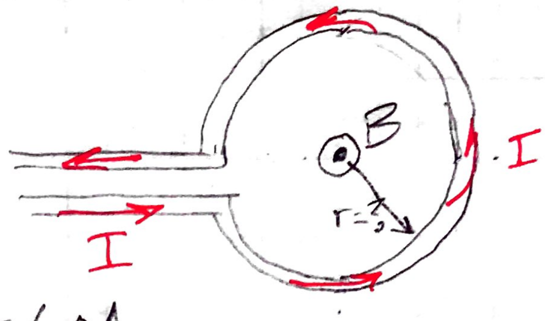
12.35




$I = 6.0 A$
$B = 2.0e-4 T$
$r = ?$

Use Biot-Savart to develop relationship:

$$B = \int \frac{kI\, d\vec{s} \times \hat{r}}{r^2}$$
$$= \frac{kI}{r^2} \int d\vec{s} \times \hat{r}$$


$$d\vec{s} \times \hat{r} = ds\,(\sin 90) = ds$$

$$\frac{kI}{r^2} \int ds$$ Sum of all ds around the circle?

$$\int ds = \int_0^{2\pi} r\, d\theta = \underline{\underline{2\pi r}}$$ circumference

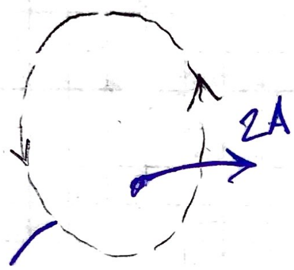
$$B = \frac{kI}{r^2}(2\pi r)$$
$$= \frac{\mu_0 I\, 2\cancel{\pi}}{4\cancel{\pi}\, r} = \frac{\mu_0 I}{2r} = B$$ (for current-carrying loop)

$$r = \frac{\mu_0 I}{2B} = \frac{(4\pi e-7)(6)}{2(2e-4T)}$$
$$= \boxed{0.0188\ m}$$

12.42 Ampere's Law: $\oint B \cdot ds = \mu_0 I$
$= \oint B \cdot dl$ (in book)

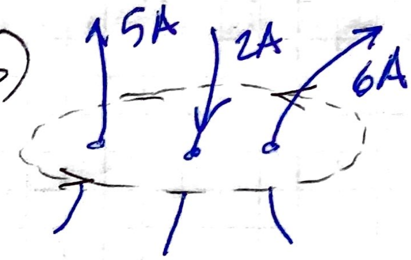
To evaluate $\oint B \cdot ds$, we just need to get $\mu_0 I$ for each situation, where $I$ = current through the "Amperian path" (closed loop).

a)



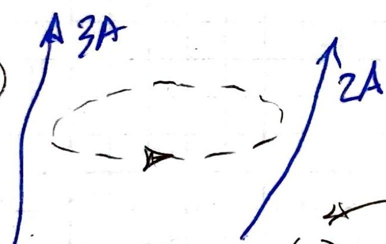

$\mu_0 I = \mu_0 2$
$= \boxed{2.51e{-6}\ T \cdot m}$

b)




$\mu_0 I = \mu_0 (5 - 2 + 6)$
$= (4\pi e{-7})(9)$
$= \boxed{1.13e{-5}\ T \cdot m}$

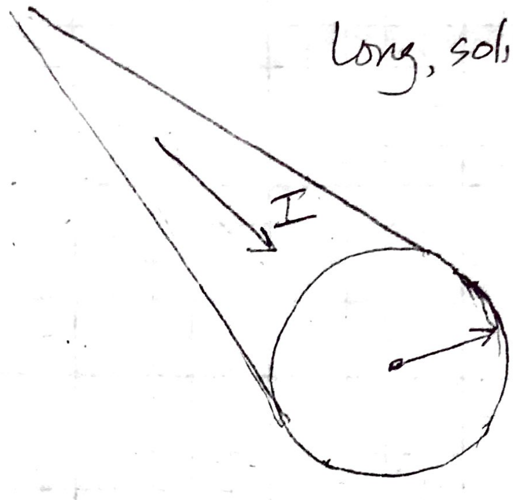
c)




No current running through Amperian path.

$\mu_0 I = \mu_0 (0)$
$= \boxed{0}$

12.47




Long, solid, cylindrical conductor

$r = 3.0e{-2}\,m$
$I = 50A$

Plot magnetic field vs a function of radius.

Outside the conductor:


$$\oint B \cdot ds = \mu_0 I$$
$$B \oint ds = \mu_0 I$$
$$B(2\pi r) = (4\pi e{-7})(60)$$
$$B = \frac{(4\cancel{\pi} e{-7})\,60}{2\cancel{\pi} r} = \frac{1.2e{-5}}{r}$$

Inside the conductor:


$$\oint B \cdot ds = \mu_0 I_{in}$$

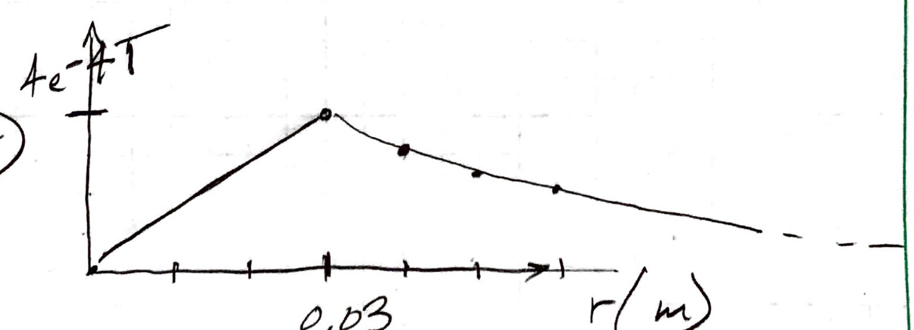
$$\frac{I_{in}}{A_{in}} = \frac{I}{A}$$

$$I_{in} = \frac{I\pi r^2}{\pi R^2}$$

Combining: $B(2\pi r) = \frac{\mu_0 I \cancel{\pi} r^2}{\cancel{\pi} R^2}$

$$B = \frac{\mu_0 (60)(r)}{2\pi (3e{-2})^2} = \boxed{0.0133\, r}$$

Plotting




For future reference:
"sketch" = no data points needed,
but identify significant locations.
"plot" = calculate representative points

12.48


$r_1$ inside

Find a) $r \leq r_1$

Using analysis in 12.47 for inside a conductor.

$$\boxed{B = \frac{\mu I r}{2\pi r_1^2}}$$

b) $r_2 \geq r \geq r_1$ (r between two conductors)

Using analysis in 12.47 for r outside conductor:

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

c) $r_3 \geq r \geq r_2$ (r in outer conductor)

Currents are in opposite directions, so we'll need to calculate the net current.

Inside is full I. In outer cylindrical shell:

$$\frac{I_{in}}{A_{in}} = \frac{I}{A} \rightarrow I_{in} = I\left(\frac{A_{in}}{A}\right)$$

$$I_{in} = I\left(\frac{\pi r^2 - \pi r_2^2}{\pi r_3^2 - \pi r_2^2}\right)$$

Only the shaded part is internal to Amperian path.

$$\oint B \cdot ds = \mu_0 I_{net}$$

$$B(2\pi r) = \mu_0\left(I - I\left(\frac{\pi r^2 - \pi r_2^2}{\pi r_3^2 - \pi r_2^2}\right)\right)$$

$$B = \boxed{\frac{\mu_0 I}{2\pi r}\left(1 - \frac{r^2 - r_2^2}{r_3^2 - r_2^2}\right)}$$ ouch!

d) $r \geq r_3$ (outside both conductors)

Here, the net current is $I_{inner} + I_{outer} = \boxed{0}$ because currents are going in opposite directions.

Therefore $B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 (0)}{2\pi r} = \boxed{0}$ !