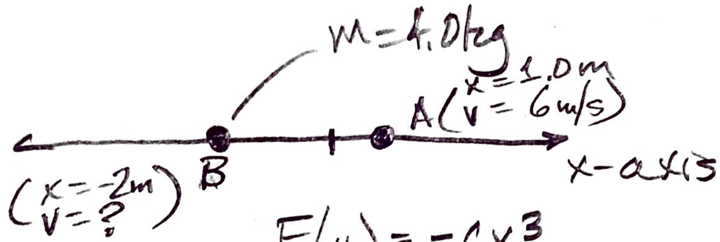
8.47



$F(x) = -cx^3$

$c = 8.0\,N/m^3$

Conservation of Mechanical Energy:

$$U_A + K_A = U_B + K_B$$

Assuming $F(x) = -cx^3$ is a conservative force;

$$U = -\int F \cdot dx$$

$$\underline{U = \frac{c}{4}x^4}$$

So...

$$\frac{c}{4}x_A^4 + \frac{1}{2}mv_A^2 = \frac{c}{4}x_B^4 + \frac{1}{2}mV_B^2$$

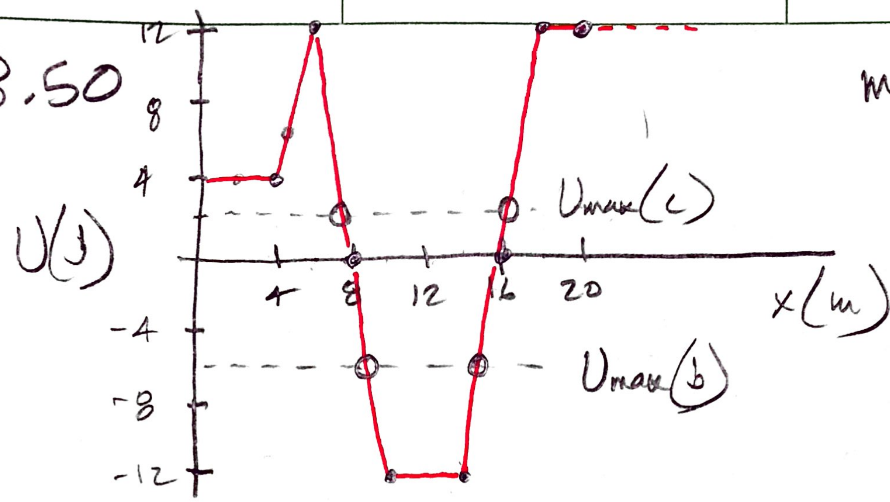
$$\cancel{2}(1m)^4 + \frac{1}{\cancel{2}}(4)(6)^2 - \cancel{2}(-2)^4 = \cancel{2}\,V_B^2$$

$$1 + 36 - 16 = V_B^2$$

$$V_B = \sqrt{21} = \pm \boxed{4.58\,m/s}$$

8.50

$m = 0.50\,kg$



a) Force $= -\frac{dU}{dx}$, so at $x = 2$, $F = \text{slope} = \underline{0}$

at $x = 5$, $F = -\left(\frac{12-4}{5-4}\right) = \underline{-8.0\,N}$

at $x = 8$, $F = -\left(\frac{0-12}{8-6}\right) = \underline{+6.0\,N}$

at $x = 12$, $F = \underline{0}$

b) Total $E_{mechanical}$ $(=K+U) = -6.0\,J$ – find x_{max} & x_{min}.

$E_{total} = K + U$, so

$U = E_{total} - K$

U is maximum $(= E_{total})$ when K is 0, & all energy is U, $= -6\,J$,

U_{max} is at $-6\,J$, & $\boxed{x_{max} = 15\,m, \; x_{min} = 9\,m}$ } looking at graph

c) If $E_{total} = 2\,J$... $\boxed{16.5\,m}$ & $\boxed{7.5\,m}$

d) If $E = 16\,J = K + U$

$K = 16 - U$

$\frac{1}{2}mv^2 = 16 - U$

$v = \sqrt{\frac{16-U}{0.25}}$

a) At $x=2$, $U=4$, so $v = \sqrt{\frac{16-4}{.25}} = \boxed{6.93\,m/s}$

b) At $x=5$, $U=6$, so $v = \sqrt{\frac{16-6}{0.25}} = \boxed{6.32\,m/s}$

c) At $x=8$, $U=0$, so $v = \sqrt{\frac{16-0}{0.25}} = \boxed{8.0\,m/s}$

d) At $x=12$, $U=-12$, so $v = \sqrt{\frac{16-{-12}}{0.25}} = \boxed{10.6\,m/s}$

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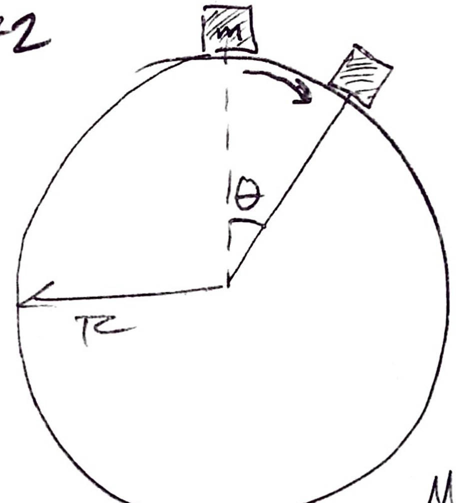
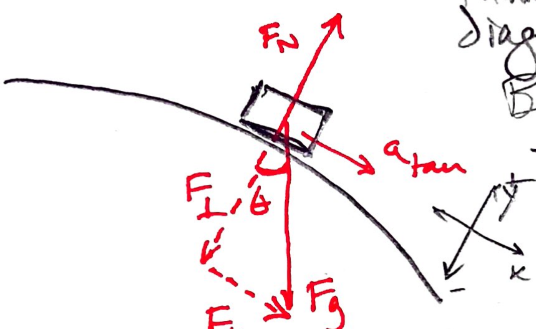
8.72



m slides down "frictionlessly"
Prove that m leaves sphere at $\theta = \cos^{-1}\left(\frac{2}{3}\right)$. (Huh??!)

Analysis. How do we know when the body leaves the sphere?
When there is no more F_{normal}.

Mass m is moving in a circle & I'm thinking about forces, so... free-body diagram!



Because directions at acceleration are tilted, I'm tilting my axes.

$$\Sigma F_y = ma_y = \frac{mv^2}{r}$$

$$F_N - F_\perp = -\frac{mv^2}{r}$$

When $F_N = 0$, $\cancel{m}g\cos\theta = \frac{\cancel{m}v^2}{r}$

$$\boxed{\cos\theta = \frac{v^2}{rg}}$$

Okay, but... I don't know v at that point. How do I get v?
Energy! $U + K = U + K$

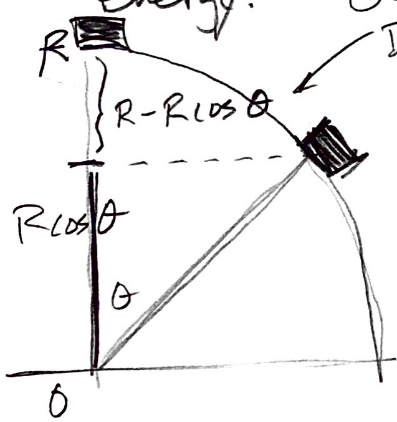


Diagram shows that block has dropped a distance R - R cos θ.

So...

$$U + K = U + K$$

$$\cancel{m}g(R - R\cos\theta) + 0 = 0 + \frac{1}{2}\cancel{m}v^2$$

$$v^2 = 2(R - R\cos\theta)$$

$$v^2 = 2Rg(1 - \cos\theta)$$

Plugging into eqn above.

$$\cos\theta = \frac{v^2}{Rg} = \frac{2\cancel{Rg} - 2\cancel{Rg}\cos\theta}{\cancel{Rg}}$$

$$\cos\theta = 2 - 2\cos\theta$$

$$3\cos\theta = 2$$

$$\cos\theta = \frac{2}{3}$$

$$\theta = \boxed{\cos^{-1}\left(\frac{2}{3}\right)}\ !$$

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