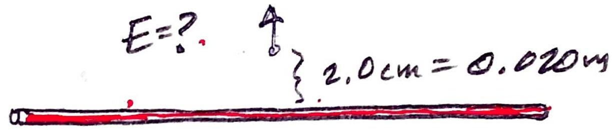
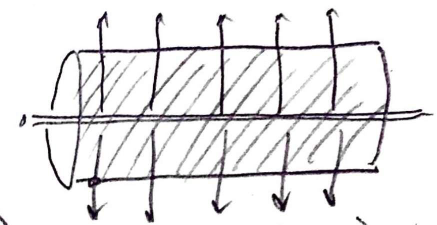
6.43



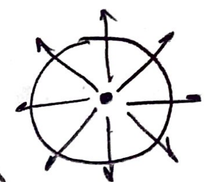
long thin wire, $\lambda = 50e{-6}\ C/m$

Find E at 2.0cm from wire.

Gauss's Law analysis:



Side view



End view

$$\int E \cdot dA = \frac{q_{in}}{\varepsilon_0}$$

$$E \oint dA = q_{in}(4\pi k)$$

$$E(2\cancel{\pi} r l) = 4\cancel{\pi} k q$$

$$E = \frac{2kq}{r\cancel{l}} = \frac{2k\lambda}{r}$$

$$E = \frac{2(9e9)(50e{-6}\ C/m)}{0.02m}$$

$$= \boxed{4.5\,e7\ N/C}$$

6.44

$Q = -30\,\mu C$ distributed uniformly

radius = 10 cm = 0.010 m.

Find E at various radii.

a) <u>0.020 m</u> from center of sphere = inside sphere.

$\oint E \cdot dA = \frac{q_{in}}{\varepsilon_0}$

$E(4\pi r^2) = q_{in} 4\pi k$

No! Negative charge so E is toward center, or $-\hat{r}$.

ρ constant $= \frac{Q}{V} = \frac{q_{in}}{V_{in}}$

$\frac{Q}{\cancel{4/3\pi} R^3} = \frac{q_{in}}{\cancel{4/3\pi} r^3} \qquad q_{in} = Q\frac{r^3}{R^3}$

$E(\cancel{4\pi} \cancel{r^2}) = \left(Q\frac{r^3}{R^3}\right)\cancel{4\pi}k = \frac{kQr}{R^3}$

$E = \frac{(9e9)(30e\text{-}6)(0.02m)}{(0.10m)^3} = \boxed{5.4e6\ N/C} \boxed{-\hat{r}}$

b) Same strategy for $r = 5.0$cm

$E = \frac{(9e9)(30e\text{-}6)(0.05)}{(0.10)^3} = \boxed{1.35e7\ N/C} \boxed{-\hat{r}}$

c) For $r = 20.0$ cm, now we're outside the sphere.

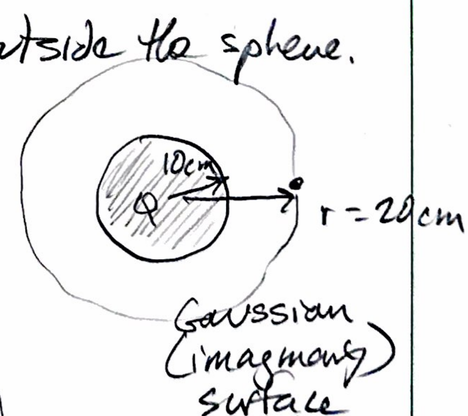
$\oint E \cdot dA = \frac{q_{in}}{\varepsilon_0}$

$E(\cancel{4\pi} r^2) = q_{in} \cancel{4\pi} k$

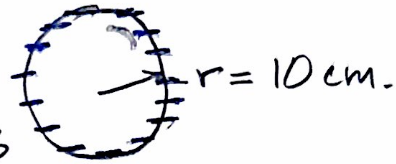
$E = \frac{k q_{in}}{r^2}$

$= \frac{(9e9)(30e\text{-}6)}{(0.20)^2}$

$= \boxed{6.75e{+}6\ N/C} \boxed{-\hat{r}}$

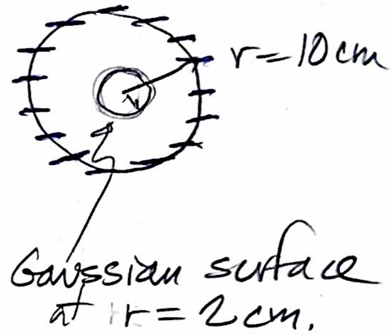


6.45 $Q = -30\mu C$



For conductors, charges move to reduce their proximity – they repel each other, increasing the distance between themselves. Thus, all the electrons are on the surface of the conducting sphere.

a) From analysis in 6.44,



$$E = \frac{k q_{in} r}{R^3}$$

$q_{in} = 0!$

$$= \frac{(9e9)(0)(0.02)}{(0.10)^3}$$

No internal charge so no E field inside the conductor.

b) For $r = 0.05m$, same reasoning. $E = 0$.

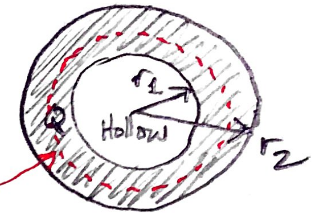
c) For points external to sphere, $E = \frac{kQ}{r^2}$

$$= \boxed{6.75e{+}6 \text{ N/C } (-\hat{r})}$$

(same as before)

6.46 Q distributed uniformly throughout a spherical shell with inner & outer radii r_1 & r_2. Identify E field at various locations.

a) For $r < r_1$, a Gaussian sphere in the hollow cavity encloses no charge. $q_{in} = 0$, so $E = 0$.



b) For $r_1 \le r \le r_2$, only a subset of Q is internal to that Gaussian surface.

$$\oint E \cdot dA = \frac{q_{in}}{\varepsilon_0}$$

$$E(\cancel{4\pi} r^2) = q_{in}(\cancel{4\pi} k)$$

$$\boxed{E = \frac{kQ}{r^2}\frac{(r^3 - r_1^3)}{(r_2^3 - r_1^3)}}$$

sub in $\frac{1}{4\pi\varepsilon_0}$ if you like!

; $\frac{q_{in}}{V_{in}} = \frac{Q}{V}$ These volumes are based on subset of $\frac{4}{3}\pi r_2^3$

$V = \frac{4}{3}\pi r_2^3 - \frac{4}{3}\pi r_1^3$, &

$V_{in} = \frac{4}{3}\pi r^3 - \frac{4}{3}\pi r_1^3$

$$q_{in} = Q\frac{\cancel{\frac{4}{3}\pi}(r^3 - r_1^3)}{\cancel{\frac{4}{3}\pi}(r_2^3 - r_1^3)}$$

c) For $r > r_2$

$$E = \frac{kq_{in}}{r^2} = \boxed{\frac{kQ}{r^2}}$$