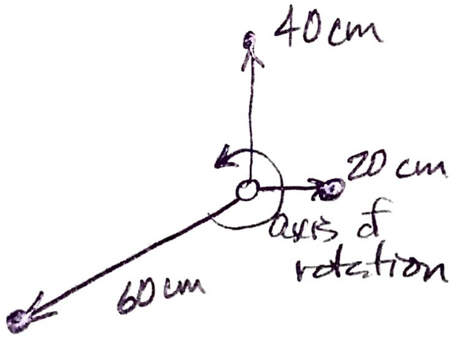
10.54



m = 0.3kg for each point mass.

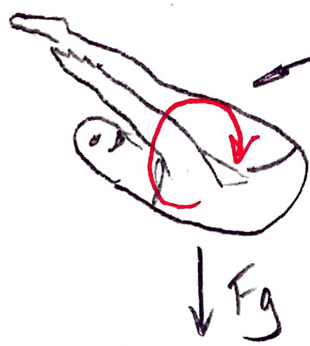
a) Find moment of inertia I for the system about the axis in the middle.

$$I = \sum r^2 m$$
$$= r_1^2 m_1 + r_2^2 m_2 + r_3^2 m_3$$
$$= (0.20\,m)^2(0.3) + (0.40)^2(0.3) + (0.60)^2(0.3)$$
$$= \boxed{0.1680\ kg \cdot m^2}$$

b) If system rotates at 5 rev/s, what is $K_{rotational}$?

$$K = \frac{1}{2} I \omega^2$$
$$\omega = \frac{5\ rev}{s} \times \frac{2\pi\ rad}{1\ rev} = 10\pi \frac{rad}{s}$$
$$K = \frac{1}{2}(0.1680\ kg \cdot m^2)\left(10\pi \frac{rad}{s}\right)^2$$
$$= 82.9\ kg \cdot m^2/s^2 = \boxed{82.9\ J}$$

10.58



This is supposed to be a diver :-(

$$K_{rotational} = 100\,J$$

&

$$K_{rotational} = \frac{1}{2} I \omega^2$$

&

$$I = 9.0\,kg\cdot m^2$$

so

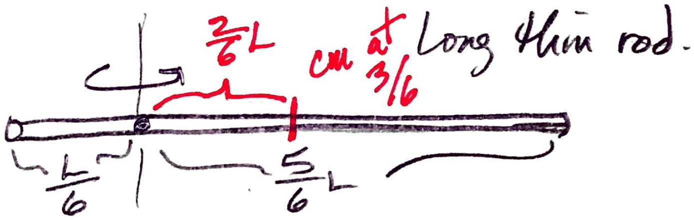
$$\omega = \sqrt{\frac{2K_{rot}}{I}}$$

$$= \sqrt{\frac{2\,100J}{9\,kg\cdot m^2}}$$

$$= \boxed{4.71\ rad/s} = \left(\times \frac{1\,rev}{2\pi}\right) = \boxed{0.75\ \frac{rev}{s}}$$

either one

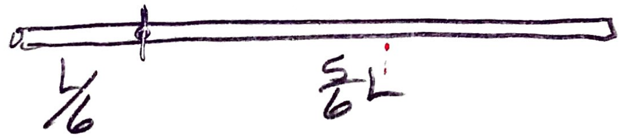
10.65



For a long, thin rod, we know $I_{cm} = \frac{1}{12}ML^2$. Here, the axis of rotation is $\frac{2}{6}L$ away from the center of mass, so

$$I_{\frac{L}{6}} = I_{cm} + MD^2$$
$$= \frac{1}{12}ML^2 + M\left(\frac{2}{6}L\right)^2$$
$$= \left(\frac{1}{12} + \frac{1}{9}\right) ML^2$$
$$= \boxed{0.194\, ML^2}$$

10.66



Calculate I at $\frac{L}{6}$ by direct integration?

$$I = \int r^2\, dm$$

$$= \int x^2\, dm, \quad \text{where } \lambda = \frac{M}{L}, \ \& \ \lambda = \frac{dm}{dl}$$

or $\lambda = \frac{dm}{dx}$

$dm = \lambda\, dx$

$$I = \int_{-L/6}^{5/6 L} x^2 \lambda\, dx$$

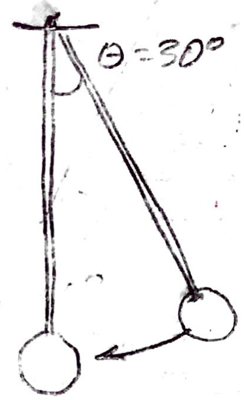
$$= \lambda \, \tfrac{1}{3} x^3 \Big|_{-L/6}^{5/6 L}$$

$$= \frac{M}{L}\frac{1}{3}\left[\left(\frac{5}{6}L\right)^3 - \left(-\frac{L}{6}\right)^3\right]$$

$$\frac{M}{L}\frac{1}{3}\left(\frac{125L^3}{216} + \frac{L^3}{216}\right)$$

$$= \boxed{0.194\, ML^2}$$

10.68



Pendulum = rod (2kg, 1m) + sphere (0.3kg, radius 0.20m)

If released from rest at 30° as shown, what is velocity (angular) at lowest point?

Strategy: Use Cons of Energy.

$$U_i = K_f$$

$$mgh = \frac{1}{2} I \omega^2$$

How high is rod-sphere assembly?

What is I for rod-sphere?

$$I = I_{rod} + I_{sphere}$$

$$= \frac{1}{3} ML^2 + ML^2$$

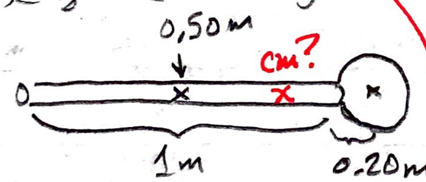
$$= \frac{1}{3}(2)(1)^2 + (0.3)(1+0.2m)^2$$

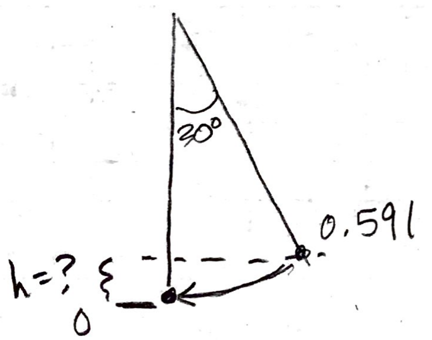
$$= \underline{1.10} \text{ kg}\cdot\text{m}^2 \checkmark$$

$$x = \frac{1}{M} \sum x_i m_i$$

$$x = \frac{1}{2.3}(0.5)(2kg) + (1.2)(0.3kg)$$

$$x_{cm} = \underline{0.591m}$$





$$h = L - L\cos\theta$$

$$= 0.591m - 0.591\cos 30$$

$$= \underline{0.0792m} \checkmark$$

$$(2+0.3kg)(9.8)(0.0792m) = \frac{1}{2}(1.10)(\omega^2)$$

$$\omega = \boxed{1.80 \text{ rad/s}}$$