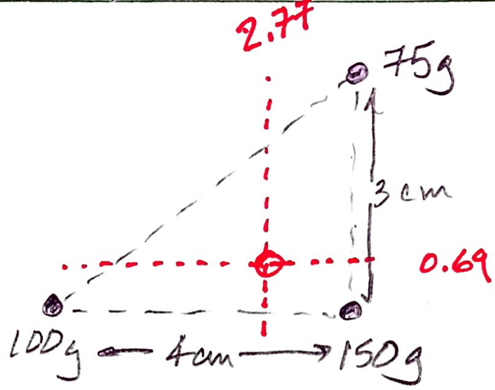
# 9.63




Find cm ("center of mass") of the system.

Analyze x & y separately:

$$x_{cm} = \frac{1}{M}\sum m_i x_i$$

$$= \frac{1}{(.10+.150+.075)}\left((.1)(0)+(.150)(0.04)+(0.075)(.04)\right)$$

$$= \frac{1}{0.325}(0+0.0060+0.0030)$$

$$= 0.0277\,m = \boxed{2.77\,cm}$$

$$y_{cm} = \frac{1}{M}\sum m_i y_i$$

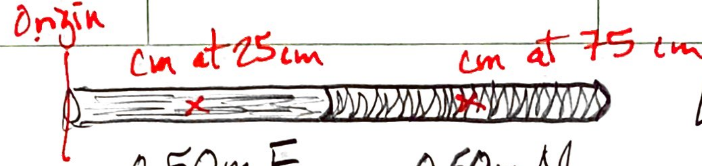
$$= \frac{1}{0.325}(0.100)(0)+(0.150)(0)+(0.075)(0.03)$$

$$= 0.0069\,m = \boxed{0.69\,cm}$$

$$r_{cm} = \boxed{(2.77\,\hat{i} + 0.69\,\hat{j})\,cm}$$

relative to 100g mass at <u>origin</u>.

9.68




Long thin rod

0,50m Fe $\rho = 8\,g/cm^3$

0.50m Al $\rho = 2.7g/cm^3$

Where is center of mass? Closer to the iron side – it's more dense, but where? And how to resolve density in $cm^3$ unit, but we only have cm in rod?

Each half has a uniform (unspecified) cross-sectional area $\underline{A}$: $V = (50\,cm \text{ length}) \times A$

$$\rho = \frac{m}{V}, \text{ so } mass = \rho V$$

$$m = \rho(50cm)A$$

Fe

$$m_F = \left(8\frac{g}{cm^3}\right)(50\,cm)A$$

Al

$$m_A = \left(2.7\frac{g}{cm^3}\right)(50\,cm)A$$

$$\text{Total mass} = m_F + m_A$$

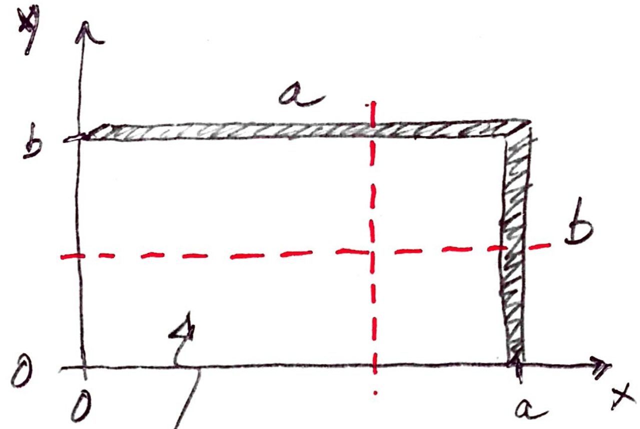
$$x_{cm} = \frac{1}{M}(m_F x_F + m_A x_A)$$

$$= \frac{1}{400\cancel{A} + 135\cancel{A}}\left((400\cancel{A})(25\,cm) + (135\cancel{A})(75\,cm)\right)$$

$$= \boxed{37.6\,cm}$$ from the left side of the rod.

There are other ways to solve this, including putting the origin at the center of mass for one of the sides...

9.70




Block has varying density, with $\rho(x,y) = \rho_0 x$, where $\rho_0$ is a constant.

Center of mass in the y-direction is easy: density in y-direction doesn't vary, so by symmetry,

$y_{cm} = \frac{b}{2}$.

What about $x_{cm}$?

$$x_{cm} = \frac{1}{M}\int_0^a x \, dm$$

$\rho = \frac{M}{V}$, or

$dm = \rho \, dV$

& $dV = A \, dx$

cross-sectional area

$dm = \rho \, dV$

$dm = \underbrace{\rho_0 x A \, dx}$

$$x_{cm} = \frac{1}{M}\int_0^a \rho_0 x^2 A \, dx$$

$$= \frac{1}{M}\rho_0 A \int_0^a x^2 dx = \frac{1}{M}\rho A \frac{1}{3}a^3$$

But what is $M$? $M = \int dm = \int \rho \, dV = \int_0^a \rho_0 x A \, dx$

$M = \rho_0 A \frac{1}{2} a^2$

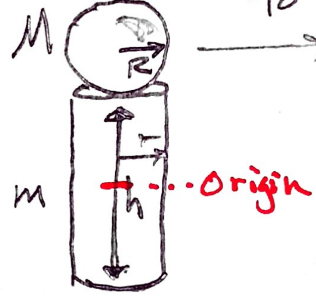
Now sub back in!

$$x_{cm} = \frac{1}{\cancel{\rho A}\frac{1}{2}\cancel{a^2}}\left(\cancel{\rho A}\,\frac{1}{3}a^{\cancel{3}}\right)$$

$$= \boxed{\frac{2}{3}a}$$

9.76 Center of mass of this system, expressed relative to origin @ center of cylinder.

a)




→ • M at $\frac{h}{2}+R$

• m at O

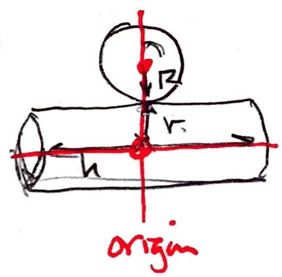
$$y_{cm} = \frac{1}{M}(m_1 y_1 + m_2 y_2)$$

$$= \frac{1}{m+M}\left(m\cdot 0 + M\left(\frac{h}{2}+R\right)\right)$$

$$= \boxed{\frac{M\left(\frac{h}{2}+R\right)}{m+M}}$$

$$\boxed{x_{cm} = 0}$$

b)




$\boxed{x_{cm} = 0}$ again

$$y_{cm} = \frac{1}{M}(m_1 y_1 + m_2 y_2)$$

$$= \frac{1}{m+M}\left(m\cdot 0 + M(2R)\right)$$

$$y_{cm} = \boxed{\frac{(R+r)M}{m+M}}$$