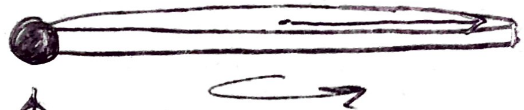
11.52

disk $m = 2.0\,kg$
$r = 0.60\,m$



attached mass $= 0.05\,kg$

$\omega = 2.0\ rev/s = \frac{2\pi\ rad}{1\ rev} = \underline{4\pi\ rad/s}$

Mass comes off. What happens to disk's rotation?

No external torque, so $\tau_{ext} = \frac{dL}{dt}$

$0 = \frac{dL}{dt}$

$L_i = L_f$

$$\underbrace{I_i \omega_i + r_i \times m v_i}_{\Sigma L_i} = \underbrace{I_f \omega_f}_{L_f}$$

$$\frac{1}{2} M \cancel{R^2} \omega_i + \cancel{R} M \cancel{R} \omega_i = \frac{1}{2} M \cancel{R^2} \omega_f$$

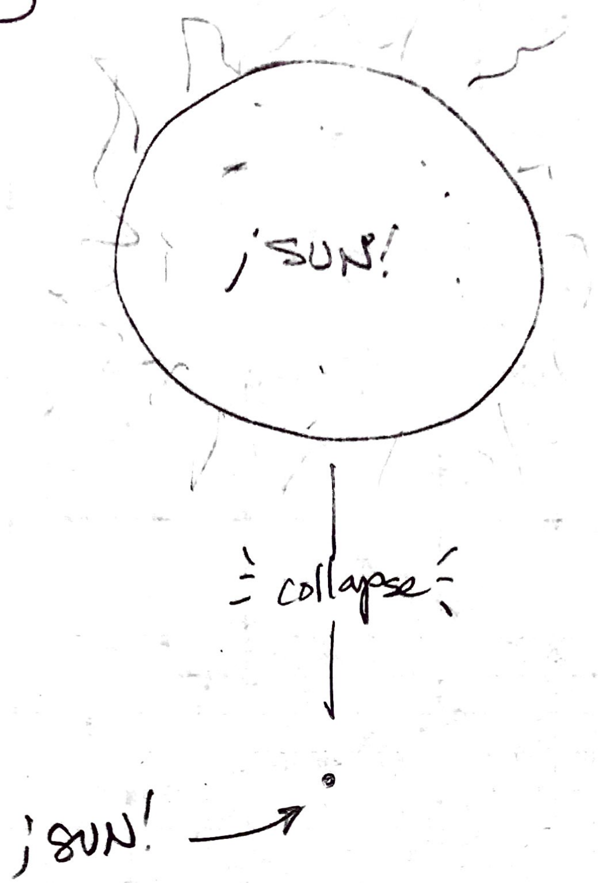
$$\cancel{\left(\tfrac{1}{2}\right)}(2)\left(4\pi \tfrac{rad}{s}\right) + (0.05)\left(4\pi \tfrac{rad}{s}\right) = \cancel{\left(\tfrac{1}{2}\right)}(2)\, \omega_f$$

$$(1.05)(4\pi) = \omega_f$$

$$\boxed{13.2\ rad/s} = \omega_f$$

It sped up!

11.53



$m = 2.0e30\ kg$
$r = 7.0e5\ km = \underline{7.0e8\ m}$
$\omega = \frac{1\ rev}{28\ days} = 2.597e\text{-}6\ rad/s$

$r_F = 3.5e3\ km = \underline{3.5e6\ m}$
$\omega_F = ?$

$L_i = L_F$ (conservation of angular mom.)

For a sphere, $L = I\omega$ & $I = \frac{2}{5}MR^2$

$I_i \omega_i = I_F \omega_F$

$(\cancel{\tfrac{2}{5}}\cancel{M}R_i^2)\omega_i = (\cancel{\tfrac{2}{5}}\cancel{M}R_F^2)\omega_F$

$\omega_F = \frac{R_i^2}{R_F^2}\omega_i$

$= \frac{(7e8)^2}{(3.5e6)^2} \frac{1\ rev}{28\ days}$

$= \boxed{1.43e3\ rev/day}$

T period = time for 1 rotation

$\frac{1\ day}{1.43e3\ rev} \times \frac{24\ hr}{1\ day} \times \frac{3600\ s}{1\ hr} = \boxed{60.5\ s/rev!}$

11.56

perigee = 500 km above surface

apogee = 2500 km above surface
$v = 6260\, m/s$

Earth's surface is at 6.37e6 m above center of gravity, so

apogee = 2.5e6 m + 6.37e6 = 8.87e6 m

perigee = 5.0e5 m + 6.37e6 = 6.87e6 m

$$L_i = L_f$$

$$r_a \times \cancel{m} V_a = r_p \times \cancel{m} V_p$$

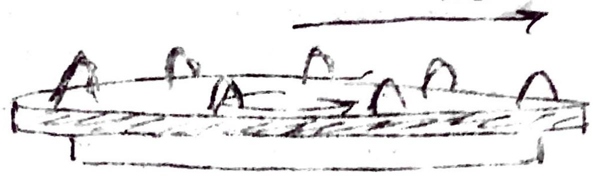
$$V_p = \frac{r_a}{r_p} V_a$$

$$= \frac{8.87e6\, m}{6.87e6\, m} 6260\, m/s$$

$$= \boxed{8.08e^3\, m/s}$$

11.63

$m = 100\,kg$
$r = 1.60\,m$

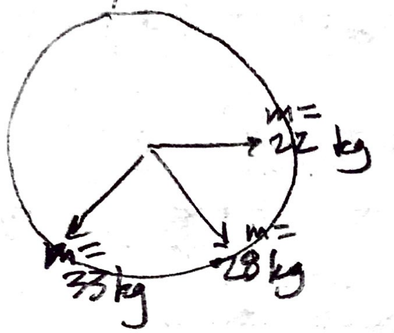


Merry-go-round (disk with $I = \frac{1}{2}MR^2$)

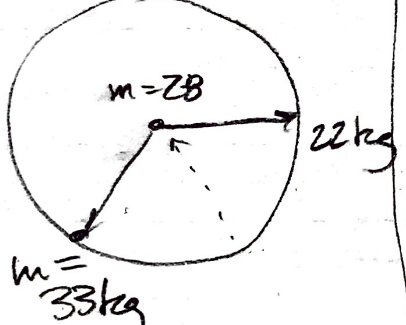
$\omega_i = 20.0 \frac{rev}{min} \times \frac{1\,min}{60} \times \frac{2\pi\,rad}{1\,rev} = 2.09\,rad/s$

Top View

initial



final



Student moving to center <u>decreases</u> the I of the system, causing it to speed up (due to Cons. of angular momentum). $\omega_f = ?$

$$\Sigma L_i = \Sigma L_f$$

$$L_{disk} + L_{22} + L_{28} + L_{33} =$$
$$I\omega + rmv + rmv + rmv$$

$v = r\omega!$

$$L_{disk}' + L_{22}' + L_{28}' + L_{33}'$$
$$I\omega' + rmv' + rmv' + rmv'$$

$$\left(\tfrac{1}{2}Mr^2\right)\omega + r^2m\omega + r^2m\omega + r^2m\omega =$$
$$\left(\tfrac{1}{2}Mr^2\right)\omega' + r'^2m\omega' + r'^2m\omega' + r'^2m\omega'$$

Combine terms, factor out ω

$$\left(\tfrac{1}{2}Mr^2 + mr^2 + mr^2 + mr^2\right)\omega =$$
$$\left(\tfrac{1}{2}(100)(1.6)^2 + 22\cdot1.6^2 + 28\cdot1.6^2 + 33\cdot1.6^2\right)(2.09\,rad/s)\ \omega_i$$
$$\left(\tfrac{1}{2}Mr^2 + mr'^2 + mr'^2 + mr'^2\right)\omega'$$
$$\left(\tfrac{1}{2}(100)(1.6)^2 + 22\cdot1.6^2 + \underline{28\cdot0^2} + 33\cdot1.6^2\right)\ \omega_f$$

$$\omega_f = \left(\frac{340\,kg\cdot m^2}{268.8}\right) 2.09\,rad/s$$

$$= \boxed{2.64\,rad/s}$$