running upwards in the front of the coils and downwards in the back.

    d. Determine the theoretical inductance of this solenoid.

    e. Calculate the amount of energy stored in the magnetic field of the inductor while it is in this circuit.
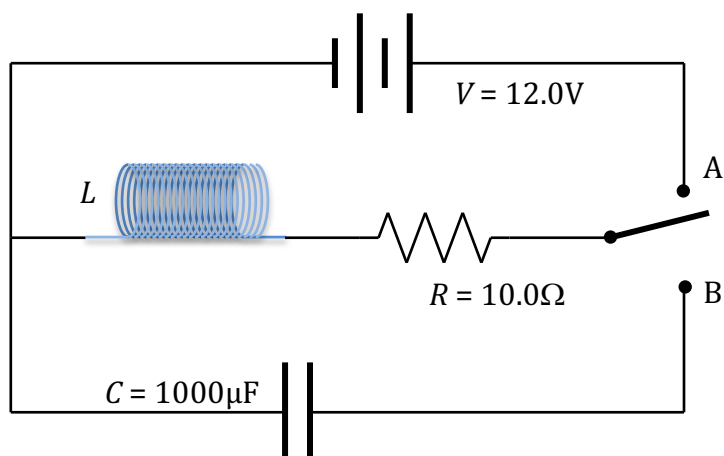
The solenoid/inductor is now placed into the circuit shown above, with the 12.0-Volt battery, the 10.0 Ω resistor, a switch, and an uncharged $1000\mu F$ capacitor as shown. The switch is initially open, and then moved to position A at time $t = 0$.

 f. Sketch a graph of the current flow through the resistor as a function of time, with current to the right considered the positive direction.
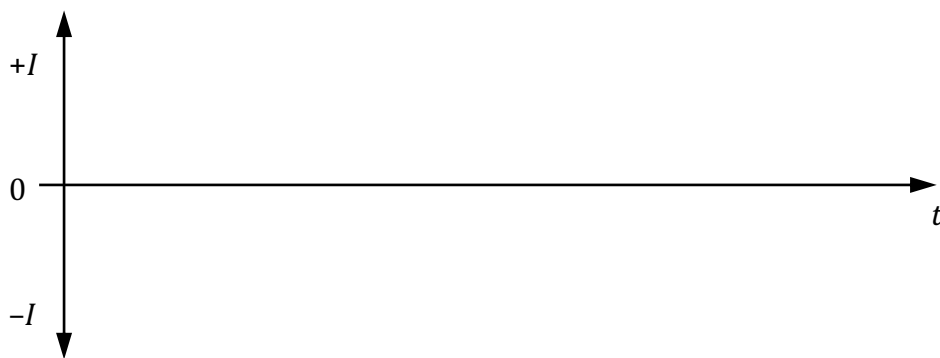
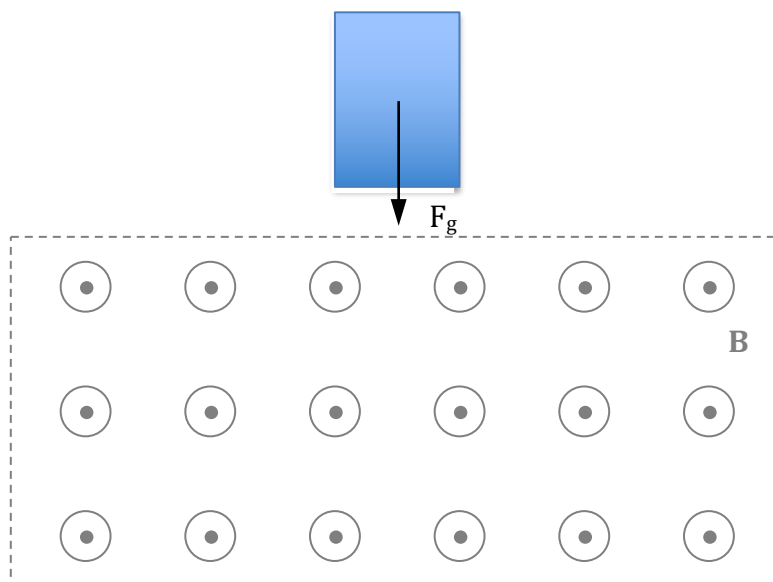After a long time has passed, the switch is quickly moved to position B.

 g. Sketch a graph of the current flow through the resistor as a function of time, with current to the right considered the positive direction.
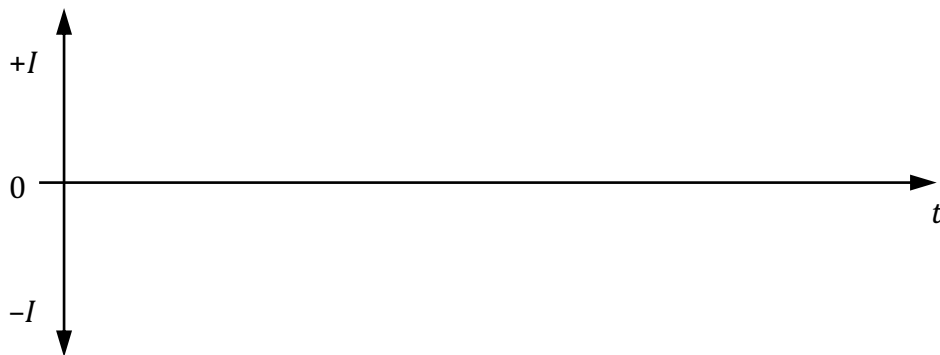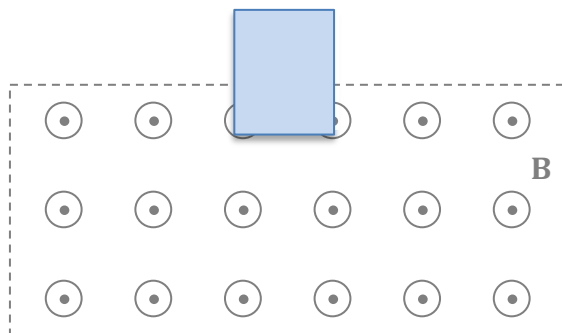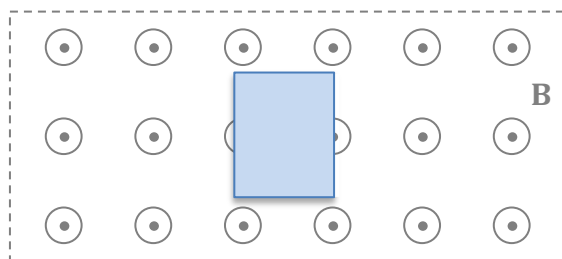
3. A constant horizontal magnetic field is directed out of the page as shown here. A thin, solid, rectangle of conducting material is held above the upper edge field and released so that it falls due to gravity.

In the diagrams below, draw the eddy current, if any, induced in the metal...
   a. as the rectangle is entering the field

   b. during the time the rectangle is in the field

c. as the rectangle is leaving the field
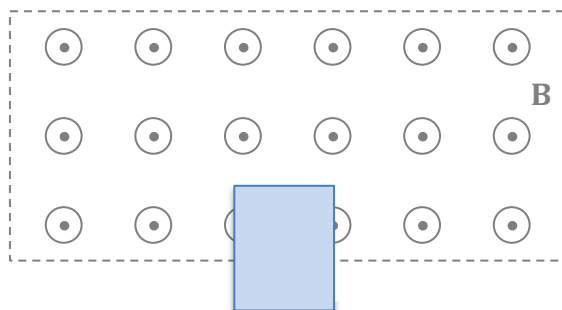
Now a rectangular conducting loop of width w and height h is dropped so that it has an initial velocity v as the lower edge of the loop enters the magnetic field at time $t = 0$. The loop has a mass m and a total resistance R.
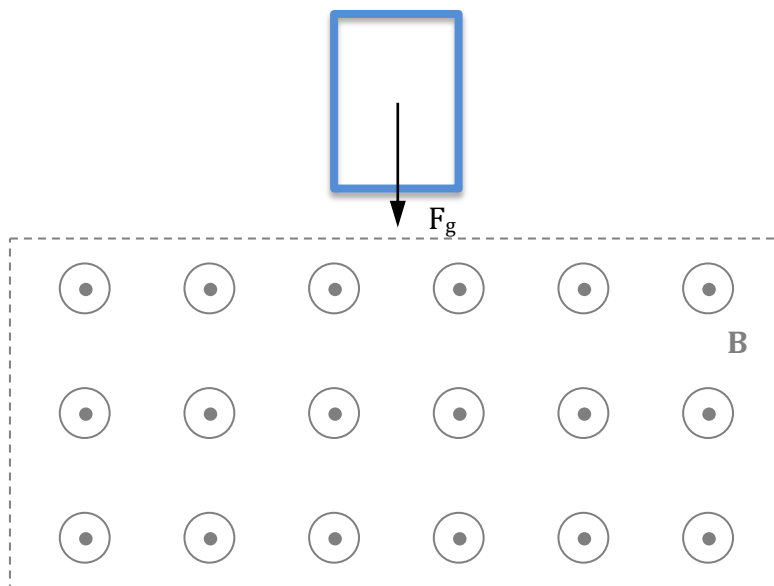
d. Develop an expression for the EMF induced in the conducting loop as the lower edge is entering the magnetic field.

e. The direction of induced current in the loop is:

_____ clockwise
_____ counterclockwise

f. Write, but do not solve, a differential equation that could be used to determine the velocity *v* as a function of time during the time period when the loop is entering the field.

g. As the loop enters the field while falling, its *speed* downwards is

_____ increasing
_____ decreasing

h. As the loop enters the field while falling, its mechanical energy

_____ increases
_____ decreases

If its mechanical energy is increasing, where does the energy come from? If its mechanical energy is decreasing, where does the energy go?

Part I. Multiple Choice

1. The correct answer is *a*. Magnetic flux in the upward direction is increasing as the magnet falls toward the loop. Lenz's law predicts that the current in the loop will have a magnet flux associated with it that *opposes* this increase in flux, ie. is in downward direction through the middle of the loop. The Right Hand Rule indicates, then, that the direction of current flow is in the clockwise direction, as viewed from the top of the loop.

   The same logic is followed to determine the current direction as the magnet falls away. Magnetic flux in the upward direction is decreasing now, inducing a current with a magnetic flux that opposes this decrease. The Right Hand Rule reveals this current to be counterclockwise in orientation.
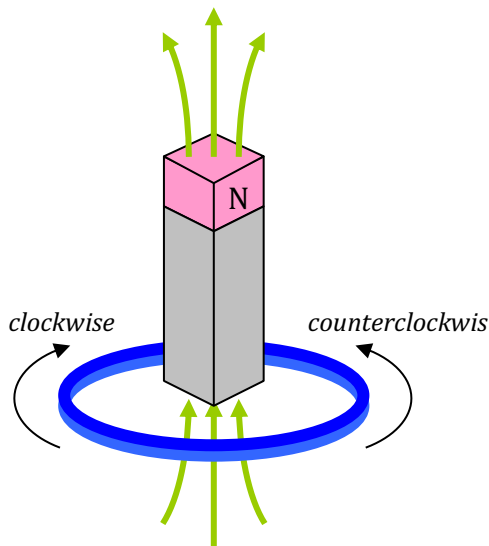
2. The correct answer is *b*. The current flowing in the circuit segment on the left produces an upward magnetic flux in the coil where there was none before. This increasing magnetic flux induces an emf in the circuit segment on the right according to Faraday's Law– $\mathcal{E} = \dfrac{-Nd\Phi_B}{dt}$ –with a current that has a flux opposing the increase in flux, according to Lenz's Law. The direction of this current flow through the resistor is to the left.

3. The correct answer is *b*. A counterclockwise current will be induced in the wire as the front edge enters the coil. This can be deduced by applying the Right Hand Rule to the free charges in the length of wire as they travel

   through the magnetic field, or by using Faraday's Law to examine the increasing magnetic flux through the area of the loop as it enters the magnetic field. This current-carrying wire, exposed to the external magnetic field *B*, experiences a magnetic force F_B in a direction opposite *v*, again determined using the Right Hand Rule, and it is this magnetic force that causes the glider to slow down.
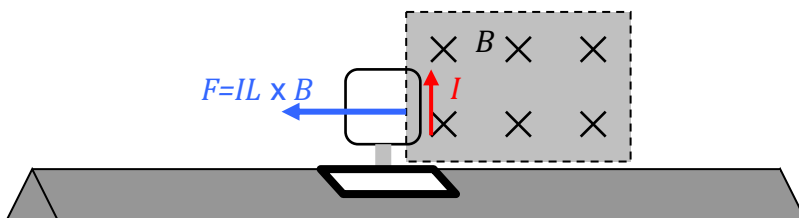
4. The correct answer is *e*. Flux through the area of the loop can be calculated as follows

$$\Phi_B = B \bullet A = BA\cos\theta$$

$$\Phi_B = (4T)\left(\pi r^2\right)$$

$$\Phi_B = (4T)\left(\pi\left(\frac{1}{2}m\right)^2\right) = \pi T \bullet m^2 = \pi W$$

   If this flux were changing, an emf would be induced in the conducting loop, causing a current to flow. Here, however, everything is constant: the magnetic field, the area of the loop exposed to the field, and the loops orientation relative to the field. Hence, there is no emf induced, and no current flow in the wire.

5. The correct answer is *a*. We can use Faraday's Law to determine the relationship between the given quantities as follows:

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

$$\varepsilon = -N\frac{d(BA\cos\theta)}{dt} = -NBA\frac{d}{dt}(\cos(\omega t))$$

$$\varepsilon = -NBA(-\omega\sin(\omega t)) = \omega NBA\sin(\omega t)$$

6. The correct answer is *c*. The inductor L in the circuit provides a "back emf" that opposes the flow of current in the circuit, according to Faraday's Law, $\varepsilon = -L\frac{dI}{dt}$. Thus, the initial flow of current in the circuit is 0.

   A long time after the switch has been connected, the current flow reaches a constant value. with no additional change in current, the magnetic field in the inductor is constant, and thus emf ε from the inductor drops to 0. Now, the current flow is simply based on the resistor, and may be calculated using Ohm's Law, $I = \frac{V}{R}$.

7. The correct answer is *d*. The inductor L has a magnetic field that begins to decrease in magnitude. The energy stored in this field, $U_L = \frac{1}{2}LI^2$, is dissipated through the right loop of the circuit as current flows through the resistor. The direction of this current flow delivered by the inductor is such that the decrease in magnetic flux is opposed–this results in a current that was in the same direction as the original current flow, ie. down through the inductor, and counterclockwise through the right loop of the circuit.

8. The correct answer is *e*. Changing flux through the area of the loop causes an emf ε to be generated according to Faraday's Law:

$$\varepsilon = -\frac{d}{dt}\Phi_B$$

$$\varepsilon = -\frac{d}{dt}(B \bullet A) = -\frac{dB}{dt} \bullet A$$

$$\varepsilon = -(0.5T)(\pi r^2) = -\left(\frac{1}{2}T\right)\left(\pi\left(\frac{1}{2}m\right)^2\right) = -\frac{\pi}{8}V$$

   We can now determine the current based on Ohm's Law, $V = IR$. In this case:

$$V = IR, \ so \ I = \frac{V}{R}$$

$$I = \frac{\pi/8\,V}{1/2\,\Omega} = \frac{\pi}{4}A$$

   The negative sign in our Faraday's Law calculation above is a reminder that the current induced will be

in a direction that is associated with a flux that *opposes* the changing flux. Here, if magnetic flux into the page is *increasing*, our current flow will have a direction consistent with magnetic field *opposing* that increase into the page. In other words, the flux will be *out of the page* (towards us), and the Right-Hand Rule indicates that the current flow will thus be in the counter-clockwise direction.

Part II. Free Response

1.

   a.   This is a motional EMF problem. As the free charges in the conducting bar move through the magnetic field, they are subjected to a magnetic force $F_B = q\mathbf{v} \times \mathbf{B}$. This force acts on the charges, causing negative charges to migrate out of the page and positive charges to migrate into the page until they achieve electrostatic equilibrium based on a corresponding electric force created by the electric field between the charges, $F_E = qE$. Under these equilibrium conditions:

$$F_B = F_E$$
$$q\mathbf{v} \times \mathbf{B} = qE$$
$$E = vB, \text{ out of the page}$$

   b.   The potential difference ΔV is determined using:
$$\Delta V = Ed$$
$$\Delta V = Bvd$$

   c.   Using Faraday's law to calculate emf based on the changing flux through the conductor:
$$\varepsilon = \frac{d\Phi_B}{dt}$$
$$\varepsilon = \frac{d}{dt} BA \cos\theta = B \frac{dA}{dt} = Bd \frac{dx}{dt} = Bvd$$

   d.   Now that a complete circuit has been formed, the potential difference ΔV causes a current to flow in the loop:
$$V = IR$$
$$Bvd = IR$$

The power dissipated in the circuit due to the resistance R may be calculated using $P = I^2R$ or any of the power equations. For example:
$$P = \frac{V^2}{R} = \frac{(Bvd)^2}{R}$$

   e.   The magnetic force $F_B = I\ell \times B$, so
$$F_B = I\ell \times B$$
$$F_B = \left(\frac{Bvd}{R}\right) d \times B$$
$$F_B = \frac{B^2 d^2 v}{R}, \text{ to the left}$$

f. A differential equation that involves **v** almost always means using $F_{net} = ma$:

$$F_{net} = m\frac{dv}{dt}$$

$$\frac{B^2 d^2 v}{R} = m\frac{dv}{dt}$$

g. Rearrange the equation so that dv and dt are on opposite sides, and integrate. As the speed goes from v to 0, time goes from 0 to t:

$$F_{net} = m\frac{dv}{dt}$$

$$\frac{-B^2 d^2 v}{R} = m\frac{dv}{dt}$$
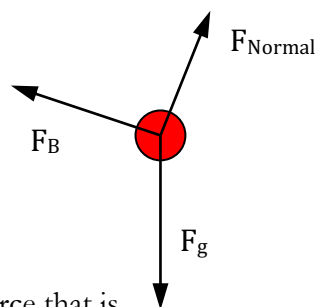
$$\frac{-B^2 d^2}{mR}dt = \frac{1}{v}dv$$

$$\int_{v}^{v_{final}} \frac{1}{v}dv = \int_{0}^{t} \frac{-B^2 d^2}{mR}dt$$

$$\ln v \Big|_{v}^{v_{final}} = \frac{-B^2 d^2}{mR}t$$

$$\frac{v_{final}}{v} = e^{\frac{-B^2 d^2}{mR}t}$$

$$v_{final} = v e^{\frac{-B^2 d^2}{mR}t}$$

h. Note that there should be no components drawn into the free-body diagram. Components are fine on working diagrams, but when drawing formal diagrams "for points," leave components off.

i. For the bar to slide at constant speed, the component of the gravity force that is parallel to the rails must equal the magnetic force opposing the bars sliding:

$$F_{net} = ma = 0$$

$$F_{//} - F_B = 0$$

$$mg\sin\theta = \frac{B^2 d^2 v}{R}$$

$$v = \frac{mgR\sin\theta}{B^2 d^2}$$

2.

    a.   The current running through the solenoid is determined using Ohm's Law:

$$V = IR$$

$$I = \frac{V}{R} = \frac{12.0V}{10.0\Omega} = 1.20A$$

    b.   The magnetic field of a solenoid is $B = \mu n I$, where n represents the number of coils per unit length of the solenoid. This equation is sometimes written as $B = \mu \frac{N}{L} I$, where N is the number of coils and L is the length of the solenoid.

$$B = \mu \frac{N}{L} I$$

$$B = (4\pi e - 7)\left(\frac{20 turns}{0.10m}\right)(1.20A) = 3.02e - 4T$$

    c.   By the Right-Hand Rule, the direction of the magnetic field inside the solenoid is to the *left*.

    d.   The inductance of a solenoid can be determined using $\mathcal{E} = -N\frac{d\Phi_B}{dt}$, Faraday's Law of Induction, and the definition of inductance, $\mathcal{E} = -L\frac{dI}{dt}$. Combining these two equations produces an equation that describes the inductance L of a solenoid:

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} \ and \ \mathcal{E} = -L\frac{dI}{dt}$$

$$-N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$

$$N\Phi_B = LI$$

$$L = \frac{N}{I}(BA)$$

$$B_{solenoid} = \mu_0 \frac{N}{L} I$$

$$L = \frac{N}{I}\left(\left(\mu\frac{N}{L}I\right)A\right) = \mu_0 N^2 \frac{A}{L}$$

$$L = (4\pi e - 7)(20 turns)^2 \left(\frac{\pi(0.01m)^2}{0.10m}\right) = 1.58e - 6H$$

e.   The potential energy in the inductor is found using $U = \frac{1}{2}LI^2$:

$$U = \frac{1}{2}LI^2$$

$$U = \frac{1}{2}(1.58e-6H)(1.2A)^2 = 1.14e-6J$$

f.   When the switch is first closed to position A, as current tries to flow in a clockwise direction there is a strong "back EMF" from the increasing magnetic flux in the inducting coil. This effect is strongest at the beginning, when the increase in current is the greatest, but the back EMF diminishes with time. Thus, the graph of current flow through the resistor is
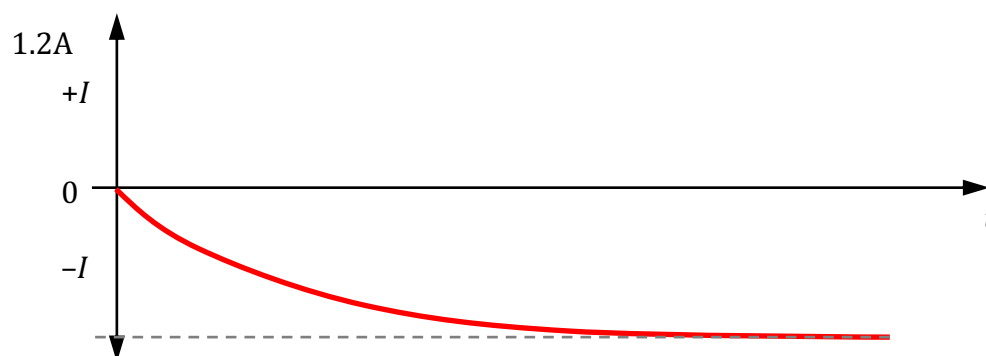
g.   When the switch is moved to position B the battery is removed from the circuit. Now the inductor L, the resistor R, and the capacitor C are all in series. The loss of current from the battery means that the energy stored in the inductor begins to be released, as the changing magnetic flux produces an EMF that opposes the decreasing field. This causes an induced EMF in the same direction that the current was flowing before, ie. to the left through the resistor.

This current causes the capacitor to charge up, with positive charges deposited on the left plate of the capacitor. Energy lost from the inductor is stored in the capacitor, with some dissipated by the resistor as well via heat.

Once the capacitor is completely charged and the inductor has no more energy, the capacitor is able to release its energy via a current that flows now clockwise in the circuit, to the right through the resistor, which again dissipates some of that energy as heat. The inductor opposes this changing current flow, however... and the cycle continues back and forth, with energy transferring from the inductor to the capacitor and back again, with some energy lost to heat in the resistor during each cycle. The graph, then, is
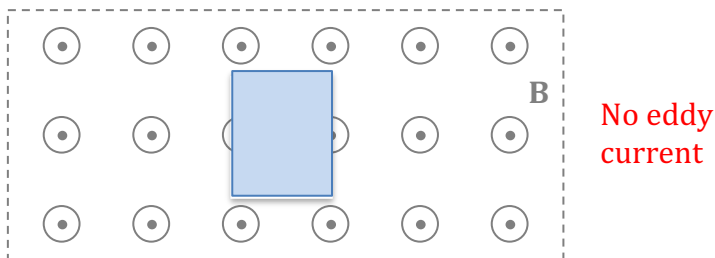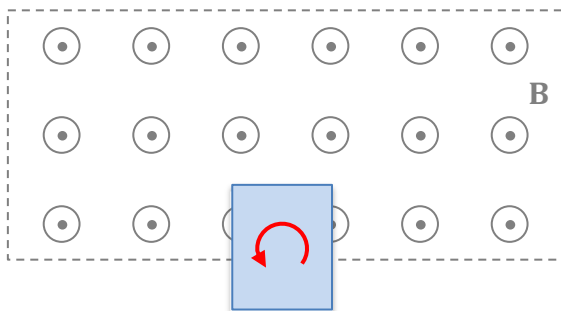
3.

a.

b.

No eddy current
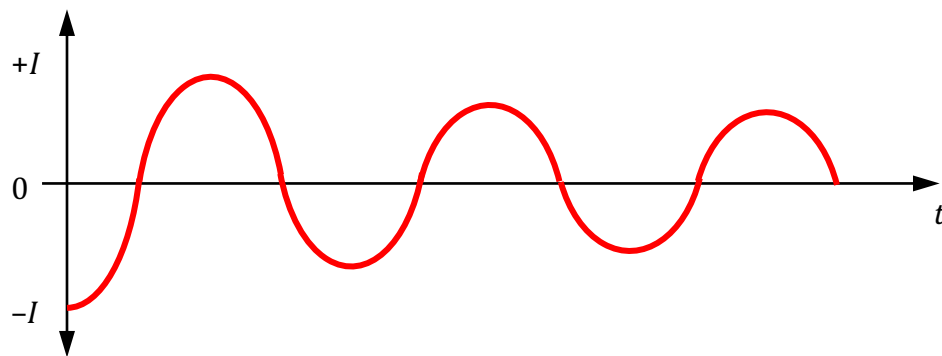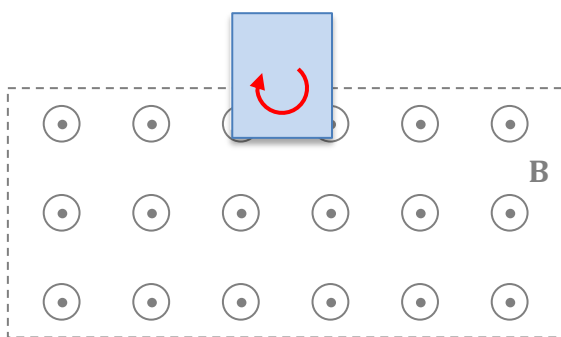
c.

d. As the loop enters the field, there is an EMF generated due to the changing flux through the center of the loop. Faraday's Law states that $\mathcal{E} = -N\dfrac{d\Phi_B}{dt}$. Although the magnetic field itself isn't changing, the flux through the conducting loop is changing, based on the increasing area inside the loop exposed to the magnetic field.

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -\frac{BdA}{dt}, \text{ where } \frac{dA}{dt} \text{ is the strip of area in the loop exposed over time}$$

$$\frac{dA}{dt} = \frac{w \bullet dy}{dt}$$

$$\mathcal{E} = Bwv$$

e. Using Lenz's law, we can see that the direction of current flow will have a magnetic field that opposes the increase in flux out of the page. Thus, it will have a magnetic field *into* the page, which implies a *clockwise* direction for current *I*.

f. The differential equation we need is based on Newton's Law, which takes into account the two forces acting on the conducting loop.

$$F_{net} = ma$$

$$F_g - F_B = m\frac{dv}{dt}$$

$$mg - IwB = m\frac{dv}{dt}$$

Using development above to get *I*:

$$I = \frac{V}{R} = \frac{Bwv}{R}$$

$$mg - \left(\frac{Bwv}{R}\right)wB = m\frac{dv}{dt}$$

$$mg - \frac{B^2w^2v}{R} = m\frac{dv}{dt}$$

g. As the loop is entering the field, its speed is *increasing*. Although there is a magnetic force upwards on the loop, the force of gravity is greater, resulting in a net force downward, and therefore a net acceleration downward, with a resulting increase in speed.

h. As the loop is entering the field, its mechanical energy is *decreasing*. We know that gravitational potential energy U_g is decreasing as it falls, but kinetic energy K is increasing by some amount. So how do we know total mechanical energy is decreasing? One explanation is that there's a magnetic force upwards that reduces the increase in Kinetic energy, so mechanical must be less. Also, we know that current is flowing in the loop with a corresponding dissipation of energy as heat in the conductor. This heat comes at the expense of the system's mechanical energy.