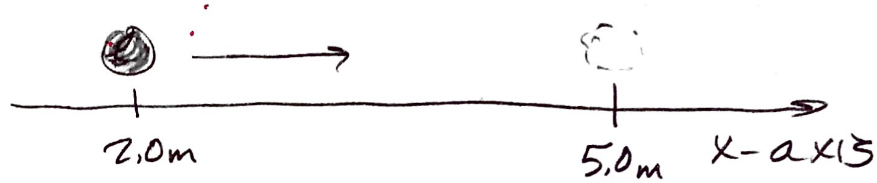
8.24



$F(x) = \frac{3}{x}$ N, so $F_{mystery}$ varies w/ position.

a) Work $= \int F\,dx = \int_{x_i}^{x_f} \frac{3}{x}\,dx = 3\ln x \Big|_{x_i}^{x_f}$

$= 3(\ln 5 - \ln 2) = \boxed{2.75\text{ J}}$

b) Don't do.

8.26

$$F = -\frac{dU}{dx} = -\frac{d}{dx}\left(-\frac{a}{x} + \frac{b}{x^2}\right)$$

$$= -\frac{d}{dx}\left(\frac{-a}{x}\right) + -\frac{d}{dx}\left(\frac{b}{x^2}\right)$$

$$= -a\left(-\frac{d}{dx}x^{-1}\right) + b\left(-\frac{d}{dx}x^{-2}\right)$$

$$= -a\left(-\left(-1x^{-2}\right)\right) + b\left(-\left(-2x^{-3}\right)\right)$$

$$= \boxed{\frac{-a}{x^2} + \frac{2b}{x^3}}$$

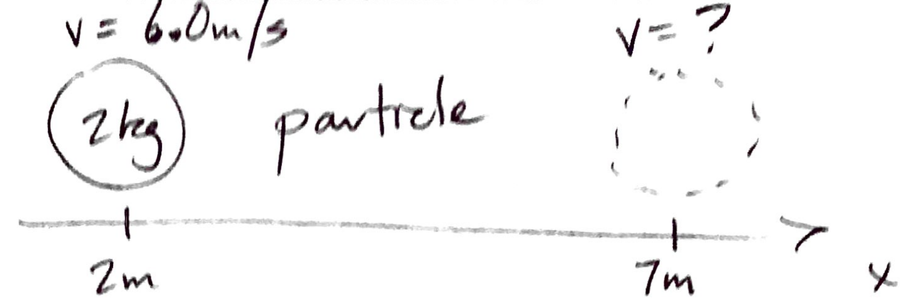
or

$$F = -\frac{dU}{dx} = \frac{d}{dx}\left(\frac{a}{x} - \frac{b}{x^2}\right)$$

$$= -ax^{-2} + 2bx^{-3}$$

$$= \boxed{\frac{-a}{x^2} + \frac{2b}{x^3}}$$

8,28



$F = \frac{3}{\sqrt{x}}$ N (??! what madness is this?)

The particle is moving under the influence of the force. We can describe this as <u>either</u>

2 ways of saying the same thing

[Work done by force $= \Delta K = K_f - K_i$
where $W = \int F \cdot dx = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

or

[$U_i + K_i = U_f + K_f$, where $\Delta U = -\int F \cdot dx$

Let's use potential energy U.

$$\Delta U = -\int F \cdot dx = -\int_{x_i}^{x_f} \frac{3}{\sqrt{x}}\, dx$$

$$U_f - U_i = -\int 3x^{-\frac{1}{2}}\, dx = -6x^{\frac{1}{2}}\Big|_{x_i}^{x_f} = -6x_f^{1/2} - -6x_i^{\frac{1}{2}}$$

So

$$U_i + K_i = U_f + K_f \quad \text{(conservation of energy)}$$

$$-6x_i^{\frac{1}{2}} + \frac{1}{2}mv_i^2 = -6x_f^{\frac{1}{2}} + \frac{1}{2}mv_f^2$$

$$-6(2)^{\frac{1}{2}} + \frac{1}{2}(2)(6\,m/s)^2 = -6(7)^{\frac{1}{2}} + \frac{1}{2}(2)v_f^2$$

$$-8{,}485 + 36 = -15{,}875 + v_f^2$$

$$v_f = \boxed{6{,}59\ m/s}$$