Ch 3 - Vectors

Vectors

Vectors have two characteristics: magnitude, and direction; and we've talked about three kinds of vector quantities so far:

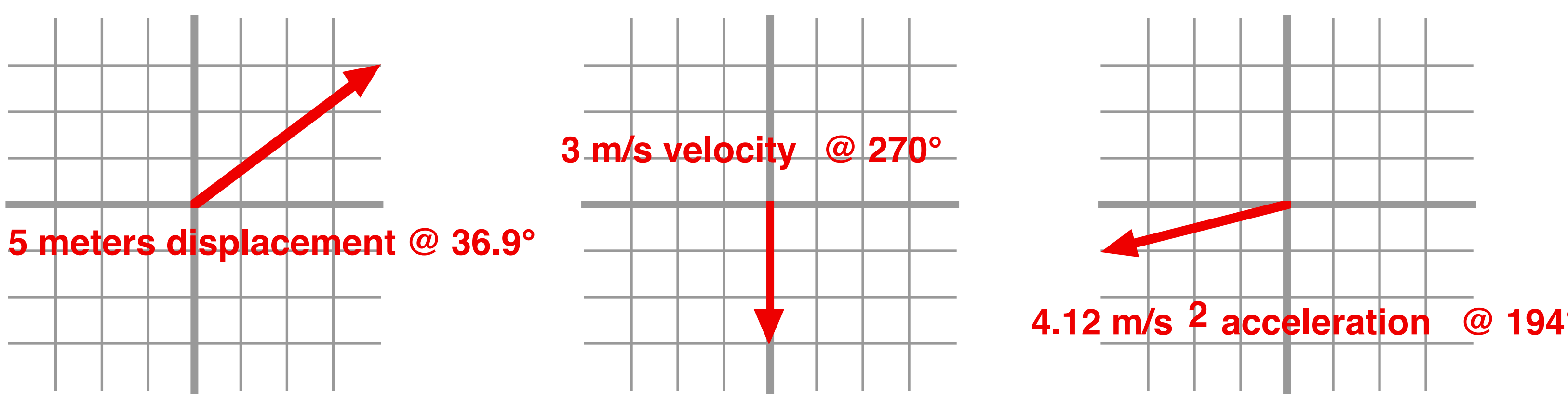
displacement, ***x***
velocity, ***v***
acceleration, ***a***

Up till now, we' ve considered very simple straight line motion, restricting ourselves to one-dimensional horizontal motion (in the **+*x*** or **-*x*** direction), or vertical motion (in the **+y** or **-y** direction). We need to expand our abilities now into additional dimensions...

Vectors – Graphical Analysis

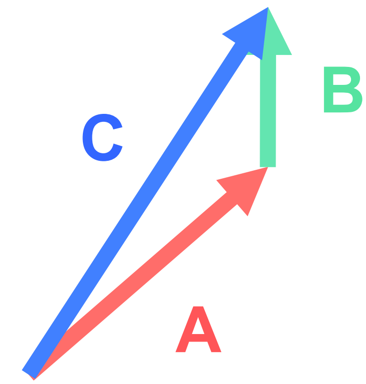
One way of analyzing 2-dimensional situations with vectors is by drawing the vector on paper, according to the following rules.

1. length of line drawn is proportional to *magnitude* of vector quantity
2. direction that arrow points (relative to an Cartesian coordinate system) indicates *direction* of vector quantity.



Vectors – Why?

It turns out that there are *lots* of things we can use vectors for: adding vectors, for example, allows us to determine the combined effects of two different motions.

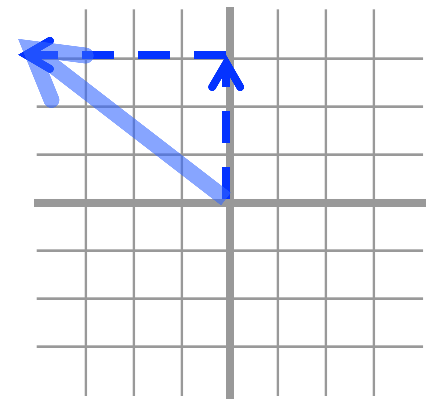


Adding two or more vectors *graphically* is accomplished by drawing the vectors such that the head (the arrow) of one vector is touching the tail of the next vector, in any order. The net, or *resultant* vector is found by drawing a line from the tail of the first vector to the head of the last vector.

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{C}}$$

Example 1 – Adding Vectors

If I walk 6 blocks north from my house, and 8 blocks west, where do I end up relative to where I started?



This technique is called the "tip-to-tail" technique, or the "triangle" technique, of vector addition.

In order to actually determine our final answer here, we'd need to use a ruler & protractor to measure the magnitude and direction of the final, *resultant*, displacement.

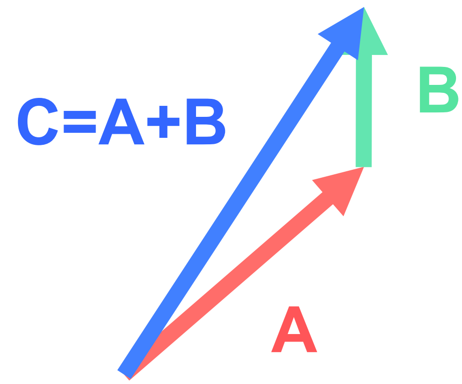
Measuring with a ruler & protractor, the displacement is:

10 blocks, 140°

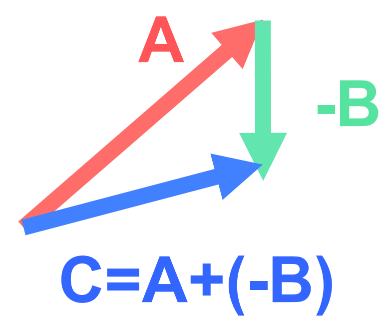
Vector Subtraction?

How would one go about graphically *subtracting* one vector from another?

$$\vec{C} = \vec{A} - \vec{B}$$

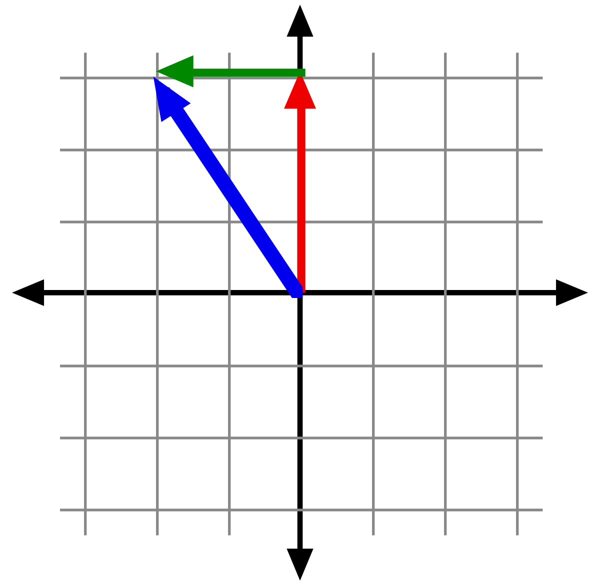


A negative vector has the same magnitude, but points in the opposite direction.



Example 2 – Subtracting Vectors

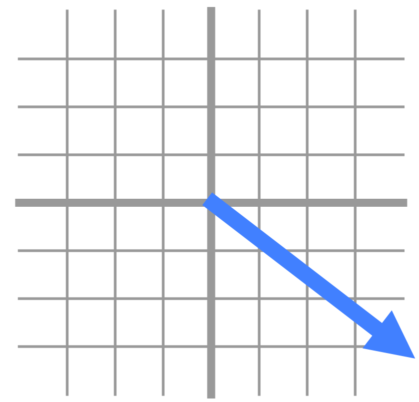
A = 3 m/s North
B = 2m/s East
What is A + -B (magnitude & direction)?



Measure the resultant vector to get your final answer, both magnitude (3-point-something m/s) and direction (~120° ?)

PI Problem

What direction is the vector pointing in this problem?



a. Southwest
b. Southeast
c. 36.9°
d. -36.9°
e. 323°
f. 53.1° East of South

Answer:

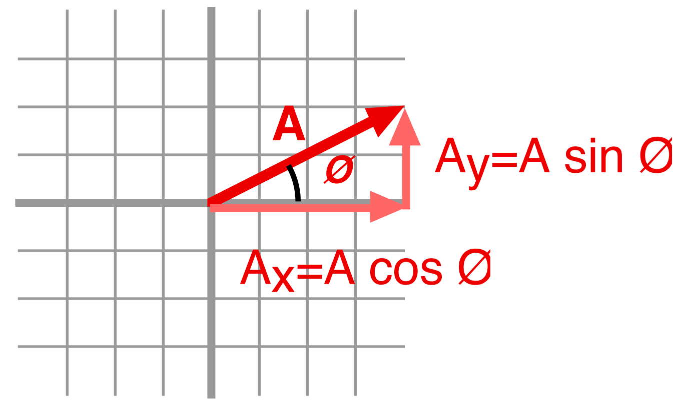
Answers *b*, *d*, e, and *f* are all correct to some extent.
The best answers are *d* and e.

Other ways of adding vectors

Obviously, graphical addition of vectors is cool for sketches, but measuring with a ruler and protractor isn't very precise when it comes to numeric solutions. For that, we have two slightly more complex, but incredibly useful, systems: the *polar-notation* system, and the *unit-vector* system.

Example 3 – Trig Review

The vector **A** shown here is pointing at some angle Ø measured relative to the *x*-axis.



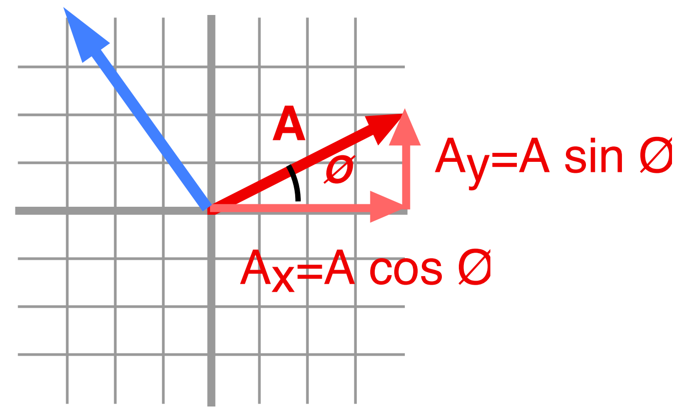
If we know A and Ø, how could we calculate A_x and A_y?

If we knew A_x and A_y, how could we calculate A's magnitude and direction?

$$A = \sqrt{A_x^{\,2} + A_y^{\,2}} \text{ and } \tan\phi = \frac{A_y}{A_x}$$

Polar Notation

This system of describing a vector **A** in terms of its magnitude A, and its polar angle Ø is called *polar notation.* Angles are typically given relatively to East, or positive-x, and can be notated as follows:



$$4.47 \text{ m/s} @ 26.6°$$

$$4.47 \text{ m/s} \angle 26.6°$$

The blue vector would be written as...

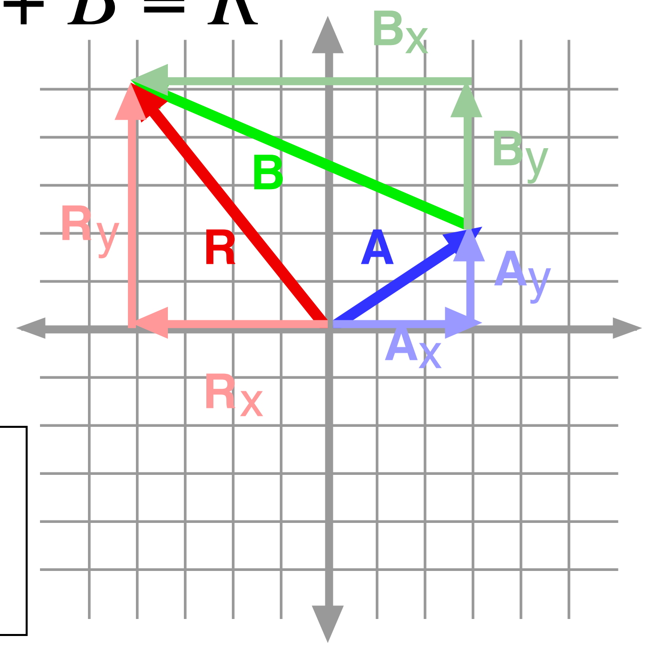
$$5 \text{ m/s} @ 126.9°$$

Adding Vectors in PolarNot

Watch the process here:

$$\vec{A} + \vec{B} = \vec{R}$$

The important thing to note here is the relationship between all of the ***x***-components of the vectors, and all of the ***y***-components of the vectors.



$$\text{x - direction: } A_x + B_x = R_x$$

$$\text{y - direction: } A_y + B_y = R_y$$

General Strategy:

1. Sketch & label vectors
2. Find ***x-*** and ***y-*** components of all vectors
3. Add ***x-***components together to get ***x-***component of Resultant. Do the same thing with the ***y-***components.
4. Use Pythagorean theorem to get Resultant's magnitude
5. Use trig to get Resultant's direction

Example 4 – Polar Notation

A tortoise crawls 10m SE, then 12m at 60° west of south.

1. Make a sketch of his journey.
2. Find components of each leg of trip
3. Calculate components of resultant
4. Calculate magnitude & direction of resultant

Solution: A_x=7.07m, A_y=-7.07m
B_x=-10.4m, B_y=-6.00m
$R_x=A_x+B_x$=-3.3m, $R_y=A_y+B_y$=-13.07m
Use Pythagoras to get R=13.5m
Use $\tan^{-1}$ to get 255°

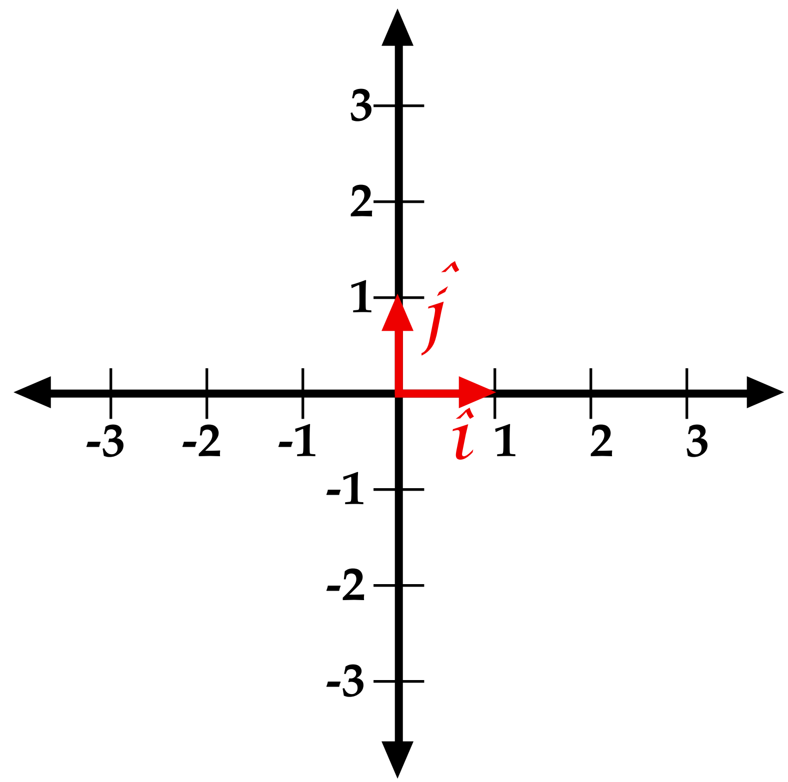
Adding 2-D Vectors

- Graphically
- Polar Notation
  - split vectors into components
  - add components separately
  - recombine component sums to get resultant magnitude and direction
- Unit Vector Notation

Unit Vectors

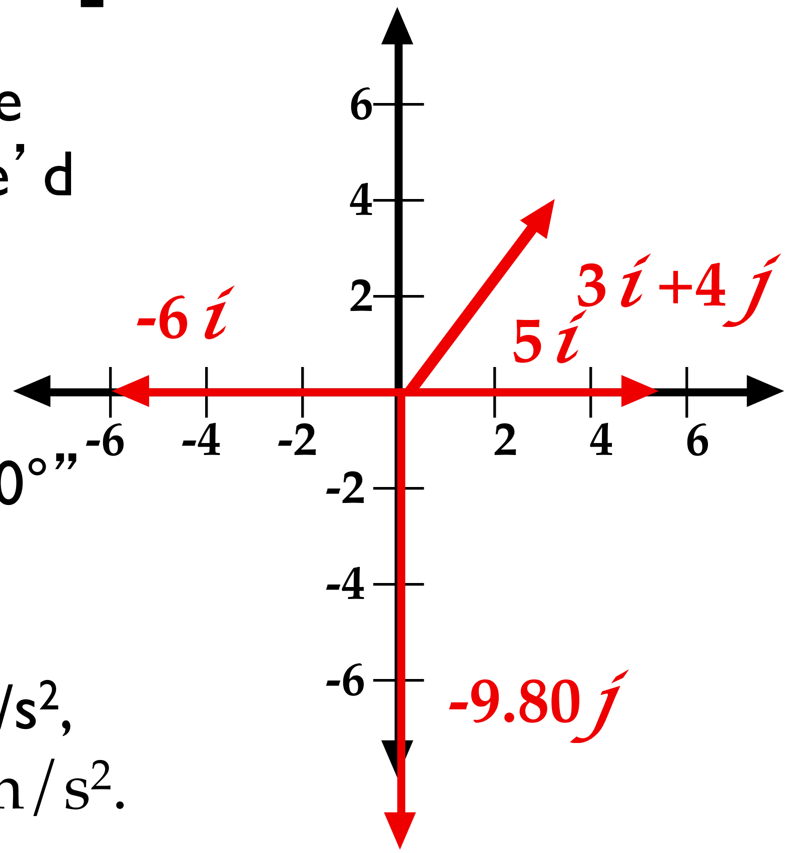
Yet another way exists to designate, and calculate with, vectors: *unit vectors*. A unit vector is simply a vector along the *x*- or *y*-axis that has a value of 1.

The unit vector in the *x*-direction is given the label ***i***, while the *y*-direction unit vector is labelled ***j*** (sometimes with a carat ^ over the top of them). These unit vectors have a unit that matches whatever unit we're talking about (meters, m/s, m/s^2, or whatever).



Unit Vector Examples

So, if we want to write the vector for the displacement "5 meters to the east," we' d write it: (5***i***) m.



The vector for the velocity "6 m/s at 180°" would be.... (-6***i***) m/s.

The vector for the acceleration "9.80 m/s²", down" would be.... (-9.80 ***j***) m/s².

And the vector for the displacement "5 m at 53.1°" is ....

(3***i***+4 ***j***) m.

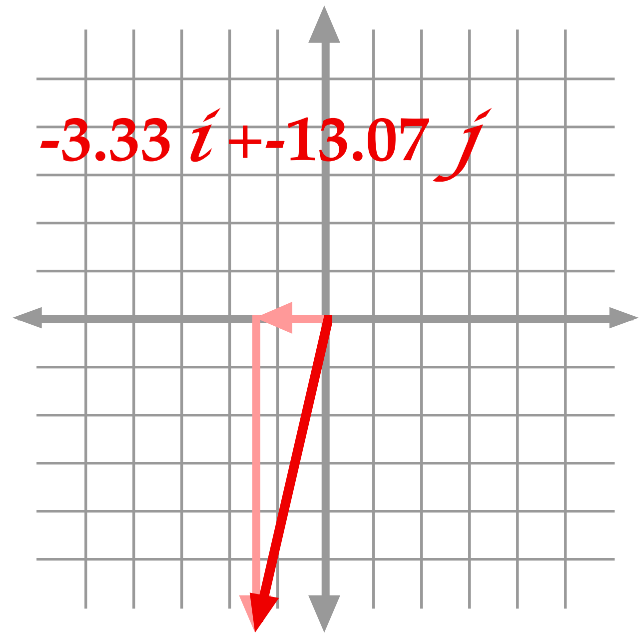
Why you love unit vectors

The **very cool** thing about unit vectors is that we' re essentially dealing with *components*: the *x*-component of the vector is given to you, and labeled with an ***i***, while the *y*-component is there, and labeled with a ***j***. So any vector addition doesn't require that the vector be split into components—it's already been *given* to you in components.

Example 5 – Another tortoise

A tortoise crawls(7.07***i***+ -7.07 ***j***)m,
then (-10.4 ***i***+ -6.00 ***j***)m.

1. Make a sketch of his journey.
2. Determine his displacement.



Solution:

(7.07***i*** + -7.07 ***j***)m
+ (-10.4 ***i*** +-6.00 ***j***)m

(-3.33***i*** -13.07***j***)m

If you're lucky, you can leave it in that form and be done with it. Sometimes, you' ll have to take those end vectors and convert them back into polar notation.

Converting from unit-vector to polar!

$$A^2 + B^2 = R^2$$

$$R = \sqrt{A^2 + B^2} = \sqrt{(-3.33)^2 + (-13.07)^2} = 13.50m$$

$$\tan^{-1}\left(\frac{13.07}{3.33}\right) = 75.7° \quad \ldots + 180° = 256°$$

Ch 4 Preview

Consider a projectile–a Nerf dart?–that is shot at some angle above the horizontal.

- How can we describe the dart's motion in the *horizontal* direction?

- How can we describe the dart's motion in the *vertical* direction?

- How can we combine our knowledge of these two situations to determine where the dart lands?